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Extensions of the model "Integrated laycan and berth allocation problem with ship stability and conveyor routing constraints in bulk ports"

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Abstract. This paper proposes significant extensions to the model presented in the paper "Integrated Laycan and Berth Allocation Problem with Ship Stability and Conveyor Routing Constraints in Bulk Ports" recently published in Computers & Industrial Engineering, mainly by considering multiple calendars of non-working periods that prevent the performance of certain tasks, complicating the scheduling of tasks and the respect of tidal constraints. A significant adaptation of this first model is proposed and tested in this paper.

Keywords. Laycan allocation, Berth allocation, Tidal bulk ports, Ship stability, Preventive maintenance, Multiple non-working calendars, Integer programming

1 Introduction

This paper proposes significant extensions to the model presented in the paper "Integrated Laycan and Berth Allocation Problem with Ship Stability and Conveyor Routing Constraints in Bulk Ports" by H. Bouzecri, G. Alpan and V. Giard, recently published in Computers & Industrial Engineering. To facilitate the presentation of the proposed extensions, this model will be referred to by the acronym BAG, made up of the initials of the authors' names. In this article, it seemed unnecessary to repeat and continue the literature review proposed in the previous CAIE article published a few months ago, since we did not find the extensions, we proposed in the existing literature. The need for these extensions became apparent during tests on the application of the BAG model to the management of the bulk port of Jorf, which mainly exports phosphate fertilizers; the failure of the BAG model to consider certain constraints prevented its relevant application.

Since we are presenting an extension of an existing model, this article will focus on adaptations to address needs that were not covered initially. It repeats the original formulation of the BGA model (notations and relations) to highlight our contribution better. To properly delimit our contribution, we need to start by listing the features considered in the BAG model.

1. 1. Heterogeneous characteristics of berths located on several quays: length, depth at low tide, loading rate of bulk handling cranes and the possibility of connecting some adjacent berths.
2. Heterogeneous characteristics of vessels: length, draught of the loaded vessel; inability of certain vessels to berth in certain positions (in addition to the constraints of length and depth) for meteorological reasons (high swell), for example.
3. Multiple types of cargo to load onto a vessel.
4. Constraints of boat stability to be considered during bulk loading.
5. Conveyor capacity constraints between warehouses and berths.
6. Maintenance operations, both on berths and on conveyors, to be carried out on fixed dates or within predefined periods, which implies that the model includes the definition of maintenance dates intending to optimize the overall use of the port.
7. Limitation of the number of vessels crossing the channel at the same time to enter or leave the port.
8. Restriction for some loaded boats to pass through the channel only at high tide.
9. Charter party clauses.
10. 10. Taking into account the availability dates of the batches to be loaded (if some are not available on the expected arrival date of the vessel).
11. Non-working periods defined at the vessel level.
12. 12. Modular time frames for decision-making without impacting the time frame used to describe the consequences of those decisions, which remain assessed at the finest level of detail.

The following items are extensions that must be considered in the BAG model before it can be used effectively.

A. Considering a docking time that may vary from ship to ship. During this docking time, it is not possible to load a batch. This docking time could be taken into account by considering the boat's characteristics and the berth, but this refinement is not of genuine interest.

B. Possible modulation of the loading speed of a vessel, depending in particular on the goods to be loaded (for example, the loading of ores is slower than the loading of fertilizers in the port of Jorf).

C. Possibility of using several non-working calendars and any combination of these calendars, defined in absolute terms, independently of the works to be carried out in the port.

D. Possible use of different non-working calendars for batches to be loaded onto a vessel instead of requiring the same non-working calendar for all batches on that vessel. This extension considers the prohibition of night loading for certain cargoes, such as fertilizers, due to safety concerns. Additionally, specific schedules can be used to maintain conveyor or berth equipment if certain operations, such as at night, cannot be performed at certain times.

E. Possibility of defining the location of a fertilizer at the level of a group of hangars, rather than at the level of a single hangar. This possibility is significant because, for vessels arriving towards the end of the selected horizon, it is often the case that the exact location of the batches is not known because the production has not yet taken place, but it has already been decided to produce these fertilizers on one of the production lines supplying a group of hangars.

F. The objective functions have been enriched with variants adapted to the needs of managers who prefer effectiveness to efficiency. In the part of the port of Jorf where the tests were carried out, short-term management concerns led the managers to give priority to an efficiency criterion since the financial impact of delays or early arrivals was more or less the same for all vessels. This is not the case in the part of the port dedicated to sulphur and acids.

Finally, this paper corrects important errors in the BAG model related (only) to the laycans for new vessels to be chartered, including the definition of the vessel departure date, the definition of the batch dates, and the constraints on conveyor occupancy.

These additions and corrections to the BAG model lead to a significant modification of the table of indices, parameters and variables, an adaptation of the table of predicates and a transformation of the model relations linked to the creation of these parameters and variables. The extent of these transformations makes it impossible to maintain a presentation based on pointing out the modifications in the first article since this approach (which has been tested) makes it very difficult to understand the final model. Therefore, in section 2, it was decided to rewrite the model presentation and use highlighting to emphasize the proposed transformations of the BAG model to extend its capabilities. Section 3 is devoted to a brief case study presentation, supplemented by other case studies in a loadable Excel file.

2 New Model formulation

This transformation of the model involves adjusting some of its parameters and creating new ones (§2.1) to describe the new model formulation (§2.2).

2.1 Creating and modifying certain BAG model parameters

2.1.1 Inclusion of a docking time before loading (*additional feature A*)

The model now includes a docking time α_v, inserted between the berthing date ε_V (intermediate variable deduced from the binary variable x_{vpt}) and the start of loading, which now begins at the date $\varepsilon_V + \alpha_V$.

Taking this berthing time into account means that the ship's status as defined in the BAG model needs to be more precisely defined, which leads to the creation of the parameter σ_v for vessel *v*. Status $\sigma_v = 1$ is given to ships which have already docked (involving $\alpha_v = 0$) or whose docking date, which is close, is considered to be irrevocable (involving $\alpha_v > 0$). Status $\sigma_v = 2$ is given to ships expected to dock with a signed contract. Status $\sigma_v = 3$ is for vessels whose contract is under negotiation.

It should be noted that the berth position p^* of a vessel v^* of status $\sigma_{v^*} = 1$ is necessarily known and unique ($G_{v*p*} = 1$ and $G_{v*p'} = 0, \forall p' \neq p*$) and that its maximum waiting time before the start of operations I_{v^*} is necessarily zero $(I_{v^*} = 0, \forall v^* | \sigma_{v^*} = 1)$.

2.1.2 Modification of the parameters used to calculate the loading time of a vessel (*additional feature B*)

In practice, the nominal loading rate of a ship is rarely reached and is reduced according to the type of cargo to be loaded (e.g. this rate is lower when loading ores than fertilizers) or for other reasons (bad weather, etc.). The model now includes the ability to modulate a berth's nominal loading rate (ρ_p) by a yield coefficient defined at the ship level τ_v, independent of the berth's loading rate (ρ_p).The loading rate for vessel *v* docked at the berth position *p* then becomes $\rho_p \cdot \tau_v$, and the loading time $\theta_{vp}^{b_v}$ for the batch b_v whose weight is $\varphi_v^{b_v}$, then increases from $\varphi_v^{b_v} / \rho_p$ to $\varphi_v^{b_v} / (\rho_p \cdot \tau_v)$. The formula for the ship's loading time Θ_{vp} $(\Theta_{vp} = \sum_{b_v \in \mathscr{B}_v} \theta_{vp}^{b_v}, \forall v \in \mathscr{V}, \forall p \in \mathscr{P})$ does not change, but it uses a different relation to calculate the loading time $\theta_{vp}^{b_v}$ of a batch b_v which becomes $(\theta_v^{b_v} = \varphi_v^{b_v} / (\rho_p \cdot \tau_v), \forall v \in \mathscr{V}, \forall p \in \mathscr{P}, \forall b_v \in \mathscr{B}_v)$.

2.1.3 Multiplication of usable working time calendars (*additional feature C*)

There is a need to generalize and rationalize the creation of these calendars of non-working hours[1] to be used for some tasks for three reasons: *i*) a calendar may correspond to a basic prohibition (e.g. no night or weekend work, etc.) or to a combination of basic prohibitions (e.g. no night and weekend work); *ii*) it must be possible to use a calendar at the level of batches to be loaded onto a vessel, in which loading constraints may vary according to the type of goods to be loaded (for example, the prohibition on night loading of potassium fertilizers); *iii*) maintenance and availability of some shared equipment are also subject to working restrictions. It is, therefore, necessary to create a set of independent reference systems to take into account all the time constraints to be respected by the vessels, the batches to be loaded and the maintenance operations. This leads to creating the index $c = 1..\mathrm{C}$ to set the calendars for taking the ban on working at certain times, as shown in Table 1 below. The concepts used to define these time frames need to be well-defined.

[1] The approach adopted to define these timing restrictions was initially proposed in the article by Azzamouri et al. (2020) and then taken up and completed in the articles by Bouzeki et al (2022 and 2023)

By convention, the ***general calendar*** refers to all existing periods without restriction. The ***baseline calendar*** is the first $(c=1)$ calendar that contains the general non-working periods for each operation considered in the model. This calendar is implicitly used to define the laydays of vessel v (L_v) and its maximum waiting time in the port (I_v).

This baseline calendar is supplemented by a set of elementary calendars (γ_{ct}) that describe, through a Boolean vector, the non-working periods from a specific point of view (e.g. non-working on weekends or at night). All these elementary calendars can be combined to allow the use of the appropriate calendar (ψ_{ct}) for a specific type of operation. Two time-reference frames are created from this vector: the first (Γ_{ct}), which we will call the ***relative calendar***, provides the worked period number from the specific retained point of view of calendar *c*, while the second (Υ_{vt}), which we will call ***reverse relative calendar***, provides the period number of the general calendar corresponding to a period number of the relative calendar. They are illustrated in Table 1, where two elementary prohibition cases are proposed (night and weekend) and combined to give four calendars ψ_{ct}, Γ_{ct} and Υ_{ct}, and whose calculation formulas are the following ones.

$$\begin{cases} \Gamma_{ct} = t - \sum_{t'=1}^{t'=t} \psi_{ct'}, \forall t \in \mathscr{T}, \forall c \in \mathscr{C} \\ \Upsilon_{ct} = \mathrm{Max}(t' \mid \Gamma_{ct'} = t), \forall t \in \mathscr{T}, \forall c \in \mathscr{C} \end{cases}$$

Table 1. *Elementary non-working periods* γ_{ct}*, combined non-working periods* ψ_{ct}*, relative calendar* Γ_{ct} *and inverse relative calendar* Υ_{ct}*.*

	Day	1								2								3								4								5							
	Span #	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
	t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
	Working prohibition cases (γ_{ct} = 1 if working prohibited during period t)																																								
Prohibition case	*a* (general)	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	b (night)	0	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1
	c (WE)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
	Ψ_{ct} (=1 if working prohibited during period t)																																								
Calendar index *c*	*c* =1 (*a*)	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	c =2 (*a&b*)	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1
	c =3 (*a&c*)	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
	c =4 (*a&b&c*)	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1
	Γ_{ct} (relative calendar)																																								
Calendar index *c*	*c* =1 (*a*)	1	2	3	4	5	6	7	8	9	9	9	9	9	9	9	9	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
	c =2 (*a&b*)	1	2	3	4	5	6	7	7	7	7	7	7	7	7	7	7	7	7	7	8	9	10	11	11	11	11	11	12	13	14	15	15	15	15	15	16	17	18	19	19
	c =3 (*a&c*)	1	2	3	4	5	6	7	8	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	10	11	12	13	14	15	16
	c =4 (*a&b&c*)	1	2	3	4	5	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8	9	10	11	11
	Υ_{ct} (inverse relative calendar)																																								
Calendar index *c*	*c* =1 (*a*)	1	2	3	4	5	6	7	8	17	17	17	17	17	17	17	17	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
	c =2 (*a&b*)	1	2	3	4	5	6	19	19	19	19	19	19	19	19	19	19	19	19	19	20	21	22	27	27	27	27	27	28	29	30	35	35	35	35	35	36	37	38	40	40
	c =3 (*a&c*)	1	2	3	4	5	6	7	8	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	34	35	36	37	38	39	40
	c =4 (*a&b&c*)	1	2	3	4	5	6	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	36	37	38	40	40

This change in calendar attachment causes the parameters ψ_{vt} , Γ_{vt} and Υ_{vt} to be replaced by ψ_{ct}, Γ_{ct} and Υ_{ct} to no longer refer to vessels in time references. The parameter γ_v no longer corresponds to a Boolean attached to vessel *v* but to the index of the default restriction calendar to be used for vessel *v*. It is also necessary to introduce the parameter $\gamma_v^{b_v}$ corresponding to the calendar index used in the handling operation for the batch subject to specific loading prohibition restrictions compared to those of vessel *v*. For simplicity, it is assumed here that all the batches to be loaded into a vessel and having a specific calendar, share the same specific calendar (removing this restriction is easy to consider but of little operational interest).

2.1.4 Consideration of specific time constraints for certain batches to be loaded and for maintenance operations (*additional feature D*)

Technical considerations (safety, etc.) may mean that certain products can only be loaded onto a vessel at certain times. For example, potassium-based fertilizer can only be loaded during the day at the port of Jorf, but this does not prevent potassium-free fertilizer from being loaded at night onto a vessel that needs to load both types of fertilizer. This means that two calendars of non-working hours have to be used to load this ship: the specific calendar $\gamma_v^{b_v}$,for some batches b_v and the calendar γ_v to be used for other batches. Note that all calendars $\gamma_v^{b_v}$ are necessarily more restrictive than the calendar γ_v (for example, in Table 1, the calendar $c=2$ is more restrictive than the calendar $c=1$). If not all batches are subject to the same loading period restrictions, the index of the most restrictive calendar for vessel *v* is noted $\overset{*}{\gamma}_v$; otherwise, one can set $\overset{*}{\gamma}_v = \gamma_v$, allowing $\overset{*}{\gamma}_v$ to be used without loss of generality.

The consequence of this generalization of loading time constraints is that the loading end date π_v of vessel *v* can only be derived from its berthing date ε_v and berthing location *p*, if all the batches to be loaded share the same calendar. Their calculation formula, which must involve a modification of the predicates, will be presented later. This implies the introduction of the binary parameter d_v which is worth 0 if all the batches to be loaded onto a ship share the same calendar $(\gamma_v^{b_v} = \gamma_v, \forall b_v \in \mathscr{R}_v \Leftrightarrow \mathrm{d}_v = 0)$.

Restrictions on working hours for maintenance operations can be easily taken into account with the new design of working time calendars, which no longer refer to vessels. This extension will be introduced later.

2.1.5 Introduction of new objective functions (*additional feature E*)

The problem of managing the port of Jorf corresponds to a simplified version of the extended model presented here, which is limited to the operational problem and, therefore, does not consider the contractual negotiation of new vessels likely to be received (status $\sigma_v = 3$). The evaluation adopted is no longer based on an economic point of view (efficiency criterion) but on the minimum occupation of the port for the same set of vessels handled (effectiveness criterion).

Several variants of the efficiency criterion can be adopted, bearing in mind that loading times can vary from one berth to berth. The criterion of minimizing the docking and loading time is possible but not very relevant if loading cannot be carried out during certain periods (nights, weekends, etc.). For this reason, it is preferable to minimize the dwell time between berthing ε_v and finishing loading π_v times, corresponding to the vessel's departure, which leads to minimize $\sum_{v \in \mathscr{V}} (\pi_v - \varepsilon_v)$. In most cases, several solutions minimize this global dwell time in the port. From this set of solutions, it is preferable to choose the one that leads to the earliest use of the port in order to maintain the maximum number of degrees of freedom to accommodate new vessels more easily. The indicator $\sum_{v \in \mathscr{V}} \pi_v$ responds well to this concern but it must be included in the objective function with a sufficiently low weighting k $(0 < k < 1)$ so that the criterion of total time in port remains predominant. Then the optimization criterion becomes $\sum_{v \in \mathscr{V}} (\pi_v - \varepsilon_v) + k \cdot \sum_{v \in \mathscr{V}} \pi_v$, (an expression that has not been simplified for ease of interpretation). It is clear that the value of the coefficient k is set arbitrarily in order to allow the indicator of total port occupancy time to dominate.

2.2 New Model formulation

2.2.1 Updated table of notations

Table 2. *New table of notations*

Index	*Description*

t	Index of time periods $\mathcal{T} = \{1,...,\mathrm{T}\}$.
v	Index of vessels $\mathcal{V} = \{1,...,\mathrm{V}\}$
i	Index status of a vessel: $i = 1 \rightarrow$ vessel already berthed or about to be berthed at a known date; $i = 2 \rightarrow$ chartered vessel; $i = 3 \rightarrow$ new vessels to charter
b_v	Index of batches to load on vessel v $\mathcal{B}_v = \{1,...,\mathrm{B}_v\}$.
p	Index of berthing positions $\mathcal{P} = \{1,...,\mathrm{P}\}$.
m_p	Index of maintenance activities to be performed at berthing position p $\mathcal{M}_p = \{1,...,\mathrm{M}_p\}$.
s	Index of sections composed of identical parallel conveyors $\mathcal{S} = \{1,...,\mathrm{S}\}$.
m_s	Index of maintenance activities to be performed at a conveyor in section s $\mathcal{M}_s = \{1,...,\mathrm{M}_s\}$.
h	Index of storage hangars $\mathcal{H} = \{1,...,\mathrm{H}\}$, where $\mathrm{H}=\mathrm{H}_0+\mathrm{H}_1$, with H_0 groups of hangars and H_1 hangars ($\rightarrow h_0 = 1..\mathrm{H}_0$ and $h_1 = \mathrm{H}_0 + 1..\mathrm{H}$).
c	Index of calendars linked to a set of non-working periods $c = 1..\mathrm{C}$. By convention, the ***baseline calendar*** is the calendar $c = 1$ that incorporates the universally applicable non-working periods.

Parameter	*Description*
<u>*Navigation channel*</u>	
M	Maximum number of vessels allowed to pass simultaneously through the navigation channel.
<u>*Time decision restriction*</u> (See Section 4.3)	
K_t	Boolean parameter that equals 1 if a decision of berthing vessels can be taken during time period *t*, 0 otherwise.
<u>*Tide cycle*</u>	
O_t	Boolean parameter that equals 1 if time period *t* is within a high tide cycle, 0 otherwise.
<u>*Time framework*</u> (see Section 4.2)	
Ψ_{ct}	Boolean parameter that equals 1 if the handling cannot be performed during the period *t* of the calendar *c*, 0 otherwise.

Γ_{ct} Relative period of the absolute period *t* of vessel *v* considering non-working periods of calendar *c* (***relative calendar***).

Υ_{ct} Absolute period of the relative period *t* of vessel *v* considering non-working periods of calendar *c* (***inverse relative calendar***).

Berthing positions

Q_p Length of berthing position *p*.

W_p Minimum water depth of berthing position *p*.

ρ_p Productivity of berthing position *p*, which equals the tonnage that can be loaded in a vessel per hour.

$E_p^{p'}$ Boolean parameter that equals 1 if berthing positions *p* and p' share a berthing position, 0 otherwise (e.g., in Figure 2 of the BAG article, "Berthing position 3" and "Berthing position 5" share "Berthing position 3"). When $p = p'$, $E_p^{p'} = 1$.

Sections

U_s Number of identical parallel conveyors in section *s*.

F_{sh} Boolean parameter that equals 1 if one of the conveyors belonging to the route that links a berthing position to storage hangar *h* belongs to section *s*, 0 otherwise.

Preventive maintenance activities

$R_p^{m_p}$ Duration of maintenance m_p to be performed at berthing position *p*.

$\underline{R}_p^{m_p}$ Earliest time to perform maintenance m_p at berthing position *p*.

$\overline{R}_p^{m_p}$ Latest time to perform maintenance m_p at berthing position *p*.

$R_s^{m_s}$ Duration of maintenance m_s to be performed at a conveyor in section *s*.

$\underline{R}_s^{m_s}$ Earliest time to perform maintenance m_s at a conveyor in section *s*.

$\overline{R}_s^{m_s}$ Latest time to perform maintenance m_s at a conveyor in section *s*.

Vessels

σ_v Index of the status of vessel *v* (1for berthed vessel or about to berth vessel; 2 for chartered vessel, and 3 for vessels to charter).

A_v Expected arrival time of chartered vessel *v* $(\sigma_v = 2)$ **;** earliest time a new vessel *v* to charter $(\sigma_v = 3)$ can arrive at the port, the cargo to load being

	available; $A_v = 1, \forall v \mid \sigma_v = 1$.
I_v	Maximum waiting time in the port of vessel *v*; $I_v = 0, \forall v \mid \sigma_v = 1$.
λ_v	Length of vessel *v*.
D_v	Draft of vessel *v* when it is fully loaded.
ω_v	Boolean parameter that equals 1 if vessel *v* is tide-dependent, 0 otherwise.
γ_v	Calendar index used in the handling operation of vessel *v* when all batches are subject to the same loading period restrictions.
d_v	Boolean parameter equals 1 if all the batches to load into the vessel *v* share the same calendar, otherwise, 0.
γ_v^*	Index of the most constraining calendar used by batches to load in vessel *v* if $d_v = 0$; otherwise $\gamma_v^* = \gamma_v$ allowing a general use of γ_v^*
α_v	Docking time of vessel *v* for vessels not already berthed (i.e. the chartered vessels $(\sigma_v = 2)$, the vessels to chart $(\sigma_v = 3)$, and some vessels whose berthing dates are known and are about to berth $(\sigma_v = 1 \wedge A_v > 1)$. As a result, loading operations for a vessel berthing in period *t* can start in period $t + \alpha_v$.
L_v	Laydays amplitude (laycan days) of a new vessel *v* to chart $(\sigma_v = 3)$; to enable the formulation of a general model, $L_v = 1, \forall v \mid \sigma_v < 3$; laydays refer to the baseline calendar.
J_v	Laytime (contractual handling time) of vessel *v*; in the tests performed, this parameter is arbitrarily calculated as the sum of the docking time plus the product of the tonnage to be loaded and the average loading rate.).
δ_v	Contractual departure time of berthed or chartered vessel *v* $\delta_v = \Upsilon_{\gamma_v^*, \{(\Gamma_{\gamma_v^*, A_v}) + J_v. + \alpha_v + 1)\}}$; for the new vessel to be charted $(\sigma_v = 3)$, this parameter is used to privilege the earliest possible first layday.
η_v	Contractual demurrage by hour of vessel *v*; in the tests performed for the new vessel to chart $(\sigma_v = 3)$, this parameter is arbitrarily set to a value favouring an early first layday without significantly affecting the efficiency criterion.
β_v	Contractual despatch by hour of vessel *v*; the remark made for η_v also

	applies to β_v.
G_{vp}	equals 1 if vessel *v* can berth at berthing position *p*, 0 otherwise.
Θ_{vp}	Loading time of vessel *v* when the latter is berthed at berthing position *p*, which equals the sum of loading times $(\theta_{vp}^{b_v})$ of all the batches to load on this vessel without downtime: $\Theta_{vp} = \sum_{b_v \in \mathscr{B}_v} \theta_{vp}^{b_v}, \forall v \in \mathscr{V}, \forall p \in \mathscr{P}$
τ_v	Coefficient of loading efficiency for vessel *v*.
<u>*Batches*</u>	
$H_v^{b_v}$	Hangar where batch b_v to load on vessel *v* is stored.
$C_v^{b_v}$	Date of availability of batch b_v to load on vessel *v*.
$N_v^{b_v}$	Level of the batch b_v used in the loading sequence of batches in vessel *v* to maintain ship stability (see illustration in Figure 1 in the BAG article). If stability constraints are not to be considered for the vessel *v*, $N_v^{b_v} = 0, \forall b_v \in \mathscr{B}_v$.
$\varphi_v^{b_v}$	Weight of batch b_v.
$\theta_{vp}^{b_v}$	Loading time of batch b_v on vessel *v* when the latter is berthed at berthing position *p*: $\theta_{vp}^{b_v} = \varphi_v^{b_v} / (\rho_p \cdot \tau_v), \forall v \in \mathscr{V}, \forall p \in \mathscr{P}, \forall b_v \in \mathscr{B}_v$.
$\gamma_v^{b_v}$	Index of the calendar used in the handling operation of the batch b_v. If all the batches to be loaded onto the vessel *v* use the same calendar $d_v = 0 \Leftrightarrow \gamma_v^{b_v} = \gamma_v, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v$, otherwise $d_v = 1$.

Decision variable	*Description*
x_{vpt}	1 if vessel *v* starts berthing at berthing position *p* in period *t*, 0 otherwise.
$y_{vpth}^{b_v}$	1 if the batch b_v stored in hangar *h* starts loading on vessel *v* at berthing position *p* in period *t*, 0 otherwise.
$z_{pt}^{m_p}$	1 if maintenance m_p starts performing at berthing position *p* in period *t*, 0 otherwise.
$z_{st}^{m_s}$	1 if maintenance m_s starts performing at a conveyor in section *s* in period *t*, 0 otherwise.
π_v	Finishing loading time of vessel *v*.

u_v	Delay of vessel *v*, which is the number of time periods exceeding its laytime, $u_v \in \mathbb{Z}^+$ (since the planning horizon is divided into equal-sized time periods).
w_v	Advance of vessel *v*, which is the number of periods saved in its laytime, $w_v \in \mathbb{Z}^+$.
Intermediary variable	*Description*
μ_v, $\tau_v^{b_v}$	Berthing position of vessel *v* in the decision variables x_{vpt} and $y_{vpth}^{b_v}$ respectively.
ε_v	Berthing time of vessel *v*.
π_v^*	Latest finishing loading date of vessel *v*
$\rho_v^{b_v}$	Loading start time of batch b_v.
$\sigma_v^{b_v}$	Loading finishing time of batch b_v.
r_{vt}	Boolean = 1 if $\pi_v = t, \forall v \in \mathscr{V} \mid d_v = 0, \forall t \in \mathscr{T}$.
ϕ_{svt}	Number of conveyors of section *s* required by batch loading into a vessel *v* of statut $(\sigma_v = 3)$ during period *t*, in case of overlapping

2.2.2 Decision time-interval (*unchanged*)

To reduce the computational complexity and consider the increasing uncertainty of inputs as the length of the planning horizon increases, we follow the rolling horizon approach proposed by Bouzekri et al. (2021), which modulates decision time interval through the planning horizon. Two considerations justify the modulation of the decision interval.

- First, the further we go away in time, the less precise are some of the available information, mainly that relating to vessel arrivals. As a result, the further in the future a vessel arrival date is, a less precise berthing date is justified in the context of a rolling horizon approach. In addition, whatever decision interval is chosen, the model calculates all the decision consequences at the hourly time unit.
- Second, most often, the decision hierarchy leads to the separation of tactical decisions from operational decisions. In the port context, this separation leads to solving the operational problem first, which defines the problem's constraints to be dealt with over the following periods, particularly in negotiations with new clients.

This sequential resolution does not allow the optimization of the use of port resources. The solution of a decision interval increasing in time avoids this pitfall and allows digital processing in times compatible with the use of a rolling horizon approach.

So, we define a Boolean parameter K_t, that equals 1 if vessels can berth during time period *t* (without considering other constraints). Thanks to this parameter, we can restrict berthing decision periods inside the planning horizon and hence change the decision time interval. The user of the model is free to define the values of K_t. For example:

- during the first week, chartered vessels can berth every hour " 1 ", hence $\mathrm{K}_t = 1, \forall t$;
- during the second week, every two hours " 0 – 1 ", hence $\mathrm{K}_t = 1, \forall t \mid (t)_{\mathrm{mod}\,2} = 0$ and $\mathrm{K}_t = 0$ otherwise;
- during the following weeks, every three hours as illustrated in the Table 1 below " 0 – 0 – 1 ", hence $\mathrm{K}_t = 1, \forall t \mid (t)_{\mathrm{mod}\,3} = 0$ and $\mathrm{K}_t = 0$ otherwise.

New vessels to charter can be planned during the second and third weeks, providing them with an estimated position in the schedule. Then, as we advance in the planning horizon, the schedule is refined: some chartered vessels $(\sigma_v = 2)$ will become berthed vessels $(\sigma_v = 1)$, and some new vessels to chart $(\sigma_v = 3)$ will become chartered vessels $(\sigma_v = 2)$, and hence their laydays will be replaced by an expected arrival time.

Modulating time intervals in this manner helps integrate short-term decisions (BAP) and medium-term decisions (LAP) in a single model. As the time approaches to the present, the decisions are taken in a finer granularity (every hour), while for decisions that concern the planning a few weeks from now, a rough decision is taken (every 8 hours). Besides facilitating the integration of the LAP and the BAP, this approach also helps control the number of variables (i.e., the number of variables is lower for medium-term decisions).

2.2.3 Predicates (*unchanged*)

A mathematical program is made of a set of variables and a set of constraints made of a linear or non-linear combination of these variables, one of them being an objective function to optimize. The validity domain of variables may be narrowed by using an Algebraic Modeling Language (AML), available in some software like Xpress (Fourer, 2013), which rests on separating a generic description of the model in the solver and the data to use. AML allows the usage of predicates to drive the creation of an instance of the problem. A predicate is a logical statement that returns either a value of "True" or "False", based on the parameter values used in the statement, which in turn binds the existence of a variable, depending on the values of parameters. Predicates can be used to restrain:

- the number of expanded constraints in relation using a universal quantifier;
- the validity domain of some variables without using a constraint, decreasing the number of constraints in a model; for example, the use of the predicate $\mathrm{D}_v \leq \mathrm{W}_p$ (which enforces the draft of vessel *v* to not exceed the water depth of berthing position *p*) in the definition of the validity domain of the variable x_{vpt} avoids creating the constraint $\mathrm{D}_v \cdot x_{vpt} \leq \mathrm{W}_p, \forall v, p, t$. Using this type of predicate has the advantage of avoiding the introduction of additional constraints when modeling a complex problem. It also avoids unnecessary calculations in the optimization search, as the predicate used in the problem guarantees respect for that (unintroduced) constraint.

The extensive use of predicates in the proposed model acts like a pre-treatment based on the problem data, reducing the number of binary variables and constraints. Consequently, problems of practical sizes can be solved in a reasonable time using off-the-shelf commercial Software based on AML.

We will use the following logical statements to describe the validity domain of decision variables. In our model, a decision variable exists only when the associated set of predicates returns "True".

2.2.4 Interpretation of the two mean decision variables

Some clarification is necessary to understand the meaning of the two main decision variables, x_{vpt} and $y_{vpth}^{b_v}$.

- The binary decision variable x_{vpt} determines the berthing time *t* and the berthing position *p* of vessels with status $\sigma_v \geq 2$. For vessels of status $\sigma_v = 1$, which are already berthed or about to be docked, *t* and *p* are necessarily known. The interpretation of the index *t* differs from status 2 and 3 to allow for a model that integrates the BAP and the LAP without creating separate decisions. For chartered vessels $(\sigma_v = 2)$, t is the berthing time.

For the vessel to charter $(\sigma_v = 3)$, considering the laydays of its laycan (L_v), t is used to define *conventionally* the last layday of its laycan, and its first layday is $(t - L_v + 1)$. *All constraints resulting from the berthing of the vessel during these laydays* $\left[(t - L_v + 1), t\right]$ *must be met for each of these periods during which the boat is allowed to berth* (however, the reverse convention for defining the laycan would have been possible, which would have required adjusting some model relations).To capture all vessel statuses $(\sigma_v = 1..3)$, a fictitious layday $\left(L_v = 1\right)$ is assigned to present and chartered vessels (which are not affected by the decision to fix laycans as they have known expected arrival times), and the modelling always uses $(t - L_v + 1)$ instead of *t*.

- The binary decision variable $y_{vpth}^{b_v}$ determines the loading start time t of the batch b_v, stored in hangar *h*, to be loaded onto vessel *v* berthed at berth *p*. For the new vessel to be chartered $(\sigma_v = 3)$, the consideration of the laycan at the batch level must be explained to justify the constraints on the use of conveyors, given that the modelling adopted is based on the continuous loading of batches (which makes it possible to know the total loading time of a ship at berth *p*). This continuity constraint implies that the Gantts of all the batches of a vessel berthed on the latest laycan ' layday will slide tightly to the left when using the laycan option, implying that the occupation of conveyors cannot be treated independently for each batch (as implicitly assumed in the BAG model). The possibility of shifting the loading of the batches in an autonomous use of the degrees of freedom offered by the laydays can lead, for example, to programming the loading start of all the batches on the same date (for example, that of the first layday), which violates the assumption of continuity of batch loading and inevitably leads to an excessive mobilization of the conveyors. It will, therefore, be necessary to deal specifically with the case of status 3 vessels in the constraint that prohibits the use of conveyors over their capacity. This point will be illustrated in Figure 2.

2.2.5 Predicates updating

This Predicate conditioning the existence of the variable x_{vpt}

The two predicates that define the conditions of the existence of the variable x_{vpt} must be adapted, if necessary.

The first three conditions ($\mathrm{G}_{vp} = 1$, $\lambda_v \leq \mathrm{Q}_p$ and $\mathrm{D}_v \leq \mathrm{W}_p$), referring to the physical possibility for the vessel *v* to berth at location *p* and collected in the predicate P_{vp} remain unchanged.

The Predicates P_{vpt} include four temporal existence conditions, and one must be added. They must be adapted to consider if the calendar γ_v, is shared by all the batches to load into $(\mathrm{d}_v = 1)$ or not $(\mathrm{d}_v = 0)$. In the last case, the more restrictive calendar assigned to batches may be noted γ_v^*. Without a loss of generality, this notation γ_v^* will also be used when all the batches share the vessel calendar (as $\gamma_v^{b_v} = \gamma_v, \forall b_v \in \mathscr{B}_v$).

- Vessel *v* can only berth (t) after its expected arrival time (A_v), without exceeding its maximum waiting time (I_v) in the harbor. This involves the condition, $\mathrm{A}_v \leq t - \mathrm{L}_v + 1 \leq \mathrm{A}_v + \mathrm{I}_v$, remembering that $\mathrm{L}_v = 1$ for present and chartered vessels $(\sigma_v \leq 2)$, and that t represents the last berth layday for vessels to be chartered $(\sigma_v = 3)$.
- The following constraint $t \leq \Upsilon_{\gamma_v^*, \{(\Gamma^*_{v,\mathrm{T}}) - \Theta_{vp} - \alpha_v + 1)\}}$ must be added to prevent vessel *v*, berthed at position *p*, from completing its docking (α_v) and loading (Θ_{vp}) operations after the horizon T, considering the calendar γ_v^* used for these operations where:
  - $\Gamma_{\gamma_v^*,\mathrm{T}}$ is the period number of T in the relative calendar of vessel *v* (which considers only the working periods);
  - $\Gamma_{\gamma_v^*,\mathrm{T}} - \Theta_{vp} - \alpha_v + 1$ is the latest possible date for starting operations for vessel *v* without exceeding the period T, in the relative calendar of vessel *v*;

- By using the reverse relative calendar, this relative date is converted into the date $\Upsilon_{\gamma_v^*,\{(\Gamma_{\gamma_v^*,T})-\Theta_{vp}-\alpha_v+1)\}}$ of the general calendar.

- Vessel *v* can only dock during time periods when a decision on docking vessels can be made: $K_{t-L_v+1}=1$.
- If vessel *v*' handling is restricted to working periods defined by the calendar γ_v^*, it can enter the port only during working periods: $\psi_{\gamma_v^*,(t-L_v+1)}=0$.
- The constraint that forces some vessels to finish loading and leave the port only at high tide $(\omega_v=1\wedge O_t=1)$ must be adapted because the finishing time π_v of vessel *v*, can only be calculated from its berthing time ε_v if all the batches to be loaded into vessel *v* share the same calendar $(d_v=1)$, considering its docking time α_v and its total loading time Θ_{vp} in the berth *p* $:(\pi_v=\varepsilon_v+\alpha_v+\Theta_{vp}-1)$. Otherwise $(d_v=0)$, this tidal constraint must be treated differently for vessels whose loading end date does not directly derive from the berthing date ε_v. Consequently, to retain its generality while effectively referring only to boats subject to the tidal constraint and using the same calendar for all the batches to load $(d_v=1\wedge\omega_v=1)$, the constraint that must be used, is: $(1\text{-}d_v\cdot\omega_v)+d_v\cdot\omega_v\cdot O_{\Upsilon_{v,\{(\Gamma_{v,t-L_v+1})+\alpha_v+\Theta_{vp}-1\}}}=1$.

  To account for the tidal constraint for the other vessels, it is necessary to introduce the binary variable $r_{vt},\forall t\in\mathcal{T},\forall v\in\mathcal{V}|d_v=0$, which is equal to 1 if and only when vessel *v* is at berth at the beginning of period *t*.

Predicate conditioning the existence of the variable $y_{vpth}^{b_v}$

The predicate P_{vp} referring to the physical possibility for the vessel *v* to berth at location *p* ($G_{vp}=1$, $\lambda_v\le Q_p$ and $D_v\le W_p$) and the predicate $P_{vh}^{b_v}$ considering the batch location in a hangar $(h=H_v^{b_v})$ also applies to the existence of the variable $y_{vpth}^{b_v}$. The conditions of the predicate P_{vpt} dealing with the period *t* of starting the loading of a batch, must be adapted due to the introduction of the calendar referential $\gamma_v^{b_v}$.

- The constraint enforcing a batch to be loaded at a later date than the date of its availability $(C_v^{b_v})$ remains unchanged $(t - L_v + 1 \geq C_v^{b_v})$.

- The constraint implying that the loading of a batch can only start at the beginning of a working period must be adapted to the new calendar referencial $(\psi_{\gamma_v^{b_v},(t-L_v+1)} = 0)$.

- The constraint on the range of allowed periods that can include a loading start is adapted to the introduction of a docking time (α_v) to be considered for the vessel to dock and the possible use of different batch calendars when loading a vessel, leading to the general use of γ_v^* for vessel *v*. An additional condition (second line of the following relation) is introduced to enforce the loading schedule to respect the horizon limit T.

$$\begin{cases} A_v + L_v - 1 + \alpha_v \leq t \leq \Upsilon_{\gamma_v^*,\left\{(\Gamma_{\gamma_v^*,(A_v+I_v+\alpha_v)}\Theta_{vp} - \theta_{vp}^{b_v} + 1\right\}} \\ t \leq \Upsilon_{\gamma_v^*,\{(\Gamma_{v,T}) - \theta_{vp}^{b_v} + 1)\}} \end{cases}$$

Predicate conditioning the existence of the variables $z_{pt}^{m_p}$ and $z_{st}^{m_s}$

The predicates $P_{pt}^{m_p}$ and $P_{st}^{m_s}$ relating to the maintenance of conveyors and berth positions, which require them to start inside predefined periods ($\underline{R}_p^{m_p} \leq t \leq \overline{R}_p^{m_p}$ and $\underline{R}_s^{m_s} \leq t \leq \overline{R}_s^{m_s}$), remain valid but must be completed later to take account of the introduction of specific time restrictions for maintenance operations.

New table of predicates

Table 3. Notation of predicates.

Predicate	*Notation*
$G_{vp} = 1 \wedge \lambda_v \le Q_p \wedge D_v \le W_p$	P_{vp}
$A_v + L_v - 1 \le t \le A_v + L_v - 1 + I_v \wedge$ $t \le \Upsilon_{\gamma_v^*, \{(\Gamma_{\gamma_v^*, T}) - \Theta_{vp} - \alpha_v + 1)\}} \wedge$ $K_{t-L_v+1} = 1 \wedge$ $\psi_{\gamma_v, (t-L_v+1)} = 0 \wedge$ $(1\text{-}d_v \cdot \omega_v) + d_v \cdot \omega_v \cdot O_{\Upsilon_{v, \{(\Gamma_{v, t-L_v+1}) + \alpha_v + \Theta_{vp} - 1\}}} = 1$	P_{vpt}
$A_v + L_v - 1 + \alpha_v \le t \le \Upsilon_{\gamma_v^*, \left\{(\Gamma_{\gamma_v^*, A_v}) + I_v + \alpha_v + \Theta_{vp} - \theta_{vp}^{b_v} - 1\right\}} \wedge$ $t \le \Upsilon_{\gamma_v^*, \{(\Gamma_{v,T}) - \theta_{vp}^{b_v} + 1)\}} \wedge$ $t - L_v + 1 \ge C_v^{b_v} \wedge$ $\psi_{\gamma_v^{b_v}, (t-L_v+1)} = 0$	$P_{vpt}^{b_v}$
$h = H_v^{b_v}$	$P_{vh}^{b_v}$
$\underline{R}_p^{m_p} \le t \le \overline{R}_p^{m_p}$	$P_{pt}^{m_p}$
$\underline{R}_s^{m_s} \le t \le \overline{R}_s^{m_s}$	$P_{st}^{m_s}$

Then, the logical conditions for the existence of the decision variables x_{vpt}, $y_{vpth}^{b_v}$, $z_{pt}^{m_p}$ and $z_{st}^{m_s}$ are the following ones.

$$x_{vpt} \in \{0,1\}, \forall v \in \mathcal{V}, \forall p \in \mathcal{P} \left| P_{vp}, \forall t \in \mathcal{T} \right| P_{vpt}$$

$$y_{vpth}^{b_v} \in \{0,1\}, \forall v \in \mathcal{V}, \forall b_v \in \mathcal{B}_v, \forall p \in \mathcal{P} \left| P_{vp}, \forall t \in \mathcal{T} \right| P_{vpt}^{b_v}, \forall h \in \mathcal{H} \left| P_{vh}^{b_v} \right.$$

$$z_{pt}^{m_p} \in \{0,1\}, \forall p \in \mathcal{P}, \forall m_p \in \mathcal{M}_p, \forall t \in \mathcal{T} \left| P_{pt}^{m_p} \right.$$

$$z_{st}^{m_s} \in \{0,1\}, \forall s \in \mathcal{S}, \forall m_s \in \mathcal{M}_s, \forall t \in \mathcal{T} \left| P_{st}^{m_s} \right.$$

2.3 Mathematical model

The (modified) predicates P_{vp}, P_{vpt}, $P_{vh}^{b_v}$, $P_{pt}^{m_p}$ and $P_{st}^{m_s}$ conditioning the existence of the decision variables x_{vpt}, $y_{vpth}^{b_v}$, $z_{pt}^{m_p}$ and $z_{st}^{m_s}$ are still used. The same applies to the intermediate variables ε_v (berthing time), and μ_v and $\tau_v^{b_v}$ (berthing position used for vessel v and batch b_v), that are derived from the decision variables. The intermediate variable π_v (loading end date) becomes a decision variable as the final loading time cannot be directly derived from the berthing time in the case of multiple calendars used by the batches to load onto a vessel. The relation defining π_v must be adapted. To do that, we need to define the latest finish time period π_v^* of vessel *v* berthing at date *t*, which is a new intermediate variable calculated with γ_v^* (the most restrictive calendar for vessel *v*).

We first describe the intermediate variables, which are defined by the decision variables x_{vpt} and $y_{vpth}^{b_v}$.

Definition of the intermediate variables

The berthing position (μ_v) and berthing time (ε_v) of vessels already berthed or about to berth $(\sigma_v = 1)$ are known and thus are parameters ($\varepsilon_v = 1$ for already berthed vessel). For chartered vessels $(\sigma_v = 2)$, these intermediate variables are defined by the following relations, which also apply for the vessel to be chartered $(\sigma_v = 3)$, but with a different meaning for ε_v, which then corresponds to the first layday of the laycan (the last one being obviously $\varepsilon_v + \mathrm{L}_v - 1$).

$$\mu_v = \sum_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum_{t \in \mathscr{T} | \mathrm{P}_{vpt}} p \cdot x_{vpt}, \forall v \in \mathscr{V}$$

$$\varepsilon_v = \sum_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum_{t \in \mathscr{T} | \mathrm{P}_{vpt}} (t - \mathrm{L}_v + 1) \cdot x_{vpt}, \forall v \in \mathscr{V}$$

Similarly, at the batch level, the berthing position $\tau_v^{b_v}$ and the loading start time $\rho_v^{b_v}$ are defined by the following relations. Note that for the vessel to be chartered $(\sigma_v = 3)$, $\tau_v^{b_v}$ is defined in a manner consistent with ε_v, i.e. in relation to the first layday.

$$\tau_v^{b_v} = \sum_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum_{t \in \mathscr{T} | \mathrm{P}_{vpt}^{b_v}} \sum_{h \in \mathscr{H} | \mathrm{P}_{vh}^{b_v}} p \cdot y_{vpth}^{b_v}, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v$$

$$\rho_v^{b_v} = \sum_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum_{t \in \mathscr{T} | \mathrm{P}_{vpt}^{b_v}} \sum_{h \in \mathscr{H} | \mathrm{P}_{vh}^{b_v}} (t - \mathrm{L}_v + 1) \cdot y_{vpth}^{b_v}, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v$$

The finishing loading time π_v of vessel *v* can be deduced from its starting time only if $\mathrm{d}_v = 1$, all the batches to be loaded onto vessel *v* use the same calendar γ_v. Using the appropriate relative and inverse relative calendars, the relation defining this intermediate variable π_v must consider the case of the vessels to be charted $(\sigma_v = 3)$, which necessitates adding $(\mathrm{L}_v - 1)$ in the formula of the ending date to cover the whole laycan (as ε_v corresponds to the first layday):

$$\pi_v = \sum\nolimits_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum\nolimits_{t \in \mathscr{T} | \mathrm{P}_{vpt}} x_{vpt} \cdot \Upsilon_{\gamma_v, \{(\Gamma_{\gamma_v, t-\mathrm{L}_v+1}) + \alpha_v + \Theta_{vp} + (\mathrm{L}_v - 1) - 1\}}, \forall v \in \mathscr{V}$$

If $\mathrm{d}_v = 0$, the finishing loading time π_v becomes a decision variable, and the upper bound π_v^* is calculated by replacing γ_v with γ_v^* in the previous relation.

Likewise, if $\mathrm{d}_v = 1$, the intermediate variable "finishing loading time $\sigma_v^{b_v}$ of batch b_v" is defined in a manner consistent with ε_v and $\tau_v^{b_v}$, i.e. in relation to the first layday. In conveyor reservation constraints for batches to be loaded into status-3 boats, it is necessary to consider the start of batch loading calculated from the first layday and its end calculated from the last layday by adding $(\mathrm{L}_v - 1)$ to $\sigma_v^{b_v}$ (remembering that $\mathrm{L}_v = 1$.if $\sigma_v < 3$).

$$\sigma_v^{b_v} = \sum\nolimits_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum\nolimits_{t \in \mathscr{T} | \mathrm{P}_{vpt}^{b_v}} \sum\nolimits_{h \in \mathscr{H} | \mathrm{P}_{vh}^{b_v}} y_{vpth}^{b_v} \cdot \Upsilon_{\gamma_v^{b_v}, \{(\Gamma_{\gamma_v, t-\mathrm{L}_v+1}) + \theta_{vp}^{b_v} + (\mathrm{L}_v - 1) - 1\}}, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v$$

It must be noted that for the boat to be chartered, the first layday date given by the optimization respects all the problem constraints, which may not be the case for the last layday date calculated by adding a constant to the first layday date; for example, the constraint of ending the boat loading at high tide which is respected for the first layday may not be respected for the last one depending on the acceptable range of high tide periods and the value of L_v. There are two possible solutions to this problem. The first would be to include in the problem definition the determination of the amplitude of the laydays under the constraint of respecting an acceptable amplitude range. The second would be to increase the contractual amplitude of the laydays so that all the constraints are respected for this new definition of the last layday (for example, if the amplitude used to define high tide is 6 hours and the ship is subject to this constraint, one can add 12 hours to the laydays to guarantee the existence of a solution, a little before this boundary). This last solution, which is easy to implement, is undoubtedly preferable.

Model

The complexity of this model has led us to precede each relation with its justification instead of sending the justification after listing all the model's relations.

The objective function (1) is based on an efficiency criterion that maximizes the difference between the despatch money and the demurrage charges of each vessel *v*. In contrast, the objective function ($1'$) is based on an effectiveness criterion that minimizes the total dwell time while searching for the earliest use of the port, with a value of k, which is low enough to privilege the first part of the function.

$$Max \sum_{v \in \mathscr{V}} \left(\beta_v \cdot w_v - \eta_v \cdot u_v \right) \tag{1}$$

$$Max \left[\sum_{v \in \mathscr{V}} \left(\pi_v - \varepsilon_v \right) + \mathrm{k} \cdot \sum_{v \in \mathscr{V}} \pi_v \right] \tag{1'}$$

Relation (2) ensures that each vessel *v* starts berthing at a unique berthing position *p* and in a unique period *t*.

$$\sum_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum_{t \in \mathscr{T} | \mathrm{P}_{vpt}} x_{vpt} = 1, \forall v \in \mathscr{V} \tag{2}$$

Relation (3) ensures that each batch b_v starts loading into a unique vessel *v*, at a unique berthing position *p*, in a unique period *t*, and is stored in a unique hangar *h*.

$$\sum_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum_{t \in \mathscr{T} | \mathrm{P}_{vpt}^{b_v}} \sum_{h \in \mathscr{H} | \mathrm{P}_{vh}^{b_v}} y_{vpth}^{b_v} = 1, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v \tag{3}$$

Relation (4) ensures that maintenance m_p to be performed at a berthing position *p* has a unique start time.

$$\sum_{t \in \mathscr{T} | \mathrm{P}_{pt}^{m_p}} z_{pt}^{m_p} = 1, \forall p \in \mathscr{P}, \forall m_p \in \mathscr{M}_p \tag{4}$$

Similarly, Relation (5) ensures that maintenance m_s to be performed at a conveyor in section *s* has a unique start time.

$$\sum_{t \in \mathscr{T} | \mathrm{P}_{st}^{m_s}} z_{st}^{m_s} = 1, \forall s \in \mathscr{S}, \forall m_s \in \mathscr{M}_s \tag{5}$$

Relation (6) ensures that berthing position p is the same in both decision variables x_{vtp} and $y_{vtph}^{b_v}$.

$$\mu_v = \tau_v^{b_v}, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v \tag{6}$$

Relations (7) ensure that the loading of batch b_v can only begin once vessel v has been berthed and all the batches that must precede b_v in the loading sequence have been loaded.

$$\begin{cases} \rho_v^{b_v} \geq \varepsilon_v + \alpha_v, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v \middle| \mathrm{N}_v^{b_v} = 0 \\ \rho_v^{b_v} > \rho_v^{b'_v}, \forall v \in \mathscr{V}, \forall b_v, b'_v \in \mathscr{B}_v \middle| \mathrm{N}_v^{b'_v} = \mathrm{N}_v^{b_v} - 1 \end{cases} \tag{7}$$

Relation (8) ensures that each vessel v can only leave the port when all batches have been loaded.

$$\sigma_v^{b_v} \leq \pi_v, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v \tag{8}$$

Relation (9) enforces that the vessel loading end date (π_v) is lower than its upper bound (π_v^*), when all batches do not share the same calendar $(\mathrm{d}_v = 0)$. Otherwise, π_v^* is derived from the berthing date and the vessel calendar γ_v.

$$\begin{cases} \pi_v = \pi_v^*, \forall v \in \mathscr{V} \,|\, \mathrm{d}_v = 1 \\ \pi_v \leq \pi_v^*, \forall v \in \mathscr{V} \,|\, \mathrm{d}_v = 0 \end{cases} \tag{9}$$

Relation (10) ensures that only one batch can be loaded at time t on vessel v.

$$\sum_{b_v \in \mathscr{B}_v} \sum_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum_{h \in \mathscr{H} | \mathrm{P}_{vh}^{b_v}} \sum_{t' \in \mathscr{T} \,\middle|\, \mathrm{P}_{vpt'}^{b_v} \wedge t' \leq t \leq \Upsilon_{\gamma_v^{b_v}, (\Gamma_{\gamma_v^{b_v}, t'} + \theta_{vp}^{b_v} - 1)}} y_{vpt'h}^{b_v} \leq 1, \forall t \in \mathscr{T}, \forall v \in \mathscr{V} \tag{10}$$

Relation (11) ensures that only one batch at most can leave at a time from each storage hangar h if the location of the batch to be loaded is known at the hangar level $(h \in \mathscr{H}_0)$ and not at the hangar group level.

$$\sum_{v \in \mathscr{V}} \sum_{b_v \in \mathscr{B}_v | \mathrm{P}_{vh}^{b_v}} \sum_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum_{t' \in \mathscr{T} \,\middle|\, \mathrm{P}_{vpt'}^{b_v} \wedge t' \leq t \leq \Upsilon_{\gamma_v^{b_v}, (\Gamma_{\gamma_v^{b_v}, t'} + \theta_{vp}^{b_v} - 1)}} y_{vpt'h}^{b_v} \leq \forall t \in \mathscr{T}, \forall h \in \mathscr{H}_0, \tag{11}$$

The number of parallel conveyors available in section s during period t equals the number U_s of existing conveyors minus the number of conveyors under maintenance during this period $U_s - \sum_{m_s \in \mathscr{M}_s} \sum_{t' \in \mathscr{T} | t - R_s^{m_s} + 1 \le t' \le t \wedge P_{st}^{m_s}} z_{st'}^{m_s}$. The number φ_{st}^0 of required conveyors of section s during period t is the sum of requirements φ_{st}^1 by vessels of status $\sigma_v < 3$, whose determination is easy:

$$\varphi_{st}^1 = \sum_{v \in \mathscr{V} | \sigma_v < 3} \sum_{b_v \in \mathscr{B}_v} \sum_{p \in \mathscr{P} | P_{vp}} \sum_{h \in \mathscr{H} | F_{sh} = 1 \wedge P_{vh}^{b_v}} \sum_{t' \in \mathscr{T} | P_{vpt'}^{b_v} \wedge t' \le t \le \Upsilon_{\gamma_v^{b_v}, (\Gamma_{\gamma_v^{b_v}, t'} + \theta_{vp}^{b_v} - 1)}} y_{npt'h}^{b_v}, \forall t \in \mathscr{T}, \forall s \in \mathscr{S}$$

and of requirements φ_{st}^2 by vessels of status $\sigma_v = 3$ whose determination is more complicated and based on the following reasoning.

- The loading, during the period t, of the batch b_v of a vessel v with status $\sigma_v = 3$, and therefore the occupancy of the conveyors carrying this batch, occurs between the earliest start period of the loading $(\rho_v^{b_v})$ and the latest end period of this loading $(\sigma_v^{b_v} + L_v - 1)$, taking into account the laycan of this vessel. If only one batch is to be loaded on this vessel, the booking of the conveyors involved in this loading must be made between these two dates, provided that the contract under negotiation allows for the possibility of berthing at any time within this time frame. In this context, the ship's loading time is equal to that of the batch $(\Theta_{vp} = \theta_{vp}^{b_v})$, and it suffices to add the laycan duration (L_v) to the ship's loading time to define the latest batch loading end date $\Upsilon_{\gamma_v^{b_v}, (\Gamma_{\gamma_v^{b_v}, t'} + \Theta_{vp} + L_v - 1)}$.
- If several batches are to be loaded on this vessel, generalizing this principle normally leads to overlapped use of conveyors of the same section at certain periods, which is prohibited by sequential batch loading. The chosen solution consists of creating the intermediate variable φ_{svt} based on a batch overlap authorization for vessels of status $\sigma_v = 3$ and the binary variable ϕ_{svt} that takes the value 0 if $\varphi_{svt} = 0$, and 1 if $\varphi_{svt} > 0$ (where parameter 10 is greater than the maximum number of parallel conveyors in a section), described by the Relation (12.)

$$\left| \begin{array}{l} \varphi_{svt} = \sum_{b_v \in \mathscr{B}_v} \sum_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum_{h \in \mathscr{H} | \mathrm{F}_{sh}=1 \wedge \mathrm{P}_{vh}^{b_v}} \sum_{t' \in \mathscr{T} | \mathrm{P}_{vpt'}^{b_v} \wedge t' \le t \le \Upsilon_{\gamma_v^{b_v}, (\Gamma_{\gamma_v^{b_v}, t'} + \theta_{vp}^{b_v} + \mathrm{L}_v - 1)}} y_{npt'h}^{b_v} \\ \phi_{svt} \le 10 \cdot \varphi_{svt} \\ \phi_{svt} \ge \varphi_{svt} / 10 \end{array} \right.$$

$$, \forall t \in \mathscr{T}, \forall s \in \mathscr{S}, \forall v \in \mathscr{V} \, | \sigma_v = 3 \qquad (12)$$

- The number φ_{st}^2 of conveyors mobilized by status-3 vessels during period *t* in section *s* is defined by Relation (13), and the total number φ_{st}^0 of required conveyors of section *s* during period *t* is $\varphi_{st}^0 = \varphi_{st}^1 + \varphi_{st}^2$. This requirement must not exceed the available conveyors in section *s* during period *t*, giving the Relation (14).

$$\varphi_{st}^2 = \sum_{v \in \mathscr{V} | \sigma_v = 3} \phi_{svt}, \forall t \in \mathscr{T}, \forall s \in \mathscr{S} \qquad (13)$$

$$\sum_{v \in \mathscr{V} | \sigma_v < 3} \sum_{b_v \in \mathscr{B}_v} \sum_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum_{h \in \mathscr{H} | \mathrm{F}_{sh}=1 \wedge \mathrm{P}_{vh}^{b_v}} \sum_{t' \in \mathscr{T} | \mathrm{P}_{vpt'}^{b_v} \wedge t' \le t \le \Upsilon_{\gamma_v^{b_v}, (\Gamma_{\gamma_v^{b_v}, t'} + \theta_{vp}^{b_v} - 1)}} y_{npt'h}^{b_v} + \sum_{v \in \mathscr{V} | \sigma_v = 3} \phi_{svt} \le \mathrm{U}_s - \sum_{m_s \in \mathscr{M}_s} \sum_{t' \in \mathscr{T} | t - \mathrm{R}_s^{m_s} + 1 \le t' \le t \wedge \mathrm{P}_{st}^{m_s}} z_{st'}^{m_s}, \forall t \in \mathscr{T}, \forall s \in \mathscr{S} \qquad (14)$$

Relation (15) avoids the overlapping of vessels in each berthing position *p*, the simultaneous use of berthing positions that share a space of the quay since the berth layout of each quay is hybrid and the use of berthing positions where maintenance activities are performed (to illustrate this point, in Tables 1, berth 3 is part of berth 5, so they cannot be used at the same time; then maintenance performed on berth 3 is implicitly performed on berth 5, and vice versa). The left part of this inequality defines the total number of vessels occupying berth *p* during period *t*; the right part defines the availability of berth p during period *t*, which is 1 if no maintenance is performed on this berth during this period and 0 otherwise.

$$\sum_{v \in \mathscr{V}} \sum_{p' \in \mathscr{P} | \mathrm{E}_p^{p'}=1 \wedge \mathrm{P}_{vp'}} \sum_{t' \in \mathscr{T} | \mathrm{P}_{vp't'} \wedge \le t' - \mathrm{L}_v + 1 \le t \le \Upsilon_{\gamma_v^{b_v}, (\Gamma_{\gamma_v^{b_v}, t'} + \theta_{vp}^{b_v} - 1)}} x_{vp't'} \le 1 - \sum_{p' \in \mathscr{P} | \mathrm{E}_p^{p'}=1} \sum_{m_{p'} \in \mathscr{M}_{p'}} \sum_{t' \in \mathscr{T} | t - \mathrm{R}_{p'}^{m_{p'}} + 1 \le t' \le t \wedge \mathrm{P}_{p't}^{m_{p'}}} z_{p't'}^{m_{p'}}, \forall t \in \mathscr{T}, \forall p \in \mathscr{P} \qquad (15)$$

Relation (16) limits the number of incoming and outgoing vessels that can pass through the navigation channel at the same time. The first part of the left side of this inequality refers to incoming vessels; the second part refers to outgoing vessels in which all the batches to be loaded have the same calendar $(\mathrm{d}_v = 1)$; the last part refers to outgoing vessels in which all the batches to be loaded do not have the same calendar $(\mathrm{d}_v = 0)$; r_{vt} is defined by Relation (18).

$$\sum_{v\in\mathscr{V}|\sigma_v>1}\sum_{p\in\mathscr{P}|\mathrm{P}_{vp}}\sum_{t'\in\mathscr{T}|\mathrm{P}_{vpt'}\wedge t'=t+\mathrm{L}_v-1} x_{vpt'} + \sum_{v\in\mathscr{V}|\mathrm{d}_v=1}\sum_{p\in\mathscr{P}|\mathrm{P}_{vp}}\sum_{t'\in\mathscr{T}|\mathrm{P}_{vpt'}\wedge t'=\Upsilon_{\gamma_v,(\Gamma_{\gamma_v,\mathrm{A}_v}-\alpha_v-\Theta_{vp}+1)}} x_{vpt'} + \sum_{v\in\mathscr{V}|\mathrm{d}_v=0} r_{vt} \le \mathrm{M}, \forall t \in \mathscr{T} \tag{16}$$

Relations (17) determine the delay and the advance of each vessel, referring to the contractual finishing time δ_v of berthed or chartered vessel v $(\sigma_v \le 2)$.

$$\begin{cases} u_v \ge \pi_v - \delta_v, \forall v \in \mathscr{V} \,|\sigma_v \le 2 \\ w_v \ge \delta_v - \pi_v, \forall v \in \mathscr{V} \,|\sigma_v \le 2 \\ u_v - w_v = \pi_v - \delta_v, \forall v \in \mathscr{V} \,|\sigma_v \le 2 \\ u_v, w_v \ge 0, \forall v \in \mathscr{V} \,|\sigma_v \le 2 \end{cases} \tag{17}$$

The predicate P_{vpt} contains the condition enforcing vessel that must leave port at high tide $(\omega_v = 1)$ and whose all batches to load share the same calendar $(\mathrm{d}_v = 1)$: $(1\text{-}\mathrm{d}_v \cdot \omega_v)+\mathrm{d}_v \cdot \omega_v \cdot \mathrm{O}_{\Upsilon_{v,\{(\Gamma_{v,t-\mathrm{L}_v+1})+\alpha_v+\Theta_{vp}-1\}}} = 1$ to respect that condition. Additional relations must be added for the vessels that are constrained by high tide and whose all batches to be loaded do not share the same calendar $(\mathrm{d}_v = 0)$. The auxiliary binary variable r_{vt} is created for vessels v that satisfy the condition $\mathrm{d}_v = 0 \wedge \omega_v = 1$. Relation (18) enforces $r_{vt} = 1$ only if t is the period of loading end, and Relation (19) enforces that period to be a high tide period.

$$\begin{cases} \sum_{t\in\mathscr{T}} t \cdot r_{vt} = \pi_v, \forall v \in \mathscr{V} \,|\mathrm{d}_v = 0 \wedge \omega_v = 1 \\ \sum_{t\in\mathscr{T}} r_{vt} = 1, \forall v \in \mathscr{V} \,|\mathrm{d}_v = 0 \wedge \omega_v = 1 \end{cases} \tag{18}$$

$$\sum_{t\in\mathscr{T}} t \cdot r_{vt} = \mathrm{O}_t, \forall v \in \mathscr{V} \,|\mathrm{d}_v = 0 \tag{19}$$

3 Computational experiments

In this section, we describe the test instances and report the computational results. The model solutions were obtained on a MacBook Air computer with an M2 chip and 24 GB of unified memory, of which 18 GB were allocated to the Parallels virtual machine running the XPress-IVE optimization software used (version 1.25.06), which is based on AML-type modeling (Fourer, 2013).

In this context, the first processing step is to combine the generic problem formulation with a dataset to create an instance of the optimization problem. This "compilation" takes just a few minutes before the actual problem-solving process begins. The results distinguish between these two processing phases. Solving time depends on the chosen stopping test, which is defined by the percentage difference between the value of the optimization criterion of the current solution of the search process and the probable theoretical optimal value calculated by the program

This paper presents only two cases, with detailed results (numerical and Gantt) in an Excel file[2]. The first case takes up the example used in the original BAG model article, which used the efficiency criterion, but here some data are modified to illustrate the interest of the proposed extensions. The second case is inspired by a real problem in the port of Jorf when solving a two-week port management problem involving only chartered or berthed vessels and using an effectiveness criterion.

3.1 First case study

During the first week, a decision to dock can be made every hour, then every two hours for the next two weeks, and every three hours thereafter (parameter K_t). The reference to the products to be loaded is of no interest here, only the indication that night loading is prohibited (indicated by a specific $\gamma_v^{b_v}$) is necessary. In this test, vessels that cannot be loaded on weekends are not restricted from loading at night. As in the case study of the BAG paper, we have kept: *i*) the five maintenance operations to be performed during this month with their possible time range for their execution; *ii*) the maximum number of vessels allowed to pass simultaneously is limited to three vessels; *iii*) the two quay configuration with hybrid berth layout and division into five berths each; *iv*) the number of hangar groups of is increased to four, with new hangars belonging to the last two groups (one for fertilizers and the other for ore), each connected to the port by new conveyor sections with two parallel conveyors for the third one and a unique conveyor

2 http://www.lamsade.dauphine.fr/~giard/Bulk_Port_Extended.xlsx

for the last one. In this case study, the vessel stability constraints are always considered. Table 4 describes the problem data, which requires the following explanations.

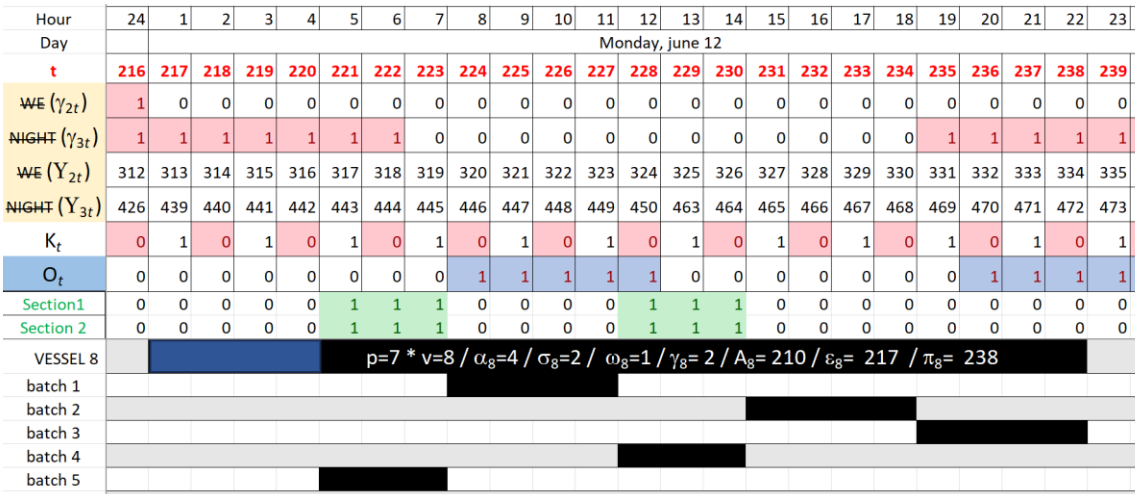
Three independent calendars have been created (*additional feature C*). The calendar index for all vessels is $\gamma_v = 1$, except for vessels 5 ($\gamma_5 = 3$, night work forbidden) and 8 ($\gamma_v = 2$ weekend work forbidden according to to a SSHEX clause). The five first batches to be loaded into vessel 14 and the two first batches to be loaded into vessel 8 cannot be handled at night (*additional feature D*), implying that their calendars $\gamma_{14}^{b_v}$ and $\gamma_8^{b_v}$ are different from the vessel's ones (γ_{14} and γ_8), making it impossible to determine mechanically the vessel' loading end π_{14} and π_8 (which become decision variables defined by relations 18 & 19). Vessel end loading must coincide with the high tide for vessels 8, 9, 11 and 14 $(\omega_9 = \omega_{11} = \omega_{14} = 1)$. Vessels 15 and 16 are to charter, with the same laycans $(L_{15} = L_{16} = 48)$, despatches $(\beta_{15} = \beta_{16} = 2)$. and demurrages $(\eta_{15} = \eta_{16} = 4)$ The docking time $\alpha_v = 4$ is used for all charter vessels and vessels to charter (*additional feature A*). The loading rate efficiency of vessel 7 is $\tau_7 = 0.8$, due to its small size, and this rate is $\tau_5 = 0.75$ for vessel 5, which has to load ore (*additional feature B*). Finally, let's add that the location of batches for vessel 15 is only known at the hangar group level (*additional feature E*), which means that for this vessel, relation 11, which aims to ensure that 2 batches cannot leave a hangar at the same time, is not mobilized for this ship.

A detailed solution with Gantts, which helps to control the respect of the time constraints, is given in the Excel file and the main results are shown in Table 4. This problem has 34,458 variables and 15,032 constraints; its compilation takes 752', and the optimization takes 969' to obtain a solution of 985.3 at 0.63% of the theoretical optimal solution (985). We can add that the solution found in 134' is at 1035, at 7.5% of the current best bound (1036).

Figure 3 shows the solution optimal solution found for vessel 8, which docks at the earliest in period 217, later than its ETA (210), which occurs during the weekend when no operations are possible. Loading can start in period 221, after the vessel has been installed on the quay. The proposed schedule respects the prohibition of night loading for batches 1 and 2 and the obligation to finish loading during a period of high tide.

Figure 4 illustrates the proposed solution for vessel 15, which is currently under negotiation. The Gantt shows how laycan has been taken into account in this solution, with a visualization of the earliest and latest schedules, as well as their impact on the reservation of the conveyor system.

Figure 3. Berthing solution for Vessel 8 in the first case study



Hour	24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Day													Monday, june 12											
t	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239
~~WE~~ (γ_{2t})	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
~~NIGHT~~ (γ_{3t})	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
~~WE~~ (Y_{2t})	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335
~~NIGHT~~ (Y_{3t})	426	439	440	441	442	443	444	445	446	447	448	449	450	463	464	465	466	467	468	469	470	471	472	473
K_t	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
O_t	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	1	1	1	1
Section1	0	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
Section 2	0	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
VESSEL 8						p=7 * v=8 / α_8=4 / σ_8=2 / ω_8=1 / γ_8= 2 / A_8= 210 / ε_8= 217 / π_8= 238																		
batch 1																								
batch 2																								
batch 3																								
batch 4																								
batch 5																								

Figure 4. Berthing solution for Vessel 15 in the first case study

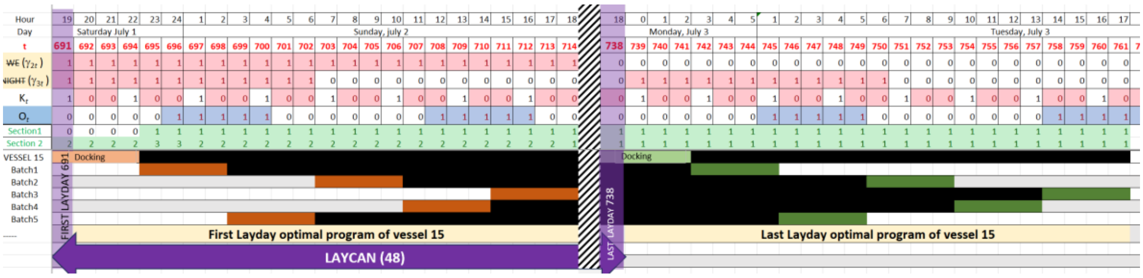


Table 4. Data of the first case study

v	σ_v	I_v	α_v	L_v	A_v	λ_v	D_v	ω_v	γ_v	η_v	β_v	τ_v	B_v
1	1	0	0	1	1	194	8	0	1	101	50.50	1.00	5
2	2	72	4	1	12	197	9	0	1	59	29.5	1.00	8
3	2	72	4	1	22	188	8	0	1	111	55.5	1.00	8
4	2	72	4	1	55	190	10	0	1	105	52.5	1.00	6
5	2	72	4	1	107	182	9	0	3	97	48.5	0.75	6
6	2	72	4	1	171	128	7	0	1	101	50.5	1.00	2
7	2	72	4	1	174	129	5	0	1	64	32.00	0.80	1
8	2	72	4	1	210	191	9	1	2	94	47.00	1.00	5
9	2	72	4	1	323	131	5	1	1	61	30.5	1.00	1
10	2	72	4	1	419	201	11	0	1	143	71.5	1.00	11
11	2	72	4	1	560	207	11	1	1	111	55.5	1.00	10
12	2	72	4	1	572	122	7	0	1	93	46.50	1.00	1
13	2	72	4	1	600	158	9	0	1	130	65	1.00	10
14	2	72	4	1	634	231	13	1	1	147	73.5	1.00	11
15	3	72	4	48	651	174	10	0	1	4	2	1.00	5
16	3	72	4	48	661	112	5	0	1	4	2	1.00	1

v	Batch weight of batch b_v										
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	2,750	2,750	3,667	3,667	3,667						
2	4,125	4,125	4,125	4,125	4,125	4,125	4,125	4,125			
3	4,450	4,450	3,867	3,867	3,867	3,844	3,844	3,844			
4	2,750	2,750	4,074	4,074	4,074	4,074					
5	4,375	4,375	4,375	4,375	4,375	4,375					
6	2,750	2,750									
7	3,245										
8	**3,767**	**3,767**	3,767	3,413	3,413						
9	3,500										
10	4,415	4,415	4,415	4,415	4,415	4,415	4,415	4,415	4,415	4,415	4,450
11	4,500	4,500	4,500	4,500	4,500	4,500	4,500	4,500	4,500	4,500	
12	3,300										
13	3,967	3,967	3,967	3,967	3,967	3,967	3,828	3,828	3,828	3,828	
14	**4,400**	**4,400**	**4,400**	**4,400**	**4,400**	4,400	4,400	4,400	4,400	4,400	4,400
15	4,400	4,400	4,400	4,400	4,400						
16	2,350										

v	Hangar of batch storage										
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	7	6	9	10	9	0	0	0	0	0	0
2	12	12	11	13	13	8	12	8	0	0	0
3	9	13	6	5	7	11	13	11	0	0	0
4	19	20	12	10	8	13	0	0	0	0	0
5	4	4	4	4	4	4	0	0	0	0	0
6	7	7	0	0	0	0	0	0	0	0	0
7	5	0	0	0	0	0	0	0	0	0	0
8	19	18	18	6	5		0	0	0	0	0
9	12	0	0	0	0	0	0	0	0	0	0
10	7	7	7	7	5	5	5	6	6	6	5
11	13	12	9	11	13	11	10	8	11	11	0
12	7	0	0	0	0	0	0	0	0	0	0
13	11	9	8	12	11	12	18	18	20	18	0
14	10	12	13	11	10	12	12	9	8	9	10
15	2	2	2	2	2	0	0	0	0	0	0
16	12	0	0	0	0	0	0	0	0	0	0

v	Availability of batch b_v										
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	1	1	1	1	1	0	0	0	0	0	0
2		1	1	1	1	1	1	1	0	0	0
3	1	1	1	1	1	1	1	1	0	0	0
4	1	1	1	1	1	1	0	0	0	0	0
5	1	31	1	31	1	31	0	0	0	0	0
6	1	22	0	0	0	0	0	0	0	0	0
7	28	0	0	0	0	0	0	0	0	0	0
8	1	1	1	1	1		0	0	0	0	0
9	299	0	0	0	0	0	0	0	0	0	0
10	341	369	395	204	232	258	122	150	176	286	313
11	372	413	124	166	206	248	290	330	455	496	0
12	548	0	0	0	0	0	0	0	0	0	0
13	393	430	466	503	540	576	600	600	600	600	0
14	326	367	407	448	489	529	570	610	204	245	285
15	537	578	618	659	699	0	0	0	0	0	0
16	661	0	0	0	0	0	0	0	0	0	0

		Hangars h																						
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Section s	U_s	JLN-I	JLN-II	JFC	Ore	JLN-I_1	JLN-I_2	JLN-I_3	JLN-I_4	JLN-I_5	JLN-I_6	JLN-I_7	JLN-II_1	JLN-II_2	JLN-II_3	JLN-II_4	JLN-II_5	JLN-II_6	JFC_1	JFC_2	JFC_3	JFC_4	JFC_5	Ore
		F_{sh}																						
1	2	1	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	3	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
3	2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0
4	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Berth	Vessel	Start	End	Dwell	Batch loading
1	6	171	180	10	*1 [178-180] *2 [175-177]
1	13	601	644	44	*1 [609-612] *2 [613-616] *3 [621-624] *4 [629-632] *5 [637-640] *6 [641-644] *7 [633-636] *8 [625-628] *9 [617-620] *10 [605-608]
2	1	1	18	18	*1 [1-3] *2 [8-10] *3 [15-18] *4 [11-14] *5 [4-7]
2	3	22	59	38	*1 [30-34] *2 [35-39] *3 [48-51] *4 [52-55] *5 [56-59] *6 [44-47] *7 [40-43] *8 [26-29]
6	7	175	182	8	*1 [179-182]
6	12	574	580	7	*1 [578-580]
6	16	661	713	6	*1 [665-713]
7	2	16	51	36	*1 [20-23] *2 [28-31] *3 [40-43] *4 [48-51] *5 [44-47] *6 [36-39] *7 [32-35] *8 [24-27]
7	4	55	80	26	*1 [59-61] *2 [70-72] *3 [73-76] *4 [77-80] *5 [66-69] *6 [62-65]
7	5	127	184	58	*1 [111-127] *2 [133-137] *3 [155-159] *4 [160-176] *5 [138-154] *6 [128-132]
7	8	217	238	22	*1 [224-227] *2 [228-231] *2 [228-231] *4 [232-234] *5 [221-223]
7	9	329	336	7	*1 [333-336]
7	10	419	466	48	*1 [427-430] *2 [431-434] *3 [443-446] *4 [451-454] *5 [455-458] *6 [463-466] *7 [459-462] *8 [447-450] *9 [439-442] *10 [435-438] *11 [423-426]
7	15	691	714	24	*1 [695-745] *2 [707-757] *3 [711-761] *4 [703-753] *5 [699-749]
10	11	568	611	44	*1 [576-579] *2 [584-587] *3 [588-591] *4 [600-603] *5 [608-611] *6 [604-607] *7 [596-599] *8 [592-595] *9 [580-583] *10 [572-575]
10	14	634	696	63	*1 [638-641] *2 [655-658] *3 [661-664] *4 [680-683] *5 [684-687] *6 [692-695] *7 [688-691] *8 [669-672] *9 [665-668] *10 [648-651] *11 [642-645]

Maintenances of Berth positions

m_p	p	$\underline{R}_p^{m_p}$	$\overline{R}_p^{m_p}$	$R_p^{m_p}$
1	1	40	51	12
2	1	328	337	10
3	2	417	423	7
4	6	266	294	29
5	6	320	333	14
6	9	367	372	6
7	9	24	33	10
8	10	306	328	23

Maintenance of conveyors

m_s	s	$\underline{R}_s^{m_s}$	$\overline{R}_s^{m_s}$	$R_s^{m_s}$
1	1	282	293	12
2	1	285	292	8
3	2	213	218	6
4	2	290	299	10
5	2	390	398	9

Maintenance solutions

Maintenance sections

s	m_s	Start
1	1	282
1	2	286
2	1	213
2	2	290
2	3	362

Maintenance Berths

p	m_p	Start
1	1	40
1	2	316
2	1	412
6	1	266
6	2	320
9	1	367
9	2	24
10	1	306

					G_{vp} (Vessel v)															
Quay	Berth p	Q_p	W_p	ρ_p	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	13.5	180	1000	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	2	13.5	255	1000	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	3	15.5	150	800	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	4	15.5	150	800	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	$5 = 3 \cup 4$	15.5	300	1200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	6	13.5	180	1200	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	7	13.5	235	1200	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	8	13.5	125	1200	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	9	13.5	125	1200	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	$10 = 8 \cup 9$	13.5	250	1200	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0

3.2 Second case study

This case corresponds to a real port management problem within a 2-weeks horizon. The efficiency criterion is retained. This problem is slightly less complex than the previous one because the quay configuration is simpler (only berths 1, 2, 5, 6, 7 and 10 remain), the availability of the batches to be loaded is guaranteed and their location is known only at the level of hangar groups. The first two batches of vessel 10 cannot be loaded at night (this restriction was not part of the original problem). The maintenance on berths 2 and 7, which take 12 and 24, must start in periods 253 and 277. No maintenance is planned on the conveyors. This problem has 35,772 variables and 6,898 constraints; its compilation takes 420', and the optimization takes 1,197' to obtain the optimal solution of 499.2 (which contains 316 hours of dwell time, including 276 hours of loading).

In practice, the solution's optimality is often less important than the speed with which it can be obtained, which, in our case, is far superior to the methods used. If we ask for a solution close to 10% of the theoretical optimal solution, the execution time drops to 15.2' for a value of 506.4, which is very close to the optimal value (499.2); this solution. It should be added that the solution obtained with a 10% criterion loses only 1 hour's stay in port for the same loading time.

Figure 5. Berthing for Vessel 10 in the second case study

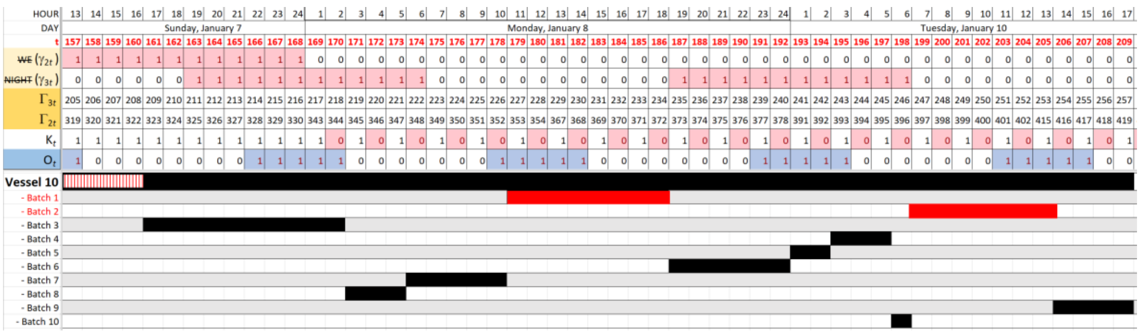


Table 5. Data and solution of the second case study

ν	σ_ν	I_ν	α_ν	L_ν	A_ν	λ_ν	D_ν	ω_ν	γ_ν	η_ν	β_ν	τ_ν	B_ν	Weight $\varphi_\nu^{b_\nu}$ of batch b_ν: 1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	0	0	1	1	250	9.7	0	1	2	1	1.00	1	1,200												
2	1	0	0	1	1	119	4.4	0	1	2	1	1.00	3	2,019	4,821	360										
3	1	0	0	1	1	179	9.5	0	1	2	1	1.00	6	2,401	1,500	3,100	2,043	1,516	2,003							
4	1	0	0	1	1	91	3.8	0	1	2	1	1.00	3	2,100	1,100	300										
5	2	72	4	1	13	106	6.5	0	3	2	1	1.00	2	2,330	670											
6	2	72	4	1	13	200	13.5	0	1	2	1	1.00	7	740	6,064	6,490	902	4,200	4,540	2,400						
7	2	72	4	1	85	143	6.3	0	1	2	1	0.80	4	650	5,350	6,823	177									
8	2	72	4	1	85	180	9.8	1	2	2	1	1.00	7	7,001	10,542	2,607	113	3,000	3,500	600						
9	2	72	4	1	109	90	3.5	1	1	2	1	1.00	2	817	2,505											
10	2	72	4	1	157	200	11.9	0	1	2	1	1.00	10	**9,300**	**8,200**	10,805	3,557	1,382	6,041	5,100	3,565	4,340	210			
11	2	72	4	1	301	95	5.5	1	1	2	1	1.00	2	1,650	2,200											
12	2	72	4	1	301	190	11.1	0	1	2	1	1.00	6	3,590	11,060	7,050	12,000	9,000	1,300							
13	2	72	4	1	349	181	10.1	0	1	2	1	1.00	10	1,000	6,670	4,900	5,887	1,413	5,187	407	3,628	1,100	2,220			
14	2	72	4	1	373	200	12.6	1	1	2	1	1.00	13	4,100	4,636	7,463	4,200	1,961	4,906	140	11,026	6,160	2,864	2,700	2074	3470

ν	Hangar $H_\nu^{b_\nu}$ of batch storage: 1	2	3	4	5	6	7	8	9	10	11	12	13
1	1												
2	1	1	1										
3	3	3	3	3	3	3							
4	1	1	1										
5	1	1											
6	3	3	3	3	3	3	3						
7	1	1	1	1									
8	3	3	3	3	3	3	3						
9	1	1											
10	1	3	1	3	3	1	3	1	3	3			
11	1	1											
12	3	3	1	3	3	3							
13	3	3	3	1	1	1	1	3	3	1			
14	1	3	1	3	1	1	1	3	1	1	3	3	1

Optimisation solution

Berth	Vessel	Start	End	Dwell	Batch loading
1	2	7	15	9	*1 [12-14] *2 [7-11] *3 [15-15]
5	1	7	7	1	*1 [7-7]
5	5	13	19	7	*1 [17-18] *2 [19-19]
5	7	85	101	17	*1 [89-89] *2 [97-101] *3 [90-95] *4 [96-96]
5	9	109	116	8	*1 [116-116] *2 [113-115]
5	12	301	343	43	*1 [333-335] *2 [311-320] *3 [305-310] *4 [321-330] *5 [336-343] *6 [331-332]
6	4	1	4	4	*1 [1-2] *2 [3-3] *3 [4-4]
6	8	85	114	30	*1 [95-100] *2 [101-109] *3 [89-91] *4 [110-110] *5 [92-94] *6 [111-113] *7 [114-114]
6	11	309	316	8	*2 [313-314]
7	6	15	42	28	*1 [42-42] *2 [33-38] *3 [23-28] *4 [41-41] *5 [29-32] *6 [19-22] *7 [39-40]
7	13	352	387	36	*1 [370-370] *2 [372-377] *3 [356-360] *4 [361-365] *5 [366-367] *6 [383-387] *7 [382-382] *8 [378-381] *9 [371-371] *10 [368-369]
10	3	7	20	14	*1 [10-12] *2 [13-14] *3 [7-9] *4 [15-16] *5 [17-18] *6 [19-20]
10	10	157	209	53	*1 [179-186] *2 [199-205] *3 [161-170] *4 [195-197] *5 [193-194] *6 [187-192] *7 [174-178] *8 [171-173] *9 [206-209] *10 [198-198]
10	14	384	441	58	*1 [397-400] *2 [406-409] *3 [416-422] *4 [393-396] *5 [423-424] *6 [401-405] *7 [425-425] *8 [426-435] *9 [410-415] *10 [439-441] *11 [388-390] *12 [391-392] *13 [436-438]

4 Conclusions:

The improvements made to the BAG model to allow it to respect the constraints encountered in the field (otherwise, a model is of no operational interest) are of great interest since the tests carried out show that the speed of obtaining a good solution is fully compatible with its inclusion in a DSS oriented to both operational and tactical decision making. Moreover, this model easily replaces the BAG model used in the approach aimed at the optimal management of fertilizer production, storage and port shipment, in a pull-flow approach, tested and described in the article "An integrated Decision Support System for planning production, storage and bulk port operations in a fertilizer supply chain", (H. Bouzekri et al., 2022).

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