PhD thesis on resilient networks design

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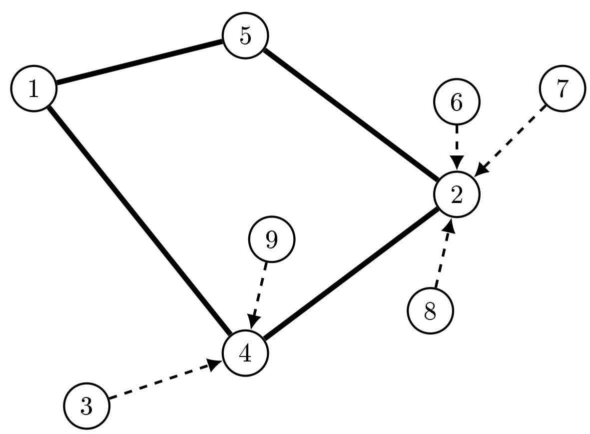
1 Scientific Context

This PhD proposal is aimed at addressing resilient network design problems. The first problem is the ring-star problem: given a complete graph, it consists in selecting a subset of nodes as hubs, in order to connect these nodes with a minimum length cycle, and then to connect each non-hub node to the closest hub, as in [1]. An additional case is presented in [2], where the number of nodes in the hub is known in advance. A ring-star network on 9 vertices is shown below as an example where nodes 1, 2, 4 and 5 have been selected as hubs, and the other ones are connected to their closest hub.



This thesis will be devoted to perform complexity analyses, and to produce exact and approximate solutions to the problem of providing a resilient response to some failures that may occur in the network.

First, we assume that at most one hub j can fail. When this happens, a backup cost is to be paid to restore the ring-star feature of the solution. More specifically, an edge joining the two neighboring hubs of j should be used to restore the cycle, and the non-hubs that were initially connected to j must be reallocated to hub nodes. For instance, if node 4 fails in the example above, then edge $(1, 2)$ should be added to the solution as a backup edge, and vertices 3 and 9 should be reallocated to hubs 1 and/or 2.

The mono-objective resilient network design problem consists in determining a ring-star network that minimizes the total cost given by the designing cost and the maximum restoration cost that can occur when a hub node fails. A bi-objective version explores the non-dominated trade-offs between these two costs. A capacitated problem version can also be considered: each hub has a limited capacity and each non-hub has a demand, hence the hub capacities should be satisfied even when the failure of a hub triggers the reallocation of its non-hub neighbors. Hence, the failure of a hub makes the reallocation of its non-hub neighbors more difficult because of the limited capacity of surviving hubs.

In a second phase, we consider the general case where k hubs can fails, for a fixed value of k.

In addition to the ring-star network design, the second problem we consider is the hub-and-spoke network design problem. This problem is of particular relevance in the context of air transport, and can be regarded as a relaxation of the ring-star network design problem. In this variant, there is no cycle to build to join the hubs, as we consider that any pair of nodes can communicate. Given a weighted complete graph, the problem consists in selecting a subset of nodes as hubs so as to minimize the exploitation costs while ensuring that each hub carries at least a given percentage of the total traffic. As with the ring-star network design problem, we plan to investigate the resilience of mono and bi-objective problem variants.

In both these problems, networks are designed knowing that some of their elements may fail. Hence, they should be easily fixable. In [3, 4, 5] the most vital elements of a network are sought. Given an optimization property defined on a graph, the problem consists in finding a subset of k edges/nodes whose deletion from the graph induces the maximum degradation of the optimal objective value. A complementary problem, called Min Blocker, aims at determining a subset of edges/nodes of minimum cardinality such that the value of the optimal solution is at least or at most a given threshold. In this thesis, we will study these aspects for the ring-star network design and hub-and-spoke problems.

The privileged tools for addressing all these problems are integer linear programming, decomposition methods [6] (in particular Benders and Dantzing-Wolfe decomposition), exact and approximate algorithms, and robust optimization.

2 Practical aspects of the PhD program

This PhD grant is funded by LAMSADE CNRS UMR 7243 Laboratory, in Université Paris-Dauphine (France). The PhD student will be primarily located in Université Paris-Dauphine, but will work in a challenging international context. He or she will be offered the opportunity to spend a couple of months in the team of Dr. Fabian Castaño in Pontificia Universidad Javeriana in Cali, Colombia, and possibly in University of Bergen (Norway) in the team of Prof. Dag Haugland. The PhD student will be proposed to teach computer science in Université Paris-Dauphine.

The expected wage is around €1600 per month if the applicant is willing to teach. As per the French standards, the PhD thesis duration is exactly three years, starting in the fall of 2020.

A very good level in English is required. In particular, the successful candidate should be able to write scientific articles in English. No skill in French language is required, but non French-speaking candidates should be prepared to learn French in order to teach, and to ease their everyday life in Paris.

The applicants are expected to have a strong background in programming in at least one of the following languages: C, C++, Python, Julia JuMP. He or she must be familiar with at least one MILP solver like IBM CPLEX, Gurobi, or GLPK, and should be able to build mathematical models, analyze them and discuss their relevance.

A good sense of organization and methodology is required, some experience of GitHub and bash programming in Linux would be appreciated. Some experience with LaTeX and in particular Beamer and TikZ would also be an advantage, as these tools will be used extensively.

3 How to apply

Applicants are expected to send their CV with their grades transcripts of the last five years, with one or more reference letters to `andre.rossi@dauphine.psl.eu`. All documents should be in PDF or JPEG format.

References

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