Thesis Poposal
Multiobjective Network Design and Polyhedra
A. Ridha Mahjoub, Hassene Aissi

Context: Polyhedral approaches are powerful techniques for solving and analysing hard combinatorial optimization problems. The purpose of this project is to develop efficient polyhedral based algorithms for some multicriteria network design problems. It also aims to demonstrate that the general framework of polyhedral techniques can be used to formulate and solve hard multicriteria combinatorial problems.

Polyhedral approaches : Pioneered by the work of Jack Edmonds, polyhedral approaches have shown to be powerful tools for formulating, analysing and solving hard combinatorial optimization problems. They may often lead to polynomial time algorithms providing exact or approximate solutions, efficiently solve hard combinatorial problems and provide nice structural min-max relations. Polyhedral approaches consist of reducing the problem to the resolution of a linear program by describing (completely or partially) the convex hull of its solutions by a linear system of inequalities. The equivalence between separation and optimization over a polyhedron and the evolution of computational tools has permitted an important development of these methods (see [7],[10]).

Multiobjective Netwok Design and Survivability: Multicriteria Combinatorial Optimization (MCO) concerns combinatorial optimization problems which involve mutually conflicting objectives. During the last decade, MCO has seen a huge development from both theoretical and practical points of view. It has applications in Operations Research, Economics, Finance and many other domains. Many MCO problems have been considered in the literature and various solution approaches have been devised (see e.g., [4]).

Survivability is the important step in the design of telecommunication and transportation networks. Network survivability means the ability to restore service in the event of a catastrophic failure of a network component, such as the complete loss of a link or the failure of a node. Designing a network meeting requirements concerning traffic survivability is a major challenge with great economic impact. Survivability requirements are generally formulated in terms of network connectivity [6].

Consider a graph G=(V,E). Suppose for each pair of nodes (i,j) there is a nonnegative integer $r(i,j)$, called connectivity type. This represents the importance of communication between i and j . Each pair of nodes (i,j) where $r(i,j) > 0$ corresponds to a demand in the network and the nodes i and j are respectively the origin and the destination of the demand. The survivability conditions require that between every pair of nodes (i,j) there are at least $r(i,j)$ disjoint paths Consider k edge cost functions $c^1,\ldots,c^k : E \to R$ and k-1 bounds $b^1,\ldots,b^{k-1}$, where k is a fixef integer. The multiobjective survivable network design problem is to determine a subgraph with minimum c^k cost satisfying both the survivability conditions and the budget constraints $c^i(x) \le b^i$ for i=1,$\ldots$,k-1. Here x is the 0-1 vector representing the solution. This problem is also called the *budgeted form* of survivable network design problem. More generally, for a combinatorial optimization problem, we can consider its budgeted form given by:

$$Q : \min c^k(x)$$
subject to
$$x \in X$$
$$c^i(x) \le b^i \text{ for } i=1,\ldots,k\text{-}1,$$

Here X is the set of the feasible solutions. Some problems have been studied in budgeted from like the minimum spanning tree and the matching problems [5,8]. Although the single-criterion version for some problem may be polynomially solvable, the new problem, with additional constraints, may be (strongly) NP-hard (since it contains a knapsack problem as a sub-problem).

Solving problem *Q* is an important ingredient in any algorithm for enumerating all the nondominated solutions. These algorithms solve iteratively Q using standard packages (like Cplex) without expoiting the combinatorial structure of problem. The polyhedral approaches could be a suitable and powerful tool for tackling such a problem (see [9] for a study on the budgeted shortest path problem) and, in consequence, enumerating all the nondominated solutions.

For the mutiobjective survivable network design problem, the set *X* is characterized by the constraints:

$$x(\delta(W)) \geq con(W) \text{ for all } W \subset V, \ W \neq \phi,$$
$$x(e) \in \{0,1\} \text{ for all } e \in E,$$

where $con(W) = \min \{r(W), r(V \backslash W)\}$ with $r(W) = \max(r(v),, v \in W)$. Optimizing over that polyhedron can be done in polynomial time.

Goals of the project : The purpose of the project is then to design effective polyhedral based algorithms for solving the survivable network design problem. The project may also be extended to deal with other network design variants considering, in addition to the survivability aspect, the routing and the dimensionning aspects with some QoS as well as more academic combinatorial optimization problem like the multiobjective minimum spanning tree problem and the multiobjective matching problem. In particular, we plan to develop the following for the problems that will be considered:
extended formulations, iterative rounding, polyhedral analysis, valid inequalities, separation algorithms, branch-and-cut-and-price algorithms, computational study.

Related work done by the supervisors: In [1], we discuss the linear relaxation of the problem. We give an excluded minor characterization of the graphs for which the linear relaxation is integral. In [2] we investigated the enumeration of the nondominated solutions of the mutiobjective global minimum cut problem. A strongly polynomial time algorithm is proposed for enumerating all the nondominated solutions for k=2. In [3] the budgeted global min cut problem is considered. We give efficient randomized algorithms for different variants of the problem.

Profile of the candidate : The candidate should have background in combinatorial optimization and mathematical programming with a high computational skill.

References

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