

**Thesis Topic :** *Extension of pre-solutions and price of extension in graphs.*

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**Key words :** Computational complexity and parameterized complexity, approximation, graph classes

## **Description of the topic :**

Most of the problems in combinatorial optimisation are quite difficult from the computation point of view. One of the principal objectives of modern theoretical computer science consists of overcoming this algorithmic barrier by proposing either approximate solution or by providing additional information on the assumptions. The objective of this thesis is to study the impact on the computational complexity (or parameterized complexity) of a problem under the additional condition of extending a particular partial solution. The extension of pre-solutions is already present in diverse techniques of computer science : a search tree algorithm which solves an optimisation problem is often based on the extension of solutions in order to prune the branches of the search tree as soon as possible. Also, for example, for the Vertex Cover problem (VC for short. Here, one tries to cover the edges of a graph with a small number of vertices), it is desirable to be able to quickly (i.e., in polynomial time) decide whether a subset of vertices (which we call a pre-solution) can form a part of a minimum solution. This type of consideration remains interesting even in the extreme case of deciding whether a particular vertex belongs to an optimal vertex cover. Moreover, this approach is already used when one wishes to enumerate the set of minimal vertex covers (with respect to inclusion) or more generally the set of minimal feasible solutions of an optimisation problem (with respect to a specific partial ordering) [9, 10]. Finally, the extension of pre-solutions play a leading role in the techniques based on dynamic programming. There exists in the literature several articles considering the extension of pre-solutions, but in a sporadic manner [3, 6, 7, 8, 11, 11, 15]. Very recently, in [?] and [16] two theoretical frameworks had been proposed for the study of extensions of pre-solutions. In these frameworks, it is assumed that the problems of optimisation are well-structured, namely, there exists a partial order on the set of realisable solutions. For example, for *hereditary* problems (resp. *anti-hereditary*), the underlying order is inclusion (resp., exclusion). In the case of inclusion, the objective is to extend a part of the solution in a manner satisfying a given criteria of robustness compatible with the partial order. The most natural criteria to satisfy is the *Pareto dominance* [?], which results in the notion of minimality. In other words, the removal of elements of a solution no longer guarantees its realisability. On the other hand, the most difficult criteria to verify is *optimality* or *quasi-optimality* [16, 11].

In this thesis, we wish to continue studying these two problems and extending them to the other types of partial orders on the set of realisable solutions such as for example lattices.

A concrete example concerns the existence of a vertex cover  $S \subseteq V$  of a graph  $G = (V, E)$  containing a specific subset  $U \subseteq V$  which is minimal by inclusion; In other words, we would like to extend  $U$  to a minimal vertex cover  $S$ . It turns out that this decision problem, called EXT VC, is **NP**-complete for very particular graph classes such as bipartite, cubic or planar, but remains decidable (in linear time) for chordal or arc-circular graphs [5]. A characterisation of forbidden structures for the existence of extensions of minimal covers is also proposed for trees in [5]. Such an approach deserves to be studied for the extension of other problems. This framework permits to study several interesting questions from the graph-theoretic point of view. For example, the existence of two maximal disjoint independent sets in graph studied in [14] can be reformulated as the extension of a vertex cover in the particular case where  $U$  is a maximal independent set.

The existence of a minimal feasible solution containing a specific subset  $U$  is not always possible since the associated decision problem is **NP**-complete. Also, the question of the existence of such extensions is interesting and leads to the study of a new type of optimisation problem whose objective consists in finding a largest subset of vertices  $U'$  included in  $U$  allowing, in polynomial time, of extending  $U'$  to a minimal feasible solution. This problem is defined in [5] as the *price of extension*, and contains as a particular case the study of the largest feasible minimal solution [1, 4, 5, 11, 12]. For example, the cost of extending VC contains the study of the approximation of UPPER VC as a particular case when  $U = V$  [2, 13].

## Conditions :

The candidate must meet the following conditions

- A master degree either in mathematics or computer science, ideally with specialisation either in algorithms or combinatorial optimisation
- A good knowledge of complexity theory, approximation algorithms and graph theory
- Good knowledge of English (written and spoken)

## Applications :

The applications with the required documents must be sent to : [jerome.monnot@dauphine.fr](mailto:jerome.monnot@dauphine.fr) and Ararat Harutyunyan before middle of April 2019.

The following documents must be attached to the application :

- A short letter of motivation
- A curriculum vitae ;
- Transcripts (including that of the Master degree studies). Bachelors and Master's diplomas
- Two letters of recommendation if possible.

*A second selection will be carried out by the PhD program and the laboratory based on an interview on May, 2019.*

## Références

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