Discrete representation of the Pareto set in multi-objective optimization

PhD thesis proposal

Place: LAMSADE, Université Paris-Dauphine, France

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Description

In multi-objective optimization, there is typically no optimal solution, *i.e.* one that is best for all the objectives. The solutions of interest are such that any solution which is better on one criterion is necessarily worse on at least one other criterion. The vectors of criterion values associated to those solutions are called *non-dominated points*. In multi-objective optimization, one of the main difficulties is the large cardinality of the set of non-dominated points (or *Pareto set*), which can be exponential with the size of the instance (see, e.g., [4]) or infinite. Therefore, it might be necessary to determine a *good representation* of the Pareto set so as to provide decision makers with a tractable set of points describing as well as possible the different choices (see, e.g., [2, 3, 6]). As outlined in [5, 9], the quality of a discrete representation of the Pareto set can be measured through three relevant dimensions: *coverage* which ensures that any non-dominated point is represented or covered by at least one point in the representation, *spacing*, also called *uniformity*, which ensures that any two points in the representation are sufficiently spaced, avoiding redundancies, and *cardinality* which should be minimal so as to make the representation as tractable as possible.

Coverage is the most important dimension for the representation to be meaningful. A natural way to deal with it is to consider the $(1+\varepsilon)$-dominance relation, which is an extension of the Pareto dominance relation including a tolerance threshold ε. Given $\varepsilon > 0$, a point y is said to $(1+\varepsilon)$-*dominate* another point y' if y is at least as good as y' within a factor $1+\varepsilon$ for all the objectives. This leads to the concept of ε-*Pareto set*, which is a set P_ε of points such that for any non-dominated point y' there exists a point $y \in P_\varepsilon$ which $(1+\varepsilon)$-dominates y' [8]. However there may exist many ε-Pareto sets, some of which can include redundancies and some of which can have a more or less small size. In order to avoid redundancies, the concept of $(\varepsilon, \varepsilon')$-kernel has been recently introduced [2]. It is a particular ε-Pareto set that satisfies an additional property of stability which imposes that the points in this set have to be pairwise independent relatively to the $(1+\varepsilon')$-dominance relation, thus controlling spacing. In the bi-objective case, it is shown that an $(\varepsilon, \varepsilon')$-kernel is a good discrete representation of the Pareto set, with guarantees on the three dimensions, which can be generated efficiently [2]. However, for more than two objectives, imposing a condition of spacing may impact negatively on the cardinality [2].

Objectives

The purpose of the PhD thesis is to produce discrete and tractable representations of the Pareto set for multi-objective optimization problems that satisfy some conditions of coverage, spacing, and cardinality. For more than two objectives, since a coverage condition must necessarily be imposed, the choice is between emphasizing spacing or cardinality. If the condition on spacing prevails it has been shown that it is possible to construct an $(\varepsilon, \varepsilon')$-kernel under certain conditions, but without any guarantee on its cardinality [2]. If the condition on cardinality prevails, known guarantees are very weak, even without any condition on spacing [1, 7]. Obtaining good representations of the Pareto set for more than two objectives is therefore a challenging open question.

From a practical point of view, it would be interesting to design efficient algorithms for more than two objectives, that would focus either on spacing or cardinality. The proposition and the implementation of such algorithms would be a part of the PhD thesis. Another extension of interest would be to integrate preference information so as to produce *personalized* representations. Determining (personalized) representations for specific problems, like multi-objective combinatorial optimization problems, is also a challenging issue.

Position

The position is fully funded for 3 years. There is no obligation to teach, but it will be possible (with additional payment). Funding for attending international conferences and visiting other research centers will also be provided. Starting date is expected in September-October 2020.

Candidate Profile

The ideal candidate has a keen interest and strong background in operations research, computer science, discrete mathematics, or related fields as evidenced by a Masters degree or equivalent. He/she should have excellent academic qualifications. Skills in programming are required. Knowledge in multiobjective optimization or approximation theory would be appreciated.

Application

Any interested candidate is invited to email his/her application (including CV, letter of motivation, academic record during the last two years and possibly the name and email of reference persons) to lucie.galand@dauphine.fr and daniel.vanderpooten@dauphine.fr by **April 19th, 2020**.

References

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