Algorithmic aspects of intersection graphs

PhD. Thesis Proposal

January 2021

Context

An intersection graph is a graph representing intersections between a family of sets. Each set is associated to a vertex of the graph and two vertices are linked if the corresponding sets intersect. Any graph can be represented as an intersection graph, but some important special classes of graphs can be defined by the types of sets that are used to form an intersection representation of them. The intersection graph of *intervals on the real line* is denoted Interval Graph and is probably one of the most well-known graph class. These graphs have been widely used to study scheduling problems (other applications include assembling contiguous subsequences in DNA mapping, and temporal reasoning) and most classical and important combinatorial NP-hard problems are solvable in polynomial time in these graphs. Other well-known intersection graphs include disk graphs, circle graphs, permutation graphs and so on.

Subject and expected results

We propose to deepen the study of variants and generalizations of interval graphs such as multiple interval graphs and multiple tracks interval graphs. Many structural questions are of interest on such graphs. For example, what is the smallest t s.t. a graph is the intersection graph of sets of at most t intervals of the real line (the interval number)? Some questions remain open [2]. Simultaneous representation contains also interesting questions [5].

Also, we propose to study classical optimization problems on such graphs. When a problem will be proven hard (i.e. no exact polynomial-time solution can be found), we will focus on approximation (i.e. a "fast" algorithm giving a solution with a guarantee on the error, like [1]) and parameterized complexity (i.e. an algorithm giving an exact solution but where the unavoidable combinatorial explosion is confined to a parameter supposed small in practice, like [4]). Finding relevant parameters for these graphs will be of particular importance here.

Even when the problem is known to be polynomial-time solvable, the complexity can be too high for practical applications (as the known $O(n^4)$ algorithm for finding the longest path in an interval graph). Again, parameterized complexity framework will be used (called "FPT in P") to reduce the polynomial in n, to the price of an higher function depending on a parameter only (see for example [3]).

Direction

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Requirements

The candidates must have :

- a Master degree in Computer Sciences or Discrete Mathematics, ideally with specialization in algorithms or combinatorial optimization.
- a good knowledge in Complexity Theory, Graph Theory, Parameterized Algorithms. Knowledge in some programming language and in mathematical programming is a plus.
- Good English skills for sciences. Knowledge in French is welcome for the teaching part.

Application (to be sent before the 30 of April by email) with :

- A short cover letter stating the motivations to apply
- A detailed CV
- Bachelor and Master grades
- 1 or 2 reference letters if possible (sent directly to the future advisors).

References

[1] Ayelet Butman, Danny Hermelin, Moshe Lewenstein, and Dror Rawitz. Optimization problems in multiple-interval graphs. *ACM Trans. Algorithms*, 6(2):40:1–40:18, 2010.

[2] Aquiles Braga de Queiroz, Valentin Garnero, and Pascal Ochem. On interval representations of graphs. *Discret. Appl. Math.*, 202:30–36, 2016.

[3] Archontia C. Giannopoulou, George B. Mertzios, and Rolf Niedermeier. Polynomial fixed-parameter algorithms: A case study for longest path on interval graphs. *Theor. Comput. Sci.*, 689:67–95, 2017.

[4] Minghui Jiang. On the parameterized complexity of some optimization problems related to multiple-interval graphs. *Theor. Comput. Sci.*, 411(49):4253–4262, 2010.

[5] Ignaz Rutter, Darren Strash, Peter Stumpf, and Michael Vollmer. Simultaneous representation of proper and unit interval graphs. In Michael A. Bender, Ola Svensson, and Grzegorz Herman, editors, *27th Annual European Symposium on Algorithms, ESA 2019, September 9-11, 2019, Munich/Garching, Germany*, volume 144 of *LIPIcs*, pages 80:1–80:15. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2019.