# SPATIAL MULTICRITERIA DECISION MAKING

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#### **SYNONYMS**

Spatial Multicriteria Decision Aid, GIS-based Multicriteria Decision Analysis

## DEFINITION

Multicriteria analysis is generally defined as "a decision-aid and a mathematical tool allowing the comparison of different alternatives or scenarios according to many criteria, often conflicting, in order to guide the decision maker towards a judicious choice" [12]. The set of decision alternatives considered in a given problem is often denoted by A and called the set of potential alternatives. A criterion is a function g, defined on A, taking its values in an ordered set and representing the decision maker's preferences according to some points of view. The evaluation of an alternative a according to criterion g is written g(a).

Spatial multicriteria decision making refers to the application of multicriteria analysis in spatial context where alternatives, criteria and other elements of the decision problem have explicit spatial dimensions. Since the late 1980s, multicriteria analysis has been coupled with *geographical information systems* (GIS) to enhance spatial multicriteria decision making.

#### HISTORICAL BACKGROUND

It is generally assumed that multicriteria analysis was born and took its actual vocabulary and form at the beginning of 1960s. In fact, most of multicriteria analysis practitioners consider that their field stems largely from the research of Simon on satisficing and the early works on goal programming. Closely related to decision-making in general and to multicriteria analysis in particular is utility theory. Although utility theory was first used to model simple individual preferences, it has been extended to the multicriteria preferences and led to the *multiattribute utility theory* [7].

The first methods in multicriteria analysis were developed during the 1960s. Goal programming, for example, uses the linear programming to resolve a multicriteria problem. In 1968, Roy conceived the initial version of ELECTRE method (see [4]).

Throughout the 1970s, the widely dispersed scientific field of multicriteria analysis started to take form. First, in 1971 Roy organized the first independent session specifically devoted to multicriteria research within the 7th Mathematical Programming Symposium, held in The Hague. Second, in 1972 Cochrane and Zeleny organized the First International Conference on multicriteria decision making at the University of South Carolina. Then in 1975, Roy organized in Brussels the first meeting of the EURO Working Group on Multi-Criteria Decision Aid. Also in 1975, Thiriez and Zionts organized the First Conference of the International Society on multicriteria analysis. In addition to these first scientific meetings, the multicriteria analysis research focused in the 1970s on the theoretical foundations of *multiobjective decision making*.

The 1980s and 1990s witnessed the consolidation and development of a great number of interactive methods. Most of these methods are oriented toward the negotiation or multiple decision makers and multicriteria decision support systems.

Multicriteria analysis has been used since its emergence to deal with spatial decision problems. The first works involving GIS-based multicriteria analysis where published in the late 1980s and the early 1990s. Currently, there are a number of relatively important devoted to GIS-based multicriteria analysis that have been published [10].

#### SCIENTIFIC FUNDAMENTALS

#### **1** General schema of multicriteria analysis methods

Different multicriteria analysis methods are available in the literature [4]. An excellent online bibliography on multicriteria analysis and its applications is available at http://www.lamsade.dauphine.fr/~mcda/biblio/. Multicriteria methods are commonly categorized as *discrete* or *continuous*, depending on the domain of alternatives. The former deals with a discrete, usually limited, number of pre-specified alternatives. The latter deals with variable decision values to be determined in a continuous or integer domain of infinite or large number of choices. Several authors classify them as (i) *multiple attribute decision-making* (MADM), and (ii) *multiple objective decision-making* (MODM). In this presentation, the discrete/continuous classification is chosen since it is in accordance with the conventional representation of data in the GIS (vector vs. raster) and it is more general than the MADM/MODM classification. Figure 1 gives the general schema of discrete and continuous multicriteria methods that will be briefly described in the two following paragraphs.



Figure 1: General schema of discrete (a) and continuous (b) multicriteria methods

## 1.1 Discrete methods

The first requirement of nearly all discrete techniques is a *performance table* containing the evaluations or *criteria* scores of a set of alternatives on the basis of a set of criteria. The next step consists of the aggregation of the different criteria scores using a specific decision rule (or aggregation procedure). It takes into account the decision maker's preferences, generally represented in terms of weights that are assigned to different criteria. The aggregation of criteria scores permits the decision maker to make a comparison between the different alternatives on the basis of these scores. The aggregation procedures represent the identities of the multicriteria analysis techniques. The discrete methods are usually categorized into two different families: (1) an outranking relation-based decision rules.

The uncertainty and the fuzziness generally associated with any decision situation require a *sensitivity/robustness* analysis enabling the decision maker(s) to test the consistency of a given decision or its variation in response to any modification in the input data and/or in the decision maker preferences.

# 1.2 Continuous methods

The starting point of most continuous methods is a set of *constraints* and a set of *objective functions*. The former set contains inequalities which reflect natural or artificial restrictions on the values of the input data. This means that *feasible solutions* are *implicitly* defined in terms of these constraints.

For the continuous methods, the decision maker's preferences generally take the form of *weights* that are assigned to different objective functions. They may also be represented as *target values* that should be satisfied with any feasible solution. The decision maker should also indicate, for each objective function, its *direction of optimization*, that is maximization or minimization. No other information than the weights and these directions of optimization are required to define the set of *non-dominated solutions*. This set contains solutions that are not dominated by any other one.

Generally, *local* and *interactive* aggregation algorithms are used to define the feasible solutions set. This permits the combination of the decision maker preferences and the computer to solve the decision problem, using methods that alternate calculation steps and dialogue steps. In reality, the local and interactive algorithms require the decision maker preferences to be expressed *progressively* during all the resolution process. The decision maker preferences, however, may be expressed *a priori* (i.e., before the resolution process) or *a posteriori* (i.e., after the resolution process).

In many practical situations, the decision maker is called upon to relax some of its constraints in order to guarantee that the set of feasible solutions is not empty or, simply, to test the stability of the results.

## 2 Spatial multicriteria decision making

A brief description of spatial multicriteria decision making concepts is provided in the following. In the rest of this entry,  $F = \{1, 2, \dots, m\}$  denotes the set of the indices of m evaluation criteria  $g_1, g_2, \dots, g_m$ . Accordingly,  $g_j$   $(j \in F)$  is the evaluation criterion number j.

## 2.1 Spatial decision alternatives

Decision alternatives can be defined as alternative courses of action among which the decision maker must choose. A spatial decision alternative consists of at least two elements [9]: *action* (what to do?) and *location* (where to do it?). The spatial component of a decision alternative can be specified *explicitly* or *implicitly* [10]. The second case holds when there is a spatial implication associated with implementing an alternative decision.

The set of spatial decision alternatives may be discrete or continuous. In the first case, the problem involves a discrete set of pre-defined decision alternatives. Spatial alternatives are then modeled through one or a combinaison of the basic spatial primitives, namely point, line, or polygon. The second case corresponds to a high or infinite number of decision alternatives, often defined in termes of constraints. For practical reasons, the set of potential alternatives is often represented in a "discretized" form where each raster represents an alternative. Alternatives may be constructed as a collection of rasters.

#### 2.2 Evaluation criteria

In the spatial context, evaluation criteria are associated with geographical entities and relationships between entities, and can be represented in the form of maps. One should distinguish a simple map layer from a *criterion map*. In fact, a criterion map models the preferences of the decision maker concerning a particular concept, while a simple map layer is a representation of some spatial real data. A criterion map represents subjective preferential information. Two different persons may assign different values to the same mapping unit in a criterion map.

# 2.3 Constraints

A constraint (or admissibility criterion) represents natural or artificial restrictions on the potential alternatives. Constraints are often used in the pre-analysis steps to divide alternatives into two categories: "acceptable" or "unacceptable". An alternative is acceptable if its performance on one or several criteria exceeds a minimum or does not exceed a maximum.

In practice, constraints are often modeled through elementary multicriteria methods like the *conjunctive* or *disjunctive* aggregation procedures. With the conjunctive method, a *minimal satisfaction level*  $\hat{g}_j$  is associated with each criterion  $g_j$ . If the performance of an alternative with respect to different criteria is equal or better to

these minimal satisfaction levels (i.e.,  $g_j(a_i) \ge \hat{g}_j, \forall j \in F$ ), the alternative is considered as acceptable. Otherwise, the alternative is considered as unacceptable. With the disjunctive method, the alternative is considered acceptable as soon as at least one satisfaction level is exceeded.

## 2.4 Quantification

The evaluation of alternatives may be quantitative or qualitative. Several methods require quantitative evaluations. In the literature, there are some totally qualitative methods such as the median ranking method. Other methods, such as the ELECTRE family of methods (see [4]), involve the two types of evaluations. When most of criteria are qualitative, quantitative criteria may be converted into qualitative ones and a qualitative method is used. Otherwise, a *quantification method* (i.e., assignment of numeric values to qualitative data) is applied; the *scaling approach* is the most used one.

Application of a quantification method requires the definition of a measurement scale. The most used measurement scale is the *Likert-type*. This scale is composed of approximatively the same number of favorable and unfavorable levels. An example with five levels is: *very unfavorable, unfavorable, neutre, favorable, very favorable.* Other more detailed measurement scales may also be used. The quantification procedure consists of constructing a measurement scale like the one with five points mentioned above. Then, numerical values are associated with each level of the scale. For instance, the numbers 1, 2, 3, 4 or 5 may be associated with the five-point scale from *very unfavorable*.

#### 2.5 Standardization

The evaluation of alternatives may be expressed according to different scales (ordinal, interval, ratio). However, a large number of multicriteria methods (including practically all the utility function-based methods) require that all of their criteria are expressed in a similar scale. Standardizing the criteria permits the rescaling of all the evaluation dimensions between 0 and 1. This allows between and within criteria comparisons.

There are a large number of standardization procedures. In all of them, standardization starts from an initial vector  $(g_j(a_1), g_j(a_2), \dots, g_j(a_m))$  to obtain a standardized vector  $(r_{1j}, r_{2j}, \dots, r_{mj})$  with  $0 \le r_{ij} \le 1$ ;  $\forall j \in F$  and  $i = 1, \dots, n$  (*n* is the number of alternatives). The most used standardization procedure in the GIS-based multicriteria decision making is the *linear transformation procedure*. It associates with each alternative  $a_i$  and for each criterion  $g_j$  the percentage of the maximum over all alternatives:

$$r_{ij} = \frac{g_j(a_i)}{\max_i g_j(a_i)}, i = 1, \cdots, n; j \in F.$$

#### 2.6 Pre-analysis of dominance

In the absence of any preferential information, the only possible operation on the performance table is to eliminate the dominated alternatives. Let a and b be two alternatives from A and F a family of criteria. The alternative adominates the alternative b in respect to F, noted  $a\Delta b$ , if and only if:

$$g_j(a) \ge g_j(b); j \in F,$$

with at least one strict inequality. Then, an alternative a from A is said to be *efficient* or *admissible* or *Pareto* optimal if and only if there is no other alternative b in A such that:  $b\Delta a$ .

#### 2.7 Criteria weights

Generally in multicriteria problems the decision maker considers one criterion to be more important than another. This *relative importance* is usually expressed in terms of numbers, often called *weights*, which are assigned to different criteria. These weights deeply influence the final choice and may lead to a non-applicable decision mainly when the interpretations of such weights are misunderstood by the decision maker.

In the literature, many direct weighting techniques have been proposed. When a simple arrangement technique is used, the decision maker sets the criteria in an order of preference. The cardinal simple arrangement technique involves each criterion being evaluated according to a pre-established scale. Some other indirect methods are also available such as the *interactive estimation method*. There are also a relatively complex weight assignment techniques such as the *indifference trade-offs* technique [7] and the *analytic hierarchy process* (AHP) [13].

#### 2.8 Preference structure and preference parameters

When comparing two alternatives a and b, the decision maker will generally have one of the three following reactions: (i) preference for one of the two alternatives, (ii) indifference between the two alternatives or (iii) impossibility to compare the alternatives. These situations are generally denoted as follows: (i) aPb if a is preferred to b (bPa if it is the opposite), (ii) aIb if there is indifference between a and b, and (iii) aRb if there is an incomparability. The binary relations of preference P, indifference I, and incomparability R are respectively the sets of tuples (a, b) such that aPb, aIb, aRb. It is generally admitted that I is reflexive and symmetric, P is asymmetric, and R is irreflexive and symmetric. The three relations (I, P, R) constitutes a structure of preference over A if and only if they have the properties mentioned above and only one of the following situations holds [14]: aPb, bPa, aIb, aRb.

Preference models require the definition of one or several thresholds, called *preference parameters*. The most used preference parameters are the *indifference*, *preference* and *veto* thresholds. These three parameters are used essentially within the outranking relation-based decision rules. The first two parameters for modeling imprecision and uncertainty in the decision maker's preferences. The latter is often used to compute the *discordance index*.

## 2.9 Decision rules

To compare alternatives in A, it is necessary to aggregate the *partial evaluations* (i.e., with respect to each criterion) into a global one by using a given *decision rule* (or *aggregation procedure*). As mentioned earlier, within the discrete family, there are usually two aggregation approaches: (i) *utility function-based approach*, and (ii) *outranking relation-based approach*. The basic principle of the first family is that the decision maker looks to maximize a utility function  $U(a) = U(g_1(a), g_2(a), \dots, g_m(a))$  aggregating the partial evaluations of each alternative into a global one. The simplest and most often used utility function has an additive form:  $U(a) = \sum_{j \in F} u_j(g_j(a))$ ; where  $u_j$  are the partial utility functions. Within this form, the preference P and indifference I binary relations are defined for two alternatives a and b as follows:

$$aPb \Leftrightarrow U(a) > U(b)$$
 and  $aIb \Leftrightarrow U(a) = U(b)$ 

In contrast with the first family, the second one uses *partial aggregation procedures*. Different criteria are aggregated into a partial binary relation S, with aSb used to indicate that "a is at least as good as b". The binary relation S is called an *outranking relation*. The most well known method in this family is ELECTRE (see, e.g., [4]). To construct the outranking relation S, for each pair of alternatives (a, b), a concordance index  $C(a, b) \in [0, 1]$ —measuring the power of criteria that are in favor of the assertion aSb—and a discordance index  $ND(a, b) \in [0, 1]$ —measuring the power of criteria that are opposed to aSb—are computed. Then, the relation S is defined as follows:

$$\begin{cases} C(a,b) \ge \hat{c} \\ ND(a,b) \le \hat{d} \end{cases}$$

where  $\hat{c}$  and d are the concordance and the discordance thresholds, respectively. Often an exploitation phase is needed to extract from S information on how alternatives compare to each other. At this phase, the concordance C(a, b) and discordance ND(a, b) indices are used to construct an index  $\sigma(a, b) \in [0, 1]$  representing the credibility of the proposition aSb,  $\forall (a, b) \in A \times A$ . The proposition aSb holds if  $\sigma(a, b)$  is greater or equal to a given cutting level,  $\lambda \in [0.5, 1]$ .

In the continuous formulation of a multicriteria problem, decision rules implicitly define the set of alternatives in terms of a set of *objective functions* and a set of *constraints* imposed on the decision variables. Here, *multiobjective mathematical programming* is often used. A multiobjective mathematical program is a problem where the aim is to find a vector  $\mathbf{x} \in \mathbf{R}^p$  satisfying constraints of type

$$h_i(\mathbf{x}) \leq 0; (i=1,2,\cdots,n),$$

respecting eventual integrity conditions and optimizing the objective functions:

$$z_j(\mathbf{x}), j = 1, 2, \cdots, m$$

The general form of a multiobjective mathematical program is as follows:

$$\begin{cases} \text{Optimize} & [z_1(\mathbf{x}), z_2(\mathbf{x}), \cdots, z_m(\mathbf{x})] \\ h_i(\mathbf{x}) \le 0 & (i = 1, \cdots, n) \\ \mathbf{x} \in X \end{cases}$$

A multiobjective mathematical program is in fact a multicriteria decision problem where [14]: (i)  $A = \{\mathbf{x} : h_i(\mathbf{x}) \leq 0, \forall i\} \subset \mathbf{R}^p$  is the set of decision alternatives, and (ii)  $F = \{z_1(\mathbf{x}), z_2(\mathbf{x}), \dots, z_m(\mathbf{x})\}$  is a set of criteria where each criterion is expressed by an objective function in terms of the decision variables.

#### 2.10 Sensitivity/Robustness analysis

The analysts should examine, through *sensitivity analysis*, the stability of results with respect to the variation of different parameters. Sensitivity analysis is the base for *robustness analysis*. There are several proposals to enhance GIS-based multicriteria decision making with sensitivity analysis procedures (e.g., [3]). Robustness analysis in multicriteria decision making is a relatively recent research topic. Proposals for enhancing GIS-based multicriteria decision making analysis are still lacking.

# 2.11 Final recommandation

The final recommendation in multicriteria analysis may take different forms, according to the manner in which a problem is stated. Roy [12] identifies four types of results corresponding to four ways for stating a problem: (i) *choice*: selecting a restricted set of alternatives, (ii) *sorting*: assigning alternatives to different predefined categories, (iii) *ranking*: classifying alternatives from best to worst with eventually equal positions or (iv) *description*: describing the alternatives and their follow-up results.

#### **KEY APPLICATIONS**

GIS-based multicriteria analysis is used in a wide range of decision and management situations. In a recent literature review, Malczewski [10] enumerates about 319 papers devoted to GIS-based multicriteria analysis between 1990 and 2004. The complete list of these papers is available at http://publish.uwo.ca/~jmalczewski/gis-mcda.htm. The following list enumerates the major domain applications of GIS-based spatial multicriteria decision making.

## Environment planning and ecology management

GIS-multicriteria evaluation have been used intensively in environment planning and ecology management. Most analyses within this application area concern land suitability, resource allocation, plan/scenario evaluation, impact assessment and site search/selection problems.

#### Transportation

Within the transportation application domain, GIS-based multicriteria evaluation is used essentially in vehicle routing and scheduling, and land suitability problems.

#### Urban and regional planning

Major uses of GIS-multicriteria analysis in urban and regional planning concern resource allocation, plan/scenario evaluation, site search/selection and land suitability problems.

#### Waste ressource management

The problems tackled in this application domain concern land suitability, plan/scenario evaluation and site search/selection.

# Hydrology and water resources

In the hydrology and water resources application domain, GIS-multicriteria analysis is used essentially for plan/scenario evaluation. There are also some works for site search/selection and land suitability problems.

#### Forestry

Major problems tackled within the forestry application domain are land suitability, site search/selection and forestry resources allocation.

## Agriculture

The problems considered here are essentially land suitability for different agricultural uses and resources allocation for agricultural activities. Some works are concerned with site seach/selection and plan/scenario evaluation problems.

## Natural hazard management

The problems considered within this application domain concern mainly land suitability and plan/scenario evaluation.

## **Recreation and tourism management**

Within this application area, the most treated problem is site search/selection.

## Health care resource allocation

Major works in this application domain concern health care site search and selection.

#### Housing and real estate

The problems that are treated here concern land suitability for habit and real estate, plan/scenario evaluation and site selection for habitation restoration.

#### FUTURE DIRECTIONS

There are many important proposals concerning GIS-based multicriteria spatial decision making. However, these proposals present some limitations that prevent them from going beyond the academic contexts. Some of these limitations are cited in the rest of this section.

## Integration of utility-based decision rules

The major part of GIS and multicriteria analysis integration works use utility-based decision rules. However, outranking relation-based decision rules are generally more appropriate to deal with ordinal aspects of spatial decision problems. The natural explanation for this is that the outranking relation-based decision rules have computational limitations with respect to the number of alternatives they consider [11]. One possible solution to facilitate the use of decision rules based outranking relation is to reduce the number of potential alternatives. The idea that is generally used consists of subdividing the study area into a set of homogenous zones which are then used as decision alternatives or as a basis for constructing these alternatives.

## Spatial and temporal dimensions in multicriteria modelling

Two points need to be addressed here: the construction of criteria involving divergent consequences and the modeling of preferences that vary across time and space. In the literature, there are some papers that deal with the construction of criteria based on divergent consequences and the modeling of time-dependent preferences. With respect to GIS-based multicriteria analysis, there are a few other papers that take these aspects into account [3].

## Fuzzy spatial multicriteria decision making

Malczewski [10] estimates that 77% of the papers that were published between 1990 and 2004 related to GIS multicriteria analysis used deterministic information. There are several plans to incorporate multicriteria methods supporting imprecision, uncertainty and fuzziness in the GIS [6]. The integration of such methods in a geographical information system has the potential to enhance its analytical strength.

## Multicriteria group spatial decision making

Spatial decision problems naturally involve several different kinds of stakeholders. However, the majority of the GIS-multicriteria articles consider individual decision maker's approaches and only a few works (e.g., [5]) are devoted to multicriteria group spatial decision making.

#### Web-based multicriteria spatial decision making

There is increasing interest in the development of Web-based GIS multicriteria evaluation systems [1]. Research on this topic is worthwhile since it promotes the sharing and access of geographical information and facilitates multicriteria collaborative spatial decision making.

#### **CROSS REFERENCES**

Decision-making Effectiveness with GIS GeoSpatial Web Services Internet GIS Modeling with Spatial and Temporal Uncertainty Multicriteria Spatial Decision Support Systems Raster Data Sensitivity Analysis Spatial Data Analysis Temporal GIS and Applications Vector Data

## **RECOMMENDED READING**

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