ELICITING INFORMATION
THE RELATIVE IMPORTANCE OF CRITERIA

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Abstract: The notion of Relative Importance of Criteria (RIC) is central in the domain of Multiple Criteria Decision Aid (MCDA). It aims at differentiating the role of each criterion in the construction of comprehensive preferences, thus allowing to discriminate among pareto-optimal alternatives. In most aggregation procedures, this notion takes the form of importance parameters.

The acquisition of information concerning the RIC may be supported by Elicitation Techniques for Importance Parameters (ETIP). The design of such techniques should account for both the meaning that each aggregation confers on its parameters and the decision makers' (DMs) understanding of the notion of RIC. More precisely, ETIPs should be able to provide a good fit between the way the analyst uses the DM's assertions in the model and the information that he/she expresses through his/her statements.

In this paper, we present an ETIP adapted to the ELECTRE methods that proceeds by means of pairwise comparisons of fictitious alternatives. Implemented in a software program called DIVAPIME, this ETIP supports the elicitation of variation intervals for the ELECTRE methods' preferential parameters.

Keywords: MCDA, Importance of Criteria, Weights, Elicitation, ELECTRE Methods.

Introduction

When the analysis of a decision problem is grounded in the definition of a set of criteria, it is difficult to discriminate between alternatives whose evaluations on several criteria are in conflict. Multiple criteria preference modelling requires that the analyst obtains from the decision maker (DM) some preference information so as to discriminate between pareto-optimal alternatives.

A classical approach to Multiple Criteria Decision Aid (MCDA) consists of linking restricted preferences (corresponding to the n criteria) with the comprehensive preferences (taking all criteria into account) through a so called Multiple Criteria Aggregation Procedure (MCAP). In the MCAP, all criteria are not supposed to play the same role; the criteria are commonly said not to have the same importance. This is why there are parameters in the MCAP that aim at specifying the role of each criterion in the aggregation of evaluations. We will call such parameters importance parameters. They aim at introducing preferential information concerning the importance that the DM attaches to the points of view modelled by the criteria.

The nature of these parameters varies across MCAPs. The way of formalising the relative importance of each criterion differs from one aggregation model to another. All this is done, for instance, by means of:

- scaling constants in Multiattribute Utility Theory (see [Keeney & Raiffa 76,93]),
- a weak-order on F in lexicographic techniques or a complete pre-order on F in the ORESTE method (see [Roubens 82]),
- intrinsic weights in PROMETHEE methods (see [Brans et al. 84]),
- intrinsic weights combined with veto thresholds in ELECTRE methods (see [Roy 91]),
- eigen vectors of a pairwise comparison matrix in AHP method (see [Saaty 80]).

Many authors have studied the problem of the elicitation of the relative importance of criteria (a critical overview may be found in [Mousseau 92]), but few of them have tried to give a precise definition of this notion. A careful analysis of this notion is still needed to build theoretically valid elicitation techniques. This paper aims at highlighting some of the difficulties that may be encountered when eliciting information concerning the relative importance of criteria and presents a way of eliciting such information when preferences are modeled through an outranking relation based on a concordance principle (see [Roy 91]).

In the first section, we will state precisely what the information underlying the notion of importance of criteria is. The second section will be devoted to the analysis of the meaning of an elicitation process. This will lead us to specify some basic requirements for importance parameters elicitation techniques. In the last section, we will present a technique, called DIVAPIME, in order to define a polyhedron of acceptable values for importance parameters in an ELECTRE type method.

1. What does the notion of the relative importance of criteria cover ?

What does a decision-maker mean by assertions such as "criterion g_j is more important than criterion g_i," "criterion g_j has a much greater importance than criterion g_i?", etc. Let us recall, by way of comparison, that the assertion "b is preferred to a" reflects the fact that, if the decision-maker must choose between the alternatives b and a, he is supposed to decide in favor of b. So, it is possible to test whether this assertion is valid or not. There is no similar possibility for comparing the Relative Importance of Criteria (RIC). Moreover, the way this notion is taken into account within the framework of the different models mentioned above by means of importance parameters reveals significant differences in what this notion deals with.

The first statement we should make when trying to analyse the notion of RIC is the following: the information underlying this notion is much richer than that contained in the importance parameters of the various multicriteria models. In fact, these parameters are mainly scalars and constitute a simplistic way of taking RIC into account, as this notion is by nature of a functional type.

In the comparison of two alternatives a and b, when one or several criteria are in favor of a and one or several others in favor of b, the way each MCAP solves this conflict and determines a comprehensive preference (i.e., taking all criteria into account) denotes the importance attached to each criterion (and to the logic of the aggregation used). Thus, the result of such conflicts (i.e., the comprehensive preference situation between a and b) constitutes the elementary data providing information on the relative importance of the criteria in conflict.

When we analyse the RIC notion, it appears that this notion represents a certain form of regularity in the link between restricted and overall preferences. In order to delimit exhaustively the importance of a criterion, we should analyse the contribution of any preference at the restricted level of a criterion to the comprehensive level (for each pair of alternatives). Nevertheless, when two alternatives are indifferent on criterion g_j, the comprehensive preference situation will generally give no significant information concerning the importance of this criterion.

Let us introduce some basic notations:
F={1,2, ..., n} a familly of n criteria g_1, g_2, ..., g_n build so as to evaluate the alternatives contained in a given set (denoted A). Considering the imprecision of the evaluations of alternatives on criteria, it is usually considered that g_1, g_2, ..., g_n are pseudo-criteria (see [Vincke 90]), i.e., such that:

$$\left[\begin{array}{c} a\,P_j\,b \Leftrightarrow g_j(a) > g_j(b) + p_j \\ a\,Q_j\,b \Leftrightarrow q_j < g_j(a) - g_j(b) \le p_j \\ a\,I_j\,b \Leftrightarrow |g_j(a) - g_j(b)| \le q_j \end{array}\right.$$

where - q_j, the indifference threshold, represents the maximum difference of evaluation compatible with an indifference situation;

- p_j, the preference threhold, represents the minimum difference of evaluation compatible with a preference situation;
- aP_jb is to be interpreted as a preference for a over b on criterion g_j;
- aI_jb is to be interpreted as indifference between a and b on criterion g_j;
- aQ_jb is to be interpreted as an hesitation between the two preceding situations.

We will call P_j the partial preference relation on the j^{th} criterion (the same terminology holds for I_j and Q_j). (I_j, Q_j, P_j) defines a pseudo-order.

The comprehensive preferences are modelled through three preference relations: P a preference relation (asymmetric, irreflexive), I an indifference relation (symmetric, reflexive), R an incomparability relation (symmetric, reflexive). For every pair of alternatives (a, b), one and only one of the four following assertions is valid:

$$\left[\begin{array}{l} a\,P\,b \\ a\,I\,b \\ b\,P\,a \\ a\,R\,b \end{array}\right. \quad [1]$$

Let us consider the outranking relation S=P∪I, aSb being interpreted as "*a is at least as good as b*". As aPb ⇒ aSb ⇒ not[bSa], the four assertions of system [1] correspond to the following assertions:

$$\left[\begin{array}{c} a\,P\,b \\ a\,S\,b \text{ and not}[a\,P\,b] \\ b\,P\,a \\ \text{not}[a\,S\,b] \text{ and not}[b\,P\,a] \end{array}\right. \quad [2]$$

When the preference model is of a (I, P, R) type, it does not seem restrictive to delimit the information concerning the relative importance of a criterion using only preference and outranking relations (see system [2]). On such a basis, we will assume that the empirical content of the RIC notion refers to the nature and variety of cases in which a partial preference on a given criterion g_j leads us (i) to accept the same preference on the comprehensive level, or (ii) to accept only an outranking in the same direction, or (iii) to refuse the inverse preference. In other words, the relative importance of criterion g_j is characterized by:

- the set of "*situations*" in which aP_jb and aPb hold simultaneously,
- the set of "*situations*" in which aP_jb and aSb hold simultaneously,
- the set of "*situations*" in which aP_jb and not[bPa] hold simultaneously.

In this characterization, the contribution of criterion g_j to the preferences at the comprehensive level is considered only through the situations of strict preference at the restricted level of criterion g_j. This does not seem restrictive when all criteria are quasi-criteria (such that $p_j=q_j$), but it can become restrictive when F contains pseudo-criteria.

On the basis of the preceding considerations [Roy & Mousseau 95] propose a formal definition of the notion of RIC and a theoretical framework to analyse it. Within this framework, it clearly appears that the importance of criteria is taken into account in very different ways in the various aggregation procedures. In particular, this means that the values attached to importance parameters are meaningless as long as the aggregation rule in which they are used is not specified. Interesting theoretical proposals can also be found in [Podinovski 88, 94].

2. What do we aim at when evaluating the importance of criteria ?

The elicitation of information about RIC is a crucial phase in a decision aid process. Basic assumptions concerning the nature of what is being done during this phase have an impact on the way to proceed to elicit such information.

2.1. The descriptivist and constructivist approaches to MCDA

The way we seek to give meaning to the notion of importance will differ according to which of these two approaches we adopt.

The *descriptivist approach* assumes that the way in which any two alternatives are compared on the comprehensive level (that is, taking all criteria into account) is well-defined in the decision-maker's mind before the modelling process begins. Moreover, it supposes that the modelling process does not modify such comparisons. The preference model chosen is intended to give an account of such pre-existing preferences as objectively as possible. Under these conditions, the role which devolves to each criterion as a function of its importance, is apprehended by the set of values attributed to the importance parameters. Thus, it is the capacity of a model to adjust to a well-defined, real-world situation which confers meaning on the notion of importance and allows to assign a numerical value to these parameters. Several authors would even talk in terms of estimating the numerical value of certain parameters, such as weight w_j. Such language is meaningless unless a true numerical value for w_j exists, in which case the goal would be to estimate this true value as precisely as possible. In such an approach, observed lability in elicited preferences (see [Fishhoff et al. 89] and [Weber & Borcherding 93]) are explained by the existence of several biases in the elicitation techniques. [Beattie & Baron 91] argues that there is a "*distinction between true and estimated weights and it is possible that subjects' true weights remain constant at all times, but become distorted in the elicitation process*."

The *constructivist approach*, on the contrary, assumes that preferences are not entirely pre-formed in the decision-maker's mind and that the very nature of the work involved in the modelling process (and, *a fortiori*, in decision-aid) is to specify and even to modify pre-existing elements. The multiple criteria aggregation procedure (MCAP), which underlies the preference model chosen, is thus nothing other than a set of rules deemed appropriate for aggregating the evaluations $g_j(a)$ and for building comprehensive preferences. Under these conditions, the numerical values assigned to importance parameters reflect a working hypothesis accepted for decision-aid. These are convenient numerical values with which it seems reasonable and instructive to work. The importance parameters, therefore, can be seen as keys which allow us to differentiate the role played by each criterion in the preference model selected for use. "True numerical values," to which we can refer to give meaning to the language of estimation, do not necessarily exist. Importance parameters and the numerical values we assign to them are, nonetheless, instruments for reasoning, investigating and communicating among the stakeholders in a decision-making process. The values (or interval of variation) for these parameters reflect, in the MCAP selected, a certain number of assertions expressed by the DM during the elicitation process (see [Roy 93], [Mousseau 93] and [Paynes et al. 92]).

2.2. Importance parameters elicitation methods

Developing Elicitation Techniques for Importance Parameters (ETIP) is an area of research that lies between the theoretical analysis of the notion of RIC and the empirical investigation of decision behavior (see figure 1). In fact, any ETIP should account both for the precise meaning of the importance parameters in the MCAP used and for the DMs behavior related in empirical studies. For this interaction to be pertinent, it is crucial that the way the analyst uses the DM's answers in the model should conform to the information that the DM expressed through his/her answers.

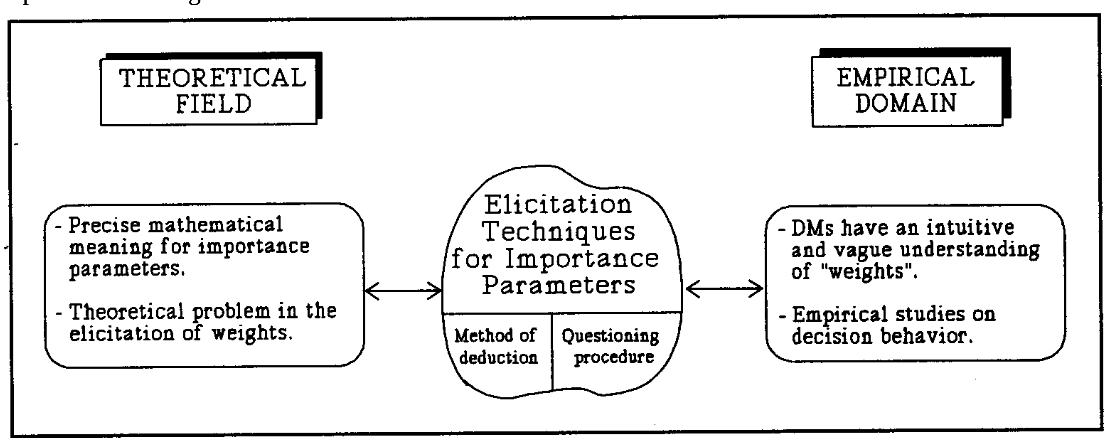


Figure 1

Schematically, two different components of an ETIP are generally distinguished: The *questioning procedure* specifies how information is collected from the DM, i.e., the questioning mode and the sequence of these questions. The *deduction method* uses information obtained by the questioning procedure, verifying if this information is compatible with the chosen MCAP so as to infer values for importance parameters.

The various ETIPs proposed in the literature (see [Mousseau 92] for a review) may be classified into two categories. In the first category, *direct ETIPs* require an information on the concept of importance from the DM (direct evaluation of parameters, comparison of criteria in term of importance, etc.). The way the values for importance parameters are derived is defined independently of the aggregation rule in which these values will be used. Proceeding in this way, these ETIPs are not able to ensure that the information expressed in the DM's answers matches the use of this information in the MCAP.

On the contrary, *indirect ETIPs* explicitly intergrate the MCAP selected for use. The interaction with the DM is not based directly on the concept of importance but on indirect information (binary comparison or ranking of alternatives, for example) from which information concerning the RIC is inferred through the aggregation rule.

It follows from the preceeding considerations that any ETIP should satisfy two types of requirements:

i) *Any ETIP should explicitly refer to the MCAP that is used to model the DM's preferences*: The logic of the various MCAP implies different meanings for their importance parameters. This means that the result of conflicts between criteria is determined both by the values assigned to importance parameters and by the logic of the MCAP. The knowledge of the value of these parameters is not sufficient to discriminate between pareto optimal alternatives; thus the meaning of importance parameters is only defined in relation to the MCAP in which they are used (see section 1).

ii) *The way an ETIP interacts with the DM should account for his perception of the notion of RIC and for its limitations in perceiving and processing information*: Any ETIP proceeds through a phase of interaction with the DM. Many studies in behavioral science and experimental psychology are useful for defining questioning procedures, pertinent from the DMs' point of view (for an overview, see [Von Winterfeld & Edwards 86], [Paynes et al. 93]).

3. DIVAPIME: a way to elicit the importance of criteria in an MCAP based on a concordance principle

In this section, we present an indirect ETIP, implemented in software called DIVAPIME (Détermination d'Intervalles de VAriation pour les Paramètres d'Importance des Méthodes Electre). This software supports the elicitation of all preferential parameters of the Electre methods (see [Roy 91]). However, the part concerning the elicitation of the importance coefficients may also apply to other MCAPs that build one or several outranking relations on a concordance concept such as PROMETHEE (see [Brans et al. 84]) or TACTIC (see [Vansnick 86]). DIVAPIME stems from the ETIP used in [Roy et al. 86] and presented in [Roy & Bouyssou 93]). Besides the implementation, our work consisted of restructuring and extending the questioning procedure to make it more precise, as well as improving the algorithmic aspects of the method.

3.1. The preferential parameters of the ELECTRE methods

ELECTRE methods build an outranking relation S, i.e., validating or invalidating, for any pair of alternatives (a,b), an assertion aSb, whose meaning is "*a is at least as good as b*". In the ELECTRE methods, preferences restricted to the significance axis of each criterion are defined through pseudo-criteria (see section 1). The indifference and preference thresholds (q_j and p_j) model the intra-criterion preferential information. They account for the imprecise nature of the evaluations $g_j(a)$. The q_j threshold specifies the largest difference of evaluation

$g_j(a)-g_j(b)$ that preserves indifference between a and b (aI_jb); p_j represents the smallest difference $g_j(a)-g_j(b)$ compatible with a preference situation in favor of a (aP_jb). The weak preference relation Q_j should be interpreted as hesitating between opting for a preference or indifference situation.

At the comprehensive level of preferences, in order for the assertion aSb to be valid, two conditions should be verified:

concordance: for an outranking aSb to be accepted, a "sufficient" majority of criteria should be in favor of this assertion,

non-discordance: when the concordance condition holds, none of the criteria in the minority should oppose to strongly to the assertion aSb

Two types of importance parameters intervene in the construction of S:

- the set of importance coefficients $(k_1,k_2, \ldots, k_n)$ takes into account the relative importance of coalitions of criteria and intervenes in the construction of a concordance index c(aSb) which is defined by:

$$c(aSb) = \frac{\sum_{i=1}^{n} k_k \cdot c_i(aSb)}{\sum_{i=1}^{n} k_i} \quad \text{with} \quad c_j(aSb) = \begin{cases} = 1 \text{ if } aP_jb \text{ or } aQ_jb \text{ or } aI_jb \\ = 0 \text{ if } bP_ja \\ \in [0,1] \text{ if } bQ_ja \end{cases} \qquad 1$$

- the set of veto thresholds $(v_1(g_1),v_2(g_2), \ldots, v_n(g_n))$ is used in the discordance. $v_j(g_j)$ represents the greatest difference of evaluation $g_j(b)-g_j(a)$ compatible with the assertion aSb.

3.2. Determination of a polyhedron of admissible values for importance parameters

In this section, we present a technique which aims at determining a non-empty polyhedron of admissible values for $k=(k_1,k_2,\ldots,k_n)$ starting from linear inequalities on these coefficients. These inequalities come from DM's answers to binary comparisons of fictitious alternatives (see 3.3). We do not aim at determining a single vector of values k, but a set of vectors consistent with assertions expressed by the DM.

3.2.1. Formulating information through a segmented description

Let us suppose that criteria can be ordered by importance and let us renumber them so that $k_1<k_2<\ldots<k_n$. We want to specify an interval $]m_i,M_i[$ containing each k_i ($\forall i\neq 1$). Each of these intervals constitutes a segment in which the value of k_i may vary. We aim at building intervals $]m_i,M_i[$ such that m_i and M_i depend only on the k_j for $j<i$; Hence we will denote them $m_i(k_1,k_2,\ldots,k_{i-1})$ and $M_i(k_1,k_2,\ldots,k_{i-1})$. The system of inequalities is then:

In what follows, $m_i(k_1,k_2,\ldots,k_{i-1})$ will be defined by the maximum of a list of lower bounds of k_i, and $M_i(k_1,k_2,\ldots,k_{i-1})$ by the minimum of a list of upper bounds of k_i. Each of these lower or upper bounds takes the form of linear combination of k_j for $j<i$[1]. For example,

[1] The representation $m_i(k_1,k_2,\ldots,k_{i-1})<k_i<M_i(k_1,k_2,\ldots,ki-1)$ of the domain of variation of $k=(k_1,k_2,\ldots,k_n)$ exists whether this domain is a polyhedron or not; the fonctions $m_i(k_1,k_2,\ldots,k_{i-1})$ and $M_i(k_1,k_2,\ldots,k_{i-1})$ may not be linear (see [Roy 70], chap. 10).

the value of $m_5(k_1,k_2,k_3,k_4)$ can be $\{k_1+k_4,k_2+k_3\}$. So as to add a new inequality to the system[2], we proceed as follows: such an inequality may always be written as $\sum_{i=1}^{n} a_i \cdot k_i > 0$, with $a_i \in$ _ . If imax is defined as the index of the greatest non-null coefficient α_i (imax = $\underset{i \in F}{\mathrm{Max}}\{ i \,/\, a_j = 0 \; \forall j > i \})$ 2,we may rewrite the preceding inequality as3According to the sign of α_{imax}, this inequality specifies a new upper or lower bound for k_{imax}. It is then necessary to check that each lower bound of k_{imax} is still lower than each upper bound of k_{imax}. This verification may lead to adding new bounds. This step is called the saturation of the segmented description.

$$
\begin{gathered}
m_2(k_1) < k_2 < M_2(k_1) \\
m_3(k_1,k_2) < k_3 < M_3(k_1,k_2) \\
m_4(k_1,k_2,k_3) < k_4 < M_4(k_1,k_2,k_3) \\
\vdots \\
m_{n-1}(k_1,k_2,\ldots,k_{n-2}) < k_{n-1} < M_{n-1}(k_1,k_2,\ldots,k_{n-2}) \\
m_n(k_1,k_2,\ldots,k_{n-1}) < k_n < M_n(k_1,k_2,\ldots,k_{n-1})
\end{gathered}
$$

3.2.2. Saturation of the segmented description

At a given stage, adding an inequality may produce supplementary inequalities. If the added inequality specifies a new upper or lower bound for k_i so that $m_i(k_1,k_2,\ldots,k_{i-1}) < k_i < M_i(k_1,k_2,\ldots,k_{i-1})$, by transitivity, it should hold:

$m_i(k_1,k_2,\ldots,k_{i-1}) < M_i(k_1,k_2,\ldots,k_{i-1})$ [i]

$$
\begin{gathered}
m_2(k_1) < k_2 < M_2(k_1) \\
m_3(k_1,k_2) < k_3 < M_3(k_1,k_2) \\
m_4(k_1,k_2,k_3) < k_4 < M_4(k_1,k_2,k_3) \\
\vdots \\
m_{n-1}(k_1,k_2,\ldots,k_{n-2}) < k_{n-1} < M_{n-1}(k_1,k_2,\ldots,k_{n-2}) \\
m_n(k_1,k_2,\ldots,k_{n-1}) < k_n < M_n(k_1,k_2,\ldots,k_{n-1})
\end{gathered}
$$

[2] This inequality comes from the answer to a pairwise comparison of alternatives (see 3.3.1).

$$
\begin{gathered}
m_2(k_1) < k_2 < M_2(k_1) \\
m_3(k_1,k_2) < k_3 < M_3(k_1,k_2) \\
m_4(k_1,k_2,k_3) < k_4 < M_4(k_1,k_2,k_3) \\
\vdots \\
m_{n-1}(k_1,k_2,\ldots,k_{n-2}) < k_{n-1} < M_{n-1}(k_1,k_2,\ldots,k_{n-2}) \\
m_n(k_1,k_2,\ldots,k_{n-1}) < k_n < M_n(k_1,k_2,\ldots,k_{n-1})
\end{gathered}
$$

If [i] is verified $\forall$ $k_1,k_2,.....,k_{i-1}$, then no additional inequality should be generated; in the opposite case, this inequality [i] should be integrated into the system, and will produce one (or several) upper and/or lower bound(s) from which one (or several) additional inequalities might be generated.

Hence, saturating a segmented description consists of producing all possible (upper or lower) bounds from inequalities similar to [i] that are not verified $\forall$ $k_1,k_2,.....,k_{i-1}$. The generation of a new bound can itself produce an additional bound. The saturation algorithm stops when no bound can be generated.

In certain cases, the addition of an inequality to the system can lead to an empty polyhedron. Such a situation occurs when a lower bound of a k_j is greater than one of the upper bounds of this k_j, $\forall$ $k_1,k_2,.....,k_{j-1}$. This situation characterizes a contradiction between the last inequality integrated into the system and one or several others.

The algorithm can detect such inconsistencies and determine which inequalities generate the contradiction[3]. In order for the polyhedron of admissible solutions not to become empty, it is necessary to delete one or several inequalities generating the inconsistency together with all bounds that have been generated by the bound(s) to be deleted.

The convergence of the saturation mechanism has been proved (see [Roy70]) ; we are then sure to reach a saturated segmented description. Let us give, for explanatory purposes, some elements that justify the convergence of the algorithm: at the saturation step, the production of an additional bound comes from the fact that the inequality $m_i(k_1,k_2,.....,k_{i-1})<M_i(k_1,k_2,.....,k_{i-1})$ is not verified, $\forall k_1,k_2,.....,k_{i-1}$. In this case, the new bound will always be added on a segment k_j such that $k_j<k_i$ (indeed, $m_i(k_1,k_2,.....,k_{i-1})$ and $M_i(k_1,k_2,.....,k_{i-1})$ are functions of k_j such that $j<i$). It follows that such deductions can be carried out only a finite number of times and that the saturation should be computed linearly, starting from the segment corresponding to the most important criterion without any backtracking to saturated segments.

3.3. Questioning procedure

3.3.1. Questioning mode: pairwise comparisons of fictitious alternatives

As this ETIP is an indirect ETIP, the questioning mode does not directly refer to the concept of importance. We can, from the answers of the DM to the questions, infer information through the MCAP used. The questioning mode selected is a pairwise comparison of fictitious alternatives. For each question, the DM has to define the comprehensive preference situation between two evaluation vectors.

The fictitious alternatives involved in the comparisons are chosen so as to provide specific information. Moreover, they should be able to correspond to real alternatives (their evaluations should be plausible and respect possible statistical links between criteria). The questioning procedure is founded upon the following fictitious alternatives:

- b_0: A reference alternative whose evaluations on each criterion are "average".
- b_i: Alternatives whose evaluations are identical to b_0 on all criteria except on criterion g_i on which its evaluation is increased by a significant amount (but not exceeding the veto threshold) relative to the scale of g_i, (b_i P_i b_0).

[3] Each upper or lower bound comes either from an original inequality, or from two "parent bounds". It is then easy to retrieve the "ancestry chain" of a bound so as to identify its origins.

$b_{i,j}$: Alternatives indentical to b_0 on all criteria except g_i and g_j on which its evaluation is increased by a significant amount (but not exceeding the veto threshold) relative to the scales of g_i and g_j, ($b_{i,j}\ P_i\ b_0$ and $b_{i,j}\ P_j\ b_0$).

b_J: Alternatives identical to b_0 on all criteria except on criteria contained in the coalition J ($J \subseteq F$) on which its evaluation is increased by a significant amount (but not exceeding the veto threshold) relative to the scales of the considered criterion ($b_J\ P_i\ b_0,\ \forall i \in J$).

If the DM and the analyst agree to ground decision aid on an ELECTRE type MCAP, then the knowledge of the comprehensive preference situation between two fictitious alternatives allows us to infer information on importance parameters of this MCAP. In fact, when the DM states $b_{J_1}\ P\ b_{J_2}$, it means that the advantages on criteria contained in J_1 loom larger than the ones on criteria contained in J_2, i.e., the coalition J_1 is more important than the coalition ($J_1 \gg J_2$). As each assertion of the relation $\gg$ (*more important than between disjoint coalitions of criteria*) is formalised, in the considered MCAP, by $\forall J_1, J_2 \subseteq F \quad J_1 \gg J_2 \Leftrightarrow \sum_{i \in J_1} k_i < \sum_{i \in J_2} k_i$, we infer from each preference relation between b_{J_1} and b_{J_2} an inequality on k_js.

Similarly, an indifference between b_{J_1} and b_{J_2} leads to stating an equality on k_js: $\sum_{i \in J_1} k_i = \sum_{i \in J_2} k_i$. So as to soften the highly reductive effect of such an equality on the set of admissible values for k_js, we propose to treat indifferences in the following way:

$$b_{J_1}\ I\ b_{J_2} \Leftrightarrow \left|\sum_{j \in J_1} k_j - \sum_{j \in J_2} k_j\right| < k_{min} \quad \text{with} \quad k_{min} = \underset{j \in F}{\mathrm{Min}}\{k_j\}$$

The "strength" of the indifference is then reduced to an equality on k_js "to within k_{min}".

Moreover, it is important to stress what underlies the technique of saturated segmented description. When the polyhedron of admissible values for k_js becomes empty, this means that it is not possible to find values for k_js compatible, in the considered model, with the assertions expressed by the DM.

3.3.2. Preliminary step: eliciting discrimination thresholds.

The role of indifference and preference thresholds q_j and p_j is to specify the preferences restricted to the significance axis of a criterion; they do not refer directly to the notion of RIC. However these thresholds interact with the inter-criteria preferential parameters and may have an indirect influence on the role of each criterion in the aggregation. Moreover, the definition of b_J requires the knowledge of p_j. So as to determine values for these thresholds, the analyst may refer to the ill-determined nature of some constituent elements of criteria (see [Roy 85], chap. 9 and [Roy et al. 86]). In certain situations, these thresholds may be elicited through an interaction with the DM.

In the implementation of the ELECTRE methods (see [Vallee & Zielniewicz 94]), p_j and q_j are defined as afine functions of evaluations, i.e., such that:

$$\begin{cases} q_j(g_j(a)) = a_q \cdot g_j(a) + b_q \\ p_j(g_j(a)) = a_p \cdot g_j(a) + b_p \end{cases}$$

So as to obtain such an information, we propose to proceed in the following way:

- Determine a neutral evaluation n_j (neither attractive, nor repulsive, neither representing an avantage, nor a drawback on the considered criterion).
- Determine an attractive evaluation a_j representing a significant advantage with regard to the neutral level.

Given a_j and n_j[4], we elicit $q_j(n_j)$, $p_j(n_j)$, $q_j(a_j)$ and $p_j(a_j)$[5]. So as to minimize the duration of the interaction, questions are asked following a dichotomic search. The following algorithm determines $q_j(n_j)$ and $p_j(n_j)$ ($q_j(a_j)$ and $p_j(a_j)$ are obtained similarly).

```
For each criterion j
Do
|       min ← nj
|       max ← aj
|       While max-min > ε                (ε: proportion of the range of the scale)
|       Do
|       |       If max+min/2 Ij nj
|       |           |  Then min ← max+min/2
|       |           |  Else max ← max+min/2
|       |       Endif
|       End
|       qj(nj) ← max+min/2
|       min ← qj(nj), max ← aj

|       While max-min > ε
|       Do
|       |       If max+min/2 Pj nj
|       |           |  Then max ← max+min/2
|       |           |  Else min ← max+min/2
|       |       Endif
|       End
|       pj(nj) ← max+min/2
End
```

Determining the discrimination thresholds in this way supposes that:

- criteria are evaluated on a continuous scale (or discrete with a large number of levels),
- criteria are not the result of a sub-aggregation,
- the DM knows precisely how evaluations are determined.

3.3.3. Step 1: Rank criteria by order of importance

The first step in the questioning procedure consists of searching for a pre-order on the k_js. To achieve this, the alternatives $b_1,b_2,\ldots,b_n$ are presented to the DM; he should then determine the alternative b_h he considers the best. Then, it holds $b_h P b_i$ $\forall i \neq h$; hence $k_h > k_i$ $\forall i \neq h$. When several alternatives $b_{h_1},b_{h_2},\ldots,b_{h_p}$ are judged to be the best and indifferent to one another, we have: $\forall h \in H, \forall h' \in F\backslash H$ $b_h P b_{h'}$ and $\forall h,h' \in H$ $b_h I b_{h'}$ with $H=\{h_1,h_2,\ldots,h_p\}$. Then $k_{h_1}=k_{h_2}=\ldots=k_{h_p}>k_{h'}$ $\forall h' \in F\backslash H$.

---

[4] The analyst should check that a_j and n_j are such that $a_j P_j n_j$.

[5] It is possible to determine more points so as to make a linear regression, but it requires more questions.

The best alternative(s) is (are) then deleted from the initial list and the DM must choose the best one in the list of remaining alternatives, etc. We then obtain a pre-order on the k_js and it is always possible to renumber the criteria so that: $k_1 \leq k_2 \leq \ldots\ldots \leq k_n$. When several criteria are of equal importance, we keep only one representative of the equivalence class, during the rest of the questioning procedure, so as to obtain, after a second renumbering: $k_1 < k_2 < \ldots\ldots < k_{n'}$.

3.3.4. Step 2: determining groups of criteria which are close in importance

The second step aims at partitioning the set of criteria (ranked by importance beforehand) according to a specific condition that can be interpreted as defining a partition into groups of "relatively close" criteria when considering their relative importance. Each group of criteria G_i is defined by the index h(i) of the least important criterion of the group. The p groups are such that:

$$\left[\begin{array}{c} \{ g_1, g_2, \ldots, g_{h(1)+i}, \ldots, g_{h(2)-1} \} \\ \{ g_{h(2)}, g_{h(2)+1}, \ldots, g_{h(2)+i}, \ldots, g_{h(3)-1} \} \\ \vdots \\ \{ g_{h(p)}, g_{h(p)+1}, \ldots, g_{h(p)+i}, \ldots, g_n \} \end{array}\right.$$

The h(i) are defined by: $h(i+1) = \min_{j \in F} \{ j \,/\, g_j \ll \{ g_{h(i)}, g_{h(i)+1} \} \}$ with $h(1) = 1$. The groups must be interpreted as: $g_{h(i+1)}$ is the least important criterion that remain more important than the coalition $\{g_{h(i)}, g_{h(i)+1}\}$. The information necessary for determining the partition is the following:

$$\left[\begin{array}{l} g_{h(i+1)} \ll \{ g_{h(i)}, g_{h(i)+1} \} \\ \text{non}\left[g_{h(i+1)-1} \ll \{ g_{h(i)}, g_{h(i)+1} \} \right] \end{array}\right. \forall i = \{ 1, 2, \ldots\ldots, p \}$$

In order to obtain this information, the DM must respond to pairwise comparisons of alternatives whose evaluations vary on 3 criteria. The protocole is the following:

- Do you prefer $b_{1,2}$ or b_n ?
  - if the answer is $b_{1,2}$ P b_n then $k_1+k_2>k_n$, we stop here for this step.
  - if the answer is $b_{1,2}$ I b_n then $|k_n-k_1-k_2| < k_1$, we stop here for this step.
  - if the answer is b_n P $b_{1,2}$ then we ask the following question.
- Do you prefer $b_{1,2}$ or b_{n-1} ?
  - if the answer is $b_{1,2}$ P b_{n-1} then $k_1+k_2>k_{n-1}$, we stop here for this step.
  - if the answer is $b_{1,2}$ I b_{n-1} then $|k_{n-1}-k_1-k_2|<k_1$, we stop here for this step.
  - if the answer is b_{n-1} P $b_{1,2}$ then we go on until we find the smallest h(2) such that not[$b_{h(2)-1}$ P $b_{1,2}$].

We then continue by asking the question:

- Do you prefer $b_{h(2),h(2)+1}$ or b_n
  - if the answer is $b_{h(2),h(2)+1}$ P b_n then $k_{h(2)}+k_{h(2)+1}>k_n$, we stop here for this step.
  - if the answer is $b_{h(2),h(2)+1}$ I b_n then $|k_n-k_{h(2)}-k_{h(2)+1}|<k_1$, we stop here for this step.
  - if the answer is b_n P $b_{h(2),h(2)+1}$ then we go on until we find the smallest h(3) such that not[$b_{h(3)-1}$ P $b_{h(2),h(2)+1}$].
- and so on until the whole familly of criteria is partitioned.

The second step of the questioning procedure is summarized in the following algorithm:

```
i←1
h(i)←1
While h(i)+2 ≤ number-of-crit
Do
|       j←number-of-crit
|       Repeat
|       |       Compare bj to bh(i),h(i)+1
|       |       j←j-1
|       Until not[bj P bh(i),h(i)+1] or h[i]+2>j
|
|       If bh(i),h(i)+1 P bj
|
|          | Then h(i+1)←j+2
|
|          | Else h(i+1)←h(i-1)+2
|       Endif
|       i←i+1
End
number-of-group ← i-1
```

So as to obtain the required information, this algorithm contains linear sequences of questions. It is possible to reduce significantly the number of questions by applying a dichotomic segmentation rule: instead of comparing $b_{h(i),h(i)+1}$ to b_n, b_{n-1}, etc. successively, it is more efficient to determine h(i+1) using a dichotomic search in the interval [h(i)+2,n].

3.3.5. Step 3: Evaluating of the "distance" between groups of criteria

At the end of the first two steps, we obtain a partition of the set of criteria (subsequently ordered) in groups of "relatively close" criteria, from the point of view of their relative importance. The third step aims at evaluating the "distance" between these groups. The procedure seeks, for each group, the coalition composed of the two least important criteria which is more important than the least important criterion of the group just above it. More precisely, for each group G_i we search for $m_1(i)$ and $m_2(i)$ such that:

$$\left[\begin{array}{ll} & \{g_{m_1(i)}, g_{m_2(i)}\} \ll g_{h(i+1)} \\ \forall\, a,b \in F & \left.\begin{array}{c} a \leq m_1(i) \text{ et } b \leq m_2(i) \\ \text{(one inequality at least being strict)} \end{array}\right] \Rightarrow g_{h(i+1)} \gg \{g_a, g_b\} \end{array}\right.$$

In order to find $m_1(i)$ and $m_2(i)$, the sequence of questions and answers is the following:

- Do you prefer $b_{h(i)+1,h(i)+3}$ or $b_{h(i+1)}$?
  - If the answer is $b_{h(i)+1,h(i)+3}$ P $b_{h(i+1)}$, then $k_{h(i)+1}+k_{h(i)+3}>k_{h(i+1)}$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+1,h(i)+3}$ I $b_{h(i+1)}$, then $|k_{h(i)+1}+k_{h(i)+3}-k_{h(i+1)}|<k_1$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i+1)}$ P $b_{h(i)+1,h(i)+3}$, then we ask the following question:
- Do you prefer $b_{h(i)+1,h(i)+4}$ or $b_{h(i+1)}$?
  - If the answer is $b_{h(i)+1,h(i)+4}$ P $b_{h(i+1)}$, then $k_{h(i)+1}+k_{h(i)+4}>k_{h(i+1)}$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i)+1,h(i)+4}$ I $b_{h(i+1)}$, then $|k_{h(i)+1}+k_{h(i)+4}-k_{h(i+1)}|<k_1$, we stop for this group and we go on to the next group.
  - If the answer is $b_{h(i+1)}$ P $b_{h(i)+1,h(i)+4}$, then we go on until we find λ such that $b_{h(i)+1,h(i)+\lambda}$ P $b_{h(i+1)}$. If it is not possible to find such a λ, then we ask the following question :

- Do you prefer $b_{h(i)+2,h(i)+3}$ or $b_{h(i+1)}$?
    - If the answer is $b_{h(i)+2,h(i)+3}$ P $b_{h(i+1)}$, then $k_{h(i)+2}+k_{h(i)+3}>k_{h(i+1)}$, we stop for this group and we go on to the next group.
    - If the answer is $b_{h(i+1)}$ I $b_{h(i)+2,h(i)+3}$, then $|k_{h(i)+2}+k_{h(i)+3}-k_{h(i+1)}|<k_1$, we stop for this group and we go on to the next group.
    - If the answer is $b_{h(i+1)}$ P $b_{h(i)+2,h(i)+3}$, then we ask the following question:
- Do you prefer $b_{h(i)+2,h(i)+4}$ or $b_{h(i+1)}$?
    - If the answer is $b_{h(i)+2,h(i)+4}$ P $b_{h(i+1)}$, then $k_{h(i)+2}+k_{h(i)+4}>k_{h(i+1)}$, we stop for this group and we go on to the next group.
    - If the answer is $b_{h(i)+2,h(i)+4}$ I $b_{h(i+1)}$, then $|k_{h(i)+2}+k_{h(i)+4}-k_{h(i+1)}|<k_1$, we stop for this group and we go on to the next group.
    - If the answer is $b_{h(i+1)}$ P $b_{h(i)+2,h(i)+4}$, then we continue until we find λ_1 and λ_2 such that not[$b_{h(i+1)}$ P $b_{h(i)+\lambda_1,h(i)+\lambda_2}$].

In the third step, the sequence of questions follows the algorithm given below:

```
For each group Gi
Do
|       m1(i)←h(i)
|       m2(i)←h(i)+1
|       stop ← false
|       Repeat
|       |       If m2(i)+1 < h(i+1)
|       |          | Then m2(i)←m2(i)+1
|       |          | Else If m1(i)+2 < h(i+1)
|       |          |         | Then m1(i)←m1(i)+1
|       |          |         |         m2(i)←m1(i)+1
|       |          |         | Else stop ← true
|       |          |         Endif
|       |       Endif
|       |       If not stop
|       |          | Then Compare b_h(i+1) to b_m1(i),m2(i)
|       |       Endif
|       Until stop  or  not[b_h(i+1) P b_m1(i),m2(i)]
End
```

3.3.6. Step 4: obtaining information so that each k_j has an upper bound

At the end of the third step, each k_j has a lower bound (at least k_{j-1}), but it does not necessarily have an upper bound. This means that the domain of admissible values for $k=(k_1,k_2,.....,k_n)$ is open. This step aims at providing an upper bound for each k_j. A sufficient condition for the polyhedron to be closed is that k_n has an upper bound and so has k_n/k_1. Indeed we may deduce from $k_n/k_1<\alpha$ that $k_j<\alpha.k_1$ ($\forall$j). So as to bound this ratio, the analyst may ask the following question: suppose that we can imagine α criteria having the same importance as g_1; let us consider the fictitious alternative b_1^{α} whose evaluations are average (i.e., identical to b_0) except on these α criteria on which its evaluation is improved by a significant amount. Formally, the comparison of b_1^{α} and b_n (α being variable) allows us to obtain an upper bound for the ratio k_n/k_1. This technique puts only few questions to the DM; however, the complexity of the required interaction can lead to an uncertain and imprecise answer.

Another technique may be used. We consider $J\subseteq F$ the set of criteria g_j for which k_j has no upper bound at the end of step 3 and such that there exists a criterion g that is less important than g_j and possesses an upper bound. The fourth step is defined by the following sequence of questions:

```
For each segment j ∈ J
Do
        Let kr+ks be an upper bound of a segment lower than j and possessing an upper
        bound (we suppose kr<ks)
        stop ← false
        Repeat
                Compare bj to br,s
                If s+1 < j
                    Then s ← s+1
                    Else If r+2 < j
                            Then r←r+1
                                 s←r+1
                            Else stop ← true
                        Endif
                Endif
        Until  stop  or  br,s P bj
End
```

An advantage of this technique lies in the fact that its interaction deals with pairwise comparisons of alternatives (which is consistent with the rest of the procedure). However, this algorithm does not always provide an upper bound to all k_j; the coefficients of some of the least important criteria may remain unbounded (this leaves the polyhedron open). For instance, it is difficult to find an upper bound for k_3 when $k_3>k_1+k_2$.

3.3.7. Step 5: adding supplementary inequalities so as to reduce the polyhedron of admissible values

The fifth step aims at reducing the polyhedron of admissible values for k. This step is necessary when the obtained polyhedron leads to considering a very large number of sets of importance coefficients to be valid.

The questions deal with two alternatives ($b_{i,j}$ et $b_{k,l}$) whose evaluations vary on 4 criteria. In order for the answers to provide supplementary information, it is necessary that $k_i<k_k,k_l<k_j$. The choice of these 4 criteria (verifying $k_i<k_k,k_l<k_j$) must be made with regard to the polyhedron obtained. It is not necessary, therefore, to automate this step (the choice of these questions is left to the analyst). It should be noted that inconsistencies usually appear at this stage (see 3.2.2 how to reduce such inconsistencies).

3.3.8. Determining several admissible weight vectors

When the polyhedron of admissible values for k is closed, an interval of variation for each k_j can easily be inferred from the saturated segmented description: each k_j lies in a segment whose bounds are functions of the k_i verifying $k_i<k_j$. These intervals are the following:

The value of the lower and upper bound of each coefficient k_j is obtained by assigning

to the k_i (such that i<j) in the function $m_j(k_1,k_2,.....,k_{j-1})$ and $M_j(k_1,k_2,.....,k_{j-1})$ their minimum or maximum value, respectively, according to the sign of the coefficient in the linear forms $m_j(k_1,k_2,.....,k_{j-1})$ and $M_j(k_1,k_2,.....,k_{j-1})$ respectively. It should be noted that these extremes values for k_j, can be reached only for particular values of k_i such that $k_i<k_j$.

Moreover, the information obtained enables us to build easily a large number of admissible vectors $k=(k_1,k_2,...,k_n)$. A "central" weight vector may be generated by assigning the central value of its own variation interval to each k_j. When we bring the value of each k_j closer to the upper bound (or to the lower bound), we obtain vectors that are "wider" (or "narrower").

$$m_2(k_1) < k_2 < M_2(k_1)$$
$$m_3(k_1, m_2(k_1)) < k_3 < M_3(k_1, M_2(k_1))$$
$$m_4(k_1, m_2(k_1), m_3(k_1, m_2(k_1))) < k_4 < M_4(k_1, M_2(k_1), M_3(k_1, M_2(k_1)))$$
$$\vdots$$
$$\text{etc.}$$

3.3.9. Elicitation of the veto thresholds

Like the coefficients k_j, the veto thresholds $v_j(g_j)$ are inter-criteria preferential parameters. They account for an aspect of the notion of RIC which is distinct from that modelled by the coefficients k_j[6]. We should recall that the veto thresholds aim at impoverishing the outranking relation through the invalidation of assertions aSb that satisfy the concordance condition.

Before trying to elicit thresholds $v_j(g_j)$, it is important to determine whether the DM wants to attach some veto power to the criteria. Moreover, the analyst must verify whether this veto power may become effective in comparing alternatives, i.e., if $v_j(g_j)<L_j$ with $L_j = \max_{a\in A}\left(g_j(a)\right) - \min_{a\in A}\left(g_j(a)\right)$. When the veto has a significant effect on the result, further interaction with the DM is necessary. The software implementation of the ELECTRE methods considers the thresholds $v_j(g_j)$ as afine functions of evaluations. Several interaction modes have been proposed so as to determine values for $v_j(g_j)$ with the DM.

It is possible to take advantage of the meaning that the DM gives to the criteria and the role he wants them to play. [Roy & Bouyssou 93] (chap. 8 et 9) propose using the ratio v_j/p_j. We will propose below a method for determining an interval of variation for v_j. We will determine values for $v_j(n_j)$ and $v_j(a_j)$ (n_j and a_j being neutral and attractive evaluations respectively).

So as to determine these values, we proceed indirectly by asking questions concerning fictitious alternatives. We denote $b^n=(n_1,n_2,.....,n_n)$ and $b^a=(a_1,a_2,.....,a_n)$; let b_J^n and b_J^a be the alternatives having the same evaluations as b^n and b^a respectively except on the criteria $j\in J$ ($J\subseteq F$) on which its evaluation is increased by p_j. We denote b^n_{j+x} and b^a_{j+x} the alternatives having the same evaluations as b^n and b^a respectively except on criterion j on which their evaluations are increased by x. So as to bound $v_j(n_j)$ and $v_j(a_j)$, the DM should compare b^n_{j+x} to b_J^n and b^a_{j+x} to b_J^a, x being variable and $J=F\setminus\{j\}$ (in what follows, we will explain only how to determine $v_j(n_j)$; we will proceed similarily with $v_j(a_j)$).

---

[6] These two facets of the importance of criteria are usually linked; however, it is formally possible for a criterion to have simultaneously a low weight and a large veto power.

It should be noted that the assertion b_J^n S b^n_{j+x} satisfies the concordance condition[7]. In the comparaison of b_J^n and b^n_{j+x}, the DM may either prefer b_J^n or hesitate because of difficulties when comparing these alternatives. In the first case, x constitutes a lower bound for $v_j(n_j)$ while in the second case, it specifies an upper bound for $v_j(n_j)$. So as to bound $v_j(n_j)$ we proceed through a dichotomic segmentation of the interval $[p_j,L_j]$ (L_j being the range of the scale); this leads to the following algorithm:

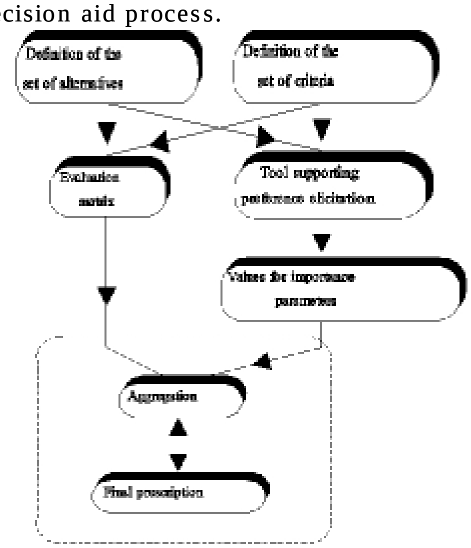
```
lower ← pj
upper ← Lj
While upper-lower > ε (ε being a proportion of Lj-pj)
Do
|     x ← upper+lower/2
|     Compare bJ^n and b^n j+x
|     If bJ^n P b^n j+x
|     |    Then lower ← upper+lower/2
|     |    Else upper ← upper+lower/2
|     Endif
End
```

Determining intervals for $v_j(n_j)$ and $v_j(a_j)$ allows us to define the coefficients of two lower and upper bound fonctions $v_j^m(g_j)$ and $v_j^M(g_j)$ (we should verify that these fonctions are such that $v_j^m(g_j(a))<v_j^M(g_j(a))\ \forall a\in A$). The remarks concerning the conditions for using the algorithm for determining discrimination thresholds (see 3.3.2) also apply for the present algorithm.

3.4. Implementation of the method

3.4.1. Role of such a tool in a decision aid process

DIVAPIME is to be inserted in a decision aid process in which a multiple criteria model is used. The elicitation of preferential information is a crucial phase of the modelisation that is often problematical. The proposed software aims at supporting this elicitation step (when an ELECTRE type MCAP is chosen to model the decision process). Figure 2 places DIVAPIME in a classical multicriteria decision aid process.



*

[7] except in very particular cases in which de k_j is very large $\left(k_j > \sum_{i\neq j} k_i\right)$.

The way this tools should be inserted in a decision process must be analysed. Several studies concerning the elicitation of the RIC have shown that there is a stumbling stock for any ETIP, namely the existence of a gap between the information underlying the answers given to the analyst by the DM, and the way these answers are interpreted in the model. We believe that the presence of an analyst may partially avoid this problem as he/she can verify whether the DM's understan-ding and interpretation of the questions is correct.

Hence, this software is essentially intended to the analysts and aims at facili-tating their work while enhancing the user-friendliness of the interaction with the DM. Although this tool has some of the characteristics of a decision support system (DSS)[8], it is not, in its present form, intended for the DM. Its role is not necessarily to transfer some of the activities of the analyst to the DM. It was not designed as a substitute for the analyst, whose role remains essential: the latter must explain to the DM the nature and the meaning of each of the preferential parameters as well as the way in which the DM's answers will be interpreted.

However, an independant use of this software by a DM with good background knowledge of the ELECTRE methods is possible. The proposed tool may then be viewed as a DSS that helps the DM in formalizing his preferences and eliciting the preferential parameters required by the ELECTRE methods. On the other hand, it may be dangerous to put such a tool in the hands of a novice in MCDA, since it would be difficult to know what the basis of the output obtained would be[9].

3.4.2. Presentation of the DIVAPIME software: general description

DIVAPIME software (Détermination d'Intervalles de VAriation des Paramètres d'Importance des Méthodes Electre) works with MS-DOS 3.2 or higher on a IBM PC with a minimum of 640 Kb memory and a VGA color monitor. It is implemented with Borland Turbo Pascal. The different options proposed are described in figure 3.

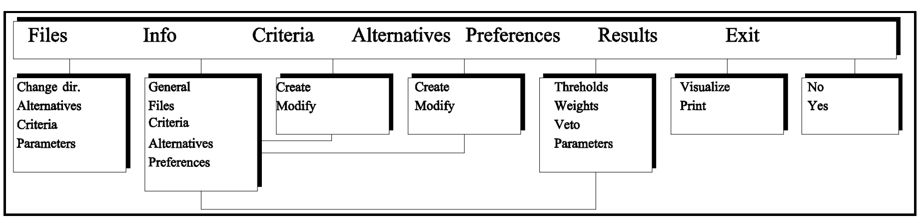


Figure 3 : DIVAPIME menu structure

The content of the options in the main menu is as follows:

Files: provide the list of criteria files, alternatives files and parameter set files.

Info: provides general information on the ETIP (in particular the way answers are used) and explanations concerning each option. This option may be activated from any option as an on-line help feature.

Criteria: input, loading and/or modification of a set of criteria.

Alternatives: input, loading and/or modification of a set of alternatives.

Preferences: Evaluation of preferential parameters through a questioning procedure.

Results: display or print obtained intervals of variation for preferential parameters.

---

[8] Assisting the DM in ill-structured tasks, helping rather than replace the DM's judgment, using the interactive possibilities of the tools (see [Sprague & Carlson 82], [Levine & Pomerol 89]).

[9] However, DMs frequently lead real world decision aid processes without the help of an analyst. The use of DIVAPIME in such a context will in any case be less controversial than the most frequently used method, i.e., direct numerical evaluation of the parameters.

The standard scheme of a DIVAPIME session is the following:

- Input (or loading/modification) of a set of criteria.
- Input (or loading/modification) of the set of alternatives[10].
- Determination of the discrimination thresholds p_j and q_j.
- Determination of intervals of variation for the importance coefficients k_j and possibly for the veto thresholds v_j[11].
- Generation of one or several sets of preferential parameters.

The interested reader will find in [Mousseau 93] an illustrative example of how DIVAPIME may be used in a decision aid process. It should also be mentioned that a new Windows version of this software will be implemented and will be adapted to a multi-actor situation.

3.5. Extensions

3.5.1. Adapting the method to the problem formulation

We believe that it is erroneous to conceive an ETIP without taking into account the problem formulation adopted in the modelling of the decision problem: the questioning procedure should correspond to the way the DM analyses the problem ($P\alpha$: choice, $P\beta$: assignment or $P\gamma$: ranking, see [Roy 85] and [Bana e Costa 93]). In our presentation, the proposed method is mainly adapted to a problem formulated in ranking terms. The questions put to the DM should be modified so as to become appropriate for the choice and assignment problem formulation ($P\alpha$ and $P\beta$).

In the case of a choice problem, the answer to a pairwise comparison of alternatives could be *"I do not want to choose either of them"*. Such an answer is hardly interpretable and appears when both alternatives are judged to be insufficiently attractive in order to be selected in the final prescription. It is then possible to "force" the DM's answer by puting him/her in a situation in which two alternatives are the only available options. Another way to avoid such a problem consists in proposing comparisons to the DM, in which both alternatives are "attractive". In the method proposed here, this may be done by changing the definition of the reference alternative b_0 (see 3.3.1) whose evaluations on all criteria should be attractive (rather than neutral).

When the decision situation is modelled through an assignment problem formulation, questioning the DM on the basis of pairwise comparisons of alternatives poses a more fundamental problem: the assignment of alternatives to a category is not founded on the comparisons of alternatives but on an absolute evaluation. In some cases, pairwise comparison of alternatives may be used. However, this mode of interaction is usually unsuitable and should be modified for the dialogue between the analyst and the DM to conform to the logic of $P\beta$.

One way to proceed is to ask the questions in terms of an assignment as follows. We denote $\{K_1,K_2,.....,K_p\}$ the ordered set of predefined categories ($i>j \Leftrightarrow$ categorie K_i is better than categorie K_j). b_0 is a fictitious alternative conceived of in such a way that b_0 is assigned to K_i (we note $b_0 \rightarrow K_i$). b_{J_1} and b_{J_2} ($J_1 \subseteq F$, $J_2 \subseteq F$ and $J_1 \cap J_2 = \emptyset$) are two fictitious alternatives defined

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[10] This option is included in the software because the DM must have a precise perception of the set of alternatives so as to express judgments on the importance of criteria.

[11] There is no constraint of precedence between the evaluation of k_js and v_js.

as in 3.3.1. If the DM assigns b_{J_1} and b_{J_2} to the categories K_{λ_1} and K_{λ_2} respectively, then it holds: $\lambda_1>\lambda_2 \Rightarrow J_1>>J_2$. From this information, we can infer values (or intervals of variation) for importance parameters through the MCAP used[12].

3.5.2. Extension to a multiple DM framework

DIVAPIME is presently intented to be used in a decision situation in which a single DM is involved. Several ways can be considered to extend this ETIP (and its implementation) to decision situations in which several DMs interact[13].

The first of these is to force the DMs to answer collectively the questions asked during the procedure. From this perspective, the multi-actor aspect of the problem is not directly managed by the method but is taken over by the DMs who must discuss their arguments before answering each question. Hence, no adaptation of the method is required[14]. This approach is particularily suitable when criteria represent viewpoints of specific actors. In this case, it is difficult for some of these actors to answer all questions as they may have a precise opinion on the importance of criteria only with regard to a subset of criteria. During a collective questioning procedure, it may occur that several DMs disagree on the answer to one (or more) question(s). It is then advisable to put aside temporarily the inequalities corresponding to these questions and to reintegrate them at the end in order to generate the corresponding polyhedrons.

A second approach consists of determining intervals of variation for importance parameters with each DM individually and trying to group DMs whose opinions are "close". The simplest situation occurs when the intersection of all polyhedrons of admissible values for k_j is non-empty; a vector $k=(k_1,k_2,.....k_n)$ can then be chosen in the polyhedron defined by this intersection. Nevertheless, this polyhedron is frequently empty and such a vector cannot be found. In this case, another way to proceed would be to take advantage of automatic classification techniques so as to constitute a partition of the set of DMs. It is then simple to generate a vector k representing each group of DMs. If the prescriptions stemming from the different sets of parameters converge, then it is not necessary to reduce divergences of opinion between DMs. In the opposite case, a discussion between DMs is imperative to reach a compromise; this discussion should not revolve directly around values for parameters but rather on the sequence of questions and answers for which a divergence has appeared.

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[12] This approach, although more suitable for the assignment problem formulation, poses problems when the number of categories is low.

[13] However such an extention is conceivable only in consensual multi-actor decision situations. In fact, in a decision problem in which opposition between actors is acute, it will be difficult for DMs to reach an agreement concerning the values of preferential parameters; seeking such values will only underline points of conflict but will not be able to orient the decision process towards a compromise solution.

[14] However, each DM should form his/her own opinion concerning the importance of criteria before the general discussion: it is possible, for example, to carry out all (or a part of) the questioning procedure with each DM individually.

Conclusion

A careful analysis of the notion of Relative Importance of Criteria (RIC) proves this notion to be more complex than is commonly assumed. We have shown (section 1) that the information that underlies this notion is much richer than that contained in the importance parameters used in the various multicriteria models. Hence, these parameters constitute a simplistic way of taking RIC into account. Moreover, the meaning of such parameters varies across models.

These considerations, together with an empirical analysis of how DMs understand the notion of RIC, constitute a basis on which Elicitation Techniques for Importance Parameters (ETIP) may be developed (section 2). In addition, the role of ETIP will differ according to whether or not preferences are assumed to exist prior to the modeling process. If we assume that preferences pre-exist, ETIP aims at estimating pre-existing information; if we assume they do not pre-exist, ETIP only provides parameters consistent (according to an aggregation rule) with some assertions stated by the DM.

Section 3 is devoted to the presentation of an ETIP adapted for the ELECTRE methods. The interaction with the DM proceeds by means of pairwise comparisons of fictitious alternatives. This technique tests the consistency of the DM's answers with the aggregation rule used and provides, as output, an interval of variation for each parameter. This output constitutes an interesting starting point for robustness analysis.

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