

ORDINAL MEASUREMENT FOR DECISION AID: A CONCEPTUAL FRAMEWORK AN RESEARCH AGENDA

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Introduction

It happens more and more often in decision aiding situations to be faced with ordinal or nominal information concerning the alternatives that are considered by a client/decision maker (hereafter we will use the term decision maker DM). By the term ordinal or nominal information we intend the fact that evaluation on attributes, descriptors, indexes, criteria etc. may be expressed on ordinal or nominal scales which, although may allow a numerical representation, do not allow the usual manipulations on numbers since they could be meaningless.

Standing on the statement that, in decision aiding, it is not possible to choose the information to work with, the necessity to have procedures, methods, models, techniques, which may be meaningful, reliable, satisfactory is clear. This paper tries to introduce a general framework in which different research projects may found their place. The paper is organized as follows. Section 1 gives some motivations for this research project. Section 2 introduces the conceptual framework and the key issues of our research. The conclusion briefly introduces the future research directions.

1. Motivations

As often happens our research project has an empirical origin. Our involvement in some real decision aiding situations (see [2] and [3]) where the presence of ordinal information was a key characteristic of the decision aiding process and the evaluation model induced us to a more general reflection around this problem. An area in which the presence of ordinal information is a common characteristic is quality evaluation. Quality is a rather fashioned concept, key issue in large sectors of industrial production and services, quite ill defined, with no consensual properties and without any consolidated methodology for its evaluation. The existence of international standards on quality (of the type ISO 9000, ISO 9001 etc.) and its derivatives in different domains (see ISO 9126 in software) does not improve the situations since such standards lack any rigorous definition and method of quality evaluation.

A result of such a situation is that quality is often evaluated either on a basis of "expert subjective

measurement" (in the sense that experts may express a rough appreciation of quality on some characteristics) or directly from observations of nominal evaluations to which an implicit value judgment is associated.

Further on a key issue to pay attention to is the fact that usually a quality evaluation is done for some decision purpose (buy a product, choose a supplier, fund a project, become a certified service provider etc.). Quality thus become a kind of value judgment often introduced in a more general evaluation model together with cost and other financial dimensions. The result is that often quality results are summed to costs in a completely meaningless and unfair way.

Another domain characterized by the presence of ordinal information is the construction of indexes. Indexes of pollution, of soil vulnerability, of flood risk, of wealth, of economic growth, of social welfare etc.. In may of such indexes the rough information available is expressed on ordinal scales (or even nominal) and no attention is given in how such information contributes in defining the overall index. The use of indexes in Geographical Information Systems is an example of this type of problem.

A third domain concerns the general problem of clustering and classification. It is more and more often the case where clusters and/or categories have to be defined on the basis of information and values which are symbolic and/or nominal. Further on such clusters or categories are defined for some purpose and not independently from a decision process. Again the problem of how such information is aggregated in order to obtain a global classification is often neglected in practice, although may greatly affect the final result.

Our list of situations in which a more rigorous treatment of ordinal and nominal information is necessary could continue. We just tried to introduce by these examples the importance of defining a common conceptual framework for such problems within which establish some research projects.

2. A conceptual framework

In this section we briefly present a framework under which the problem of "ordinal measurement" can be analysed. In such a framework we distinguish very simply three parts:

- the input of the problem and how can be classified;
- the output of the problem and its characteristics;
- what occurs among input and output.

We will limit ourselves in listing some key issues we consider relevant without claiming our list being neither exhaustive nor definitive. The order of the listing is casual.

2.1 The input

Besides a classification of the information on the basis of the nature of the rough information available (which limits itself in distinguishing the type of scale associated to each attribute) we can consider some issues that may help in characterizing the problem.

- Presence of an hierarchical structure in the evaluation model. A lot of quality models have such a structure and some interesting questions concerning the propagation of information along the hierarchy can be settled.
- The "ordinal measurement" concerns a single decision problem (for instance, choose an offer or a supplier following a call for tenders in a bid) or is going to be used in a repeated way (for instance, establish a procedure for the choice of suppliers)?
- How many "clients" or decision makers are involved in the "ordinal measurement"? Who asked for the analysis? How many are involved in the decision process? Who is going to use the result? Supporting a single DM or a committee can make a big difference.
- What kind of uncertainty can be associated to the available information (if any)? Lack of knowledge, imprecision, ambiguous definition, ill defined concepts, all such problems require specific representation tools and it should be clear whether such tools exist or not and to what extent.
- Do they exist any global, intuitive, a priori evaluations or priorities that the DM can express on some of the objects to evaluate? If it is the case both the problem of learning from such examples and of being coherent is open.
- Is the DM able to express any general or partial classification rules although they may not correspond (immediately) to a rigorous evaluation model? Does the DM feel more comfortable with a model based on rules or with a model based on a classification algorithm (independently from the fact that they could be equivalent)?

2.2 The Output

Two major distinctions on the output can be observed besides some open theoretical problems concerning the concept of absolute evaluation.

What type of evaluations are expected by the DM as a result? From our experience three principal cases may occur:

- nominal evaluation: objects classified in categories which have no order among them; a problem well known in pattern recognition, clustering and classification (numerical and/or symbolic);

- ordinal evaluation: objects classified in categories which are just ordered on an ordinal scale; problem handled by the sorting problem statement procedures;
- interval evaluation: objects classified in categories for which is possible not only to order them, but also to measure the "distance" among them (on an interval or an absolute scale).

What kind of classification is pursued? Crisp (objects belong to one and only one category) or continuous (objects belong to some degree, possibly zero, to some categories)? In the second case the membership degree can be due to:

- imperfect knowledge on the objects, while the categories are well defined;
- to ill definition of the categories, while the objects are perfectly known;
- to both the above cases.

An open theoretic problem, since the sixties, concerns the concept of "absolute evaluation". Papers like [4] already pointed out that value functions widely used in decision making do not enable to express judgments of the type "good" or "bad" and therefore to classify objects in an "absolute" sense. Two approaches have been pursued in this direction (for a first introduction see [5]):

- the first which we may call the "deontic logic" approach tries to define an appropriate formalism by which express the concept "good", "good" being a predicate in a particular first order language (see [6] [7], and [1]);
- the second which try to define codomains for value functions equipped with a "neutral value", the one for which the DM is indifferent, he does not care for objects having such a value (see [4]).

We consider necessary to situate our research with respect to such approaches. For instance what does it mean a "neutral value" on an ordinal scale? Is it possible to combine the formalisms adopted for uncertainty modeling with the ones used in deontic considerations?

2.3 Between input and output

We consider that there exist three areas of interest in the way by which we manipulate the information in an "ordinal measurement" model.

The first concerns the concept of meaningfulness. To our opinion such a concept can be used in two different ways (not incompatible). Meaningfulness from the point of view of measurement theory, thus the conditions under which the manipulation of the information respects the nature and structure of the information itself. We may emphasize that the concept of meaningfulness is well

defined as far as the transformations of regular scales are concerned, but is less well defined when the problem of aggregation of measures is on the hand. Meaningfulness from the point of view of the decision aiding, thus the conditions under which the manipulation of the information results in something sound and clear for the DM and his problem. Meaningful (for measurement theory) manipulations can result in totally meaningless (for the DM) information. Such a concept of meaningfulness is still very little studied and not at all defined in literature.

The second concerns the problem of "learning". Often an evaluation model (and is more and more the case when models based on rules are considered) is induced by examples which help to learn about the DM preferences, the parameters of the model etc. Conventional approaches to learning focus their attention in the capacity of the induced model to explain the examples and reproduce them as well as possible. However, in a decision aiding context, learning is a reciprocal procedure for both the DM and the analyst. Under such a perspective a "non reproduction" of the examples may help the DM to detect inconsistencies, to enhance his perception of the objects, to modify his opinion and judgments. We call such a situation "constructive learning". It is not clear however, what a constructive learning algorithm should be and how it can be characterized.

The third concerns the uncertainty created by the manipulation itself. This is a common situation in all modeling activities since the construction of a model induces an abstraction of the reality which naturally generates some uncertainty at least under form of more or less confident to the model. In many cases (as in hierarchical models) the problem can be extremely severe. We claim that specific uncertainty modeling formalisms and procedures should be adopted for such particular situations.

Conclusions

From this brief introduction to the subject of ordinal measurement is clear that much of the research is yet to be done. It is out of the scope of this paper to indicate any priorities among the subjects introduced. We may just emphasize that is possible to identify specific research scenarii of the type "classification to ordinal categories by rules induced by examples" or "continuous classification to nominal categories" which may affect several issues among the ones raised in the previous sections. Further on we claim that more and more application fields will require the use of tools, methods and methodologies developed under the "ordinal measurement" approach.

References

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