USING ASSIGNMENT EXAMPLES TO INFER WEIGHTS FOR ELECTRE TRI METHOD: SOME EXPERIMENTAL RESULTS

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Abstract

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Given a finite set of alternatives A, the sorting (or assignment) problem consists in the assignment of each alternative to one of the predefined categories. In this paper, we are interested in multiple criteria sorting problems and, more precisely, in the existing method ELECTRE TRI. This method requires the elicitation of preferential parameters (weights, thresholds, category limits,...) in order to construct a preference model with which the Decision Maker (DM) accept as a working hypothesis in the decision aid study. A direct elicitation of these parameters requiring a high cognitive effort from the DM, [MS98] proposed an interactive aggregation-disaggregation approach that infer ELECTRE TRI parameters indirectly from holistic information, i.e., assignment examples. In this approach, the determination of ELECTRE TRI parameters that best restitute the assignment examples is formulated through a non-linear optimization program.
In this paper, we consider the subproblem of the determination of the weights only (the thresholds and category limits being fixed). This subproblem leads to solve a linear program (rather than non-linear in the global inference model). Numerical experiments were conducted so as to check the abilities of this disagregation tool. Results showed that this tool is able to infer weights that restitutes in a stable way the assignment examples and that it is able to identify "inconsistencies" in the assignment examples.

Key words: *Sorting Problematic, Preference Disaggregation, Weight Elicitation, Numerical Experiments*

1 Introduction

When modeling a real world decision problem, we can face situations in which the decision can be formulated in terms of the assignment of a set of potential alternatives $A = \{a_1, a_2, ..., a_l\}$ to one of pre-defined categories. The assignment of an alternative a to the appropriate category should rely on the intrinsic value of a (and not on the comparison of a to other alternatives from A).

In this paper, we are interested in the multiple criteria sorting problematic and, more precisely, in an existing multiple criteria method called ELECTRE TRI (see [Yu92a], [MSZ99a], [MSZ99b] and [RB93]). When using this method, the analyst must determine values of several parameters (profiles that define the limits between the categories, weights, discrimination thresholds, ...). The set π of these parameters is used to construct a preference model with which the Decision Maker (DM) accept as a working hypothesis in the decision aid study. Apart from some very specific cases, it is not realistic to assume that the DM would be able to give explicitly the values of each parameter in π. They are far different from the natural terms in which the DM usually expresses his/her preferences and expertise.

A realistic approach consists in infering the model parameters of ELECTRE TRI through an analysis of assignment examples given by the DM, i.e., from hollistic information on his/her judgments. This approach aims at substituting assignment examples for direct elicitation of the model parameters. The values of the parameters will be inferred through a certain kind of regression on assignment examples.

[MS98] proposed an approach that infer all ELECTRE TRI parameters simmultaneaously starting from assignment examples. In this approach, the determination of the parameter's values that best fit the assignment examples given by a decision maker (DM) stems from the resolution of a non-linear mathematical program. This optimization procedure is integrated in an interactive tool that enable the DM (or anyone acting on his/her account) to react on the set of obtained parameters and to get insights on his/her preferences.

Although [MS98] proposed to infer simmultaneously weights, profiles and thresholds, we consider in this paper the problem of the inference of the weight vector only (in this particular case the mathematical program to be solved becomes linear). Our paper presents numerical results obtained in a laboratory experiment aiming at validating the practical usefullness of the weight inference procedure in an interactive process (preliminary results may be found in [Nau96]). The experimental questions are the following (they are operationalized through the experimental design described in section 4.):

- Let w^{opt} be the weight vector obtained using the linear optimization procedure on the basis of the assignment to categories of alternatives from a set $A^* \subset A$. Let w^{dm} be a weight vector expressed by the DM or infered by the analyst from DM's assertions. Are the assignments of alternatives from A^* more "stable" when using w^{opt} than when considering w^{dm} (the term stable is used as unsensitive of the assignments to changes of the weight vector). In other words, is the tool able to increase the "stability" of assignments of alternatives in a set A^*?

- The obtained weight vector w^{opt} depend on the information given as input, i.e., on the set of assignment examples. What is the average amount of information necessary to "calibrate" the model in a satisfactory way? How large should A^* be in order to derive w^{opt} in a reliable manner?

- In practical decision situations, real DMs do not always provide reliable information. Due to time constraints and cognitive limitations, DMs express contradictory information, their preferences change over time... The optimization procedure should be able to highlight the assignment examples that are contradictory or not representable through the ELECTRE TRI preference model. This experiment aims at investigating the ability of the tool to "identify" the inconsistencies in the DM's statements in order to help him/her in revising the preference information. How reliable is the optimization procedure to identify inconsistencies in the DM's judgments?

- The output of the optimization phase rely on the choice of an objective function. As different objective functions can be considered, it is important to check the variability of the output to the different functions. Does the choice of a specific objective function strongly impact the results?

The paper is organized as follows. In the next section, we present the general approach used by the inference tool. A brief description of the ELECTRE TRI method is given in section 3. Section 4 describes the experimental design. The final section groups results and conclusions.

2 General Scheme of the Approach

The general scheme of our inference procedure is presented in Figure 1. Its aim is to find an ELECTRE TRI model as compatible as possible with the assignment examples given by the user (the user being either the DM himself/herself or anyone acting on his/her account). The assignment examples concern a subset $A^* \subset A$ of alternatives for which the user has clear preferences, i.e., alternatives that the user can easily assign to a category, taking into account their evaluation on all criteria. The compatibility between the ELECTRE TRI model and the assignment examples is understood as an ability of the ELECTRE TRI method using this model to reassign the alternatives from A^* in the same way as the user did.

Figure 1: General scheme of the inference procedure

In order to minimize the differences between the assignments made by ELECTRE TRI and the assignments made by the user, an optimization procedure is used. The resulting ELECTRE TRI model is denoted by M_π. The user can tune the model in the course of an interactive procedure. He/she may either revise the assignment examples or fix values (or intervals of variation) for some model parameters. In the former case, the user may:

- remove and/or add some alternatives from/to A^*,
- change the assignment of some alternatives from A^*.

In the latter case, the user can give additional information on the range of variation of some model parameters basing on his/her own intuition. For example, he/she may specify:

- ordinal information on the importance of criteria,
- noticeable differences on the scales of criteria,

- incomplete definition of some profiles defining the limits between categories.

When the model is not perfectly compatible with the assignment examples, the procedure should be able to detect all "hard cases", i.e., the alternatives for which the assignment computed by the model strongly differs from the user's assignment. The user could then be asked to reconsider his/her judgment.

Inferring a form of knowledge from examples of expert's decisions is a typical approach of artificial intelligence. Induction of rules or decision trees from examples in machine learning (see [Mic83], [Qui86]), knowledge acquisition based on rough sets (see [GB92], [PS94], [Slo92]), supervised learning of neural nets (see [Gal93], [WK91]) are well-known representatives of this approach. The appeal of this approach is that the experts are typically more confident exercising their decisions than explaining them.

In Multiple Criteria Decision Analysis, this approach is concordant with the principle of posterior rationality (see [Mar88]) and with the aggregation-disaggregation logic used for the construction of a preference model in UTA-like procedures (see [JLS82], [JLMS87], [JL90], [NMK92], [Slo91]). It has been also applied for the elicitation of weights used for the construction of an outranking relation in the DIVAPIME method (see [Mou95] and [Mou93]).

3 Presentation of the ELECTRE TRI method

ELECTRE TRI is a multiple criteria assignment method, i.e., a method that assigns alternatives to predefined ordered categories. The limit between two consecutive categories is formalized by what we call a profile (see Figure 2). The assignment of an alternative a results from the comparison of a with the profiles defining the limits of the categories. Let F denote the set of the indices of the criteria $g_1, g_2, ..., g_m$ ($F = \{1, 2, ..., m\}$) and B the set of indices of the profiles defining $p+1$ categories (B=$\{1, 2, ..., p\}$), b_h being the upper limit of category C_h and the lower limit of category C_{h+1}, $h = 1, 2, ..., p$ (see Figure 2). In what follows, we will assume, without any loss of generality, that preferences increase with the value on each criterion.

ELECTRE TRI uses an outranking relation S (see [Roy91]), i.e., validates or invalidates the assertion aSb_h (and b_hSa), whose meaning is "a is at least as good as b_h". Preferences restricted to the significance axis of each criterion are defined through pseudo-criteria (see [Roy96], [RV84] for details on this double-threshold preference representation). The indifference and preference thresholds, $q_j(b_h)$ and $p_j(b_h)$, constitute the intra-criterion preferential information.

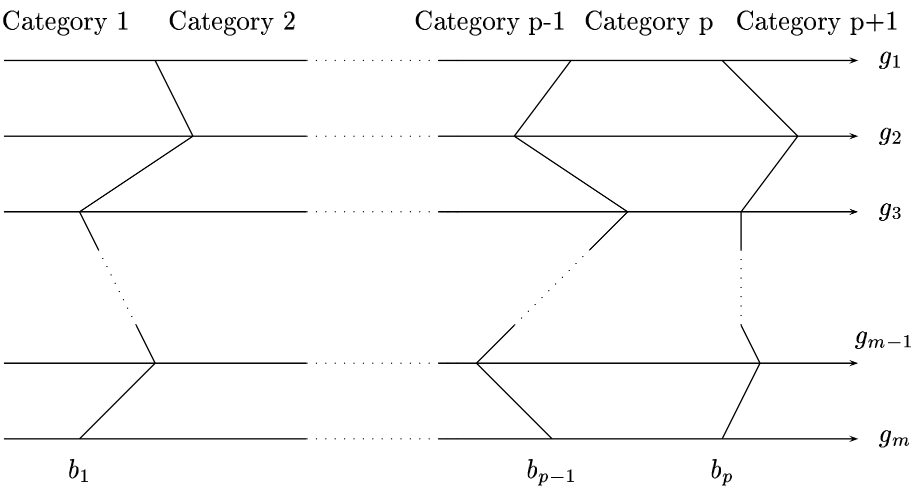

Figure 2: Definition of categories using limit profiles

They account for the imprecise nature of the evaluations g_j(a) (see [Roy89]). The indifference threshold $q_j(b_h)$ specifies the largest difference $g_j(a) - g_j(b_h)$ that preserves indifference between a and b_h on criterion g_j while the preference threshold $p_j(b_h)$ represents the smallest difference $g_j(a)-g_j(b_h)$ compatible with a preference in favor of a on criterion g_j.

At the comprehensive level of preferences, in order to validate the assertion aSb_h (or b_hSa), two conditions should be verified:

- concordance: for an outranking aSb_h (or b_hSa) to be accepted, a "sufficient" majority of criteria should be in favor of this assertion,
- non-discordance: when the concordance condition holds, none of the criteria in the minority should oppose to the assertion aSb_h (or b_hSa) in a "too strong way".

Two types of inter-criteria preference parameters intervene in the construction of S:

- the set of weight-importance coefficients $(w_1, w_2, ..., w_m)$ is used in the concordance test when computing the relative importance of the coalitions of criteria being in favor of the assertion aSb_h,
- the set of veto thresholds $(v_1(b_h), ..., v_j(b_h), ..., v_m(b_h))$ is used in the discordance test; $v_j(b_h)$ represents the smallest difference $g_j(b_h) - g_j(a)$ incompatible with the assertion aSb_h.

ELECTRE TRI builds an index $\sigma(a, b_h) \in [0, 1]$ ($\sigma(b_h, a)$, resp.) that represents the degree of credibility of the assertion aSb_h (b_hSa, resp.), $\forall a \in A, \forall h \in$

B. The assertion aSb_h (b_hSa, resp.) is considered to be valid if $\sigma(a,b_h) \geq \lambda$ ($\sigma(b_h,a) \geq \lambda$, resp.), λ being a "cutting level" such that $\lambda \in [0.5, 1]$ (see [RB93] for a justification of the construction of this index).

Determining $\sigma(a,b_h)$ consists of the following steps (the value of $\sigma(b_h,a)$ is computed analogously):

1 - compute the partial concordance indices $c_j(a,b_h)$, $\forall j \in F$:

$$c_j(a,b_h) = \begin{cases} 0 \text{ if } g_j(b_h) - g_j(a) \geq p_j(b_h) \\ 1 \text{ if } g_j(b_h) - g_j(a) \leq q_j(b_h) \\ \frac{p_j(b_h)+g_j(a)-g_j(b_h)}{p_j(b_h)-q_j(b_h)} \text{ otherwise} \end{cases} \quad (1)$$

2 - compute the comprehensive concordance index c(a,b_h):

$$c(a,b_h) = \frac{\sum_{j\in F} w_j c_j(a,b_h)}{\sum_{j\in F} w_j} \quad (2)$$

3 - compute the discordance indices $d_j(a,b_h), \forall j \in F$:

$$d_j(a,b_h) = \begin{cases} 0 \text{ if } g_j(a) \leq g_j(b_h) + p_j(b_h) \\ 1 \text{ if } g_j(a) > g_j(b_h) + v_j(b_h) \\ \frac{g_j(b_h)-g_j(a)-p_j(b_h)}{v_j(b_h)-p_j(b_h)} \text{ otherwise} \end{cases} \quad (3)$$

4 - compute the credibility index $\sigma(a,b_h)$ of the outranking relation:

$$\sigma(a,b_h) = c(a,b_h) \prod_{j\in \overline{F}} \frac{1-d_j(a,b_h)}{1-c(a,b_h)}, \quad (4)$$

$$\text{where } \overline{F} = \{j \in F : d_j(a,b_h) > c(a,b_h)\}$$

The values of $\sigma(a,b_h)$, $\sigma(b_h,a)$ and λ determine the preference situation between a and b_h:

- $\sigma(a,b_h) \geq \lambda$ and $\sigma(b_h,a) \geq \lambda \Rightarrow aSb_h$ and $b_hSa \Rightarrow aIb_h$, i.e., a is indifferent to b_h,
- $\sigma(a,b_h) \geq \lambda$ and $\sigma(b_h,a) < \lambda \Rightarrow aSb_h$ and not $b_hSa \Rightarrow a \succ b_h$, i.e., a is prefered to b_h (weakly or strongly),
- $\sigma(a,b_h) < \lambda$ and $\sigma(b_h,a) \geq \lambda \Rightarrow$not aSb_h and $b_hSa \Rightarrow b_h \succ a$, i.e., b_h is prefered to a (weakly or strongly),
- $\sigma(a,b_h) < \lambda$ and $\sigma(b_h,a) < \lambda \Rightarrow$not aSb_h and not $b_hSa \Rightarrow aRb_h$, i.e., a is incomparable to b_h.

Two assignment procedures are then available (the role of these exploitation procedures is then to analyse the way in which an alternative a compare to the profiles so as to determine the category to which a should be assigned) :

Pessimistic (or conjunctive) procedure:

a) compare a successively to b_i, for i=p,p-1, ..., 1,
b) b_h being the first profile such that aSb_h,
assign a to category C_{h+1} ($a \to C_{h+1}$).

Optimistic (or disjunctive) procedure:

a) compare a successively to b_i, i=1, 2, ..., p,
b) b_h being the first profile such that $b_h \succ a$,
assign a to category C_h ($a \to C_h$).

If b_{h-1} and b_h denote the lower and upper profile of the category C_h, the pessimistic (or conjunctive) procedure assigns alternative a to the highest category C_h such that a outranks b_{h-1}, i.e., aSb_{h-1}. When using this procedure with $\lambda = 1$, an alternative a can be assigned to category C_h only if $g_j(a)$ equals or exceeds $g_j(b_h)$ (up to a threshold) for each criterion (conjunctive rule).

The optimistic (or disjunctive) procedure assigns a to the lowest category C_h for which the lower profile b_h is prefered to a, i.e., $b_h \succ a$. When using this procedure with $\lambda = 1$, an alternative a can be assigned to category C_h when $g_j(b_h)$ exceeds $g_j(a)$ (up to a threshold) at least for one criterion (disjunctive rule). When λ decreases, the conjunctive and disjunctive characters of these rules are weakened.

4 The Optimization Procedure

The set π of parameters of an ELECTRE TRI model are:

- the profiles defined by their evaluations $g_j(b_h)$, $\forall j \in F$, $\forall h \in B$,
- the importance coefficients w_j, $\forall j \in F$,
- the indifference and preference thresholds $q_j(b_h)$, $p_j(b_h)$, $\forall j \in F$, $\forall h \in B$,
- the veto thresholds $v_j(b_h)$, $\forall j \in F$, $\forall h \in B$.

The conducted experiment considered the case where the profiles and thresholds are known and where the weights are to be inferred. Moreover, we will confine our analysis to the case were the pessimistic assignment procedure is used.

4.1 Variables of the problem

In ELECTRE TRI pessimistic assignment procedure, an alternative a_k is assigned to category C_h (b_{h-1} and b_h being the lower and upper profiles of C_h, respectively) iff $\sigma_\pi(a_k, b_{h-1}) \geq \lambda$ and $\sigma_\pi(a_k, b_h) < \lambda$ (where σ_π is the credibility index related to the set of parameters π).
Let us suppose that the DM has assigned the alternative $a_k \in A^*$ to category C_{h_k} ($a_k \rightarrow C_{h_k}$). Let us define the slack variables x_k and y_k such that $\sigma_\pi(a_k, b_{h_k-1}) - x_k = \lambda$ and $\sigma_\pi(a_k, b_{h_k}) + y_k = \lambda$.

The optimization problem will include the following variables:

x_k, y_k, $\forall k$ such that $a_k \in A^*$	slack variables ($2n$)
λ	cutting level (1)
w_j, $\forall j \in F$	importance coefficients (m)

4.2 An accuracy criterion

If the values of the slack variables x_k and y_k are both positive then ELECTRE TRI pessimistic assignment procedure will assign alternative a_k to the "correct" category. If, however, one or both of these values are negative, the ELECTRE TRI pessimistic assignment procedure will assign alternative a_k to a "wrong" category. The lower the minimum of these two values, the less adapted is the model M_π to give an account of the assignment of a_k made by the DM. Moreover, if x_k and y_k are both positive then a_k is assigned consistently with the DM's statement for all $\lambda' \in [\lambda - y_k, \lambda + x_k]$.

Let us consider now the set of alternatives $A^* \subset A$ where $card(A^*) = n$ and suppose that the DM has assigned the alternative a_k to the category C_{h_k}, $\forall a_k \in A^*$. The model M_π will be consistent with the DM's assignments iff $x_k \geq 0$ and $y_k \geq 0$, $\forall k$ such that $a_k \in A^*$.

Consistently with the preceding argument, an accuracy criterion can be defined as:

$$\max_{k:a_k \in A^*} \{\min \{x_k, y_k\}\} \tag{5}$$

We obtain a standard MaxMin problem. If the accuracy criterion takes a non-negative value then all alternatives contained in A^* are "correctly" assigned for all $\lambda' \in [\lambda - min_{k:a_k \in A^*}\{y_k\}, \lambda + min_{k:a_k \in A^*}\{x_k\}]$.

This criterion, however, takes into account the "worst case" only, i.e., the alternative for which the ELECTRE TRI model gives the most different assignment from the DM. An accuracy criterion should be able to take into account an average information concerning the accuracy of the model, i.e., its overall ability to assign the alternatives from A^* to the "correct" category. Hence, we propose to replace criterion (5) by the following one:

$$\max_{k:a_k \in A^*} \{\min\{x_k, y_k\} + \epsilon \sum_{k:a_k \in A^*} (x_k + y_k)\} \tag{6}$$

where ϵ is a small positive value. (6) can be rewritten as :

$$\max\{\alpha + \epsilon \sum_{k:a_k \in A^*} (x_k + y_k)\} \tag{7}$$

$$s.t. \quad \alpha \leq x_k, \quad \forall k \text{ such that } a_k \in A^* \tag{8}$$

$$\alpha \leq y_k, \quad \forall k \text{ such that } a_k \in A^* \tag{9}$$

4.3 Constraints of the problem

The constraints of the optimization problem are the following:

$\sigma_\pi(a_k, b_{h_k-1}) - x_k = \lambda, \forall k : a_k \in A^*$	definition of the slack variables x_k (n)
$\sigma_\pi(a_k, b_{h_k}) + y_k = \lambda, \forall k : a_k \in A^*$	definition of the slack variables y_k (n)
$\alpha \leq x_k, \alpha \leq y_k, \forall k : a_k \in A^*$	definition of α $(2n)$
$\lambda \in [0.5, 1]$	interval of variation for λ (2)
$w_j \geq 0, \forall j \in F$	non-negativity constraints (m)

Additional constraints can be added in the course of the interactive procedure in order to take into account an intuitive view of the DM on the value of some parameters. For instance, if the DM does not consider any criterion as a dictator, an appropriate constraint is: $w_j \leq \frac{1}{2} \sum_{i=1}^{m} w_i, \quad \forall j \in F$

4.4 Optimization problem to be solved

The basic form of the optimization problem to be solved is the following:

$$\max \quad \{\alpha \quad + \quad \epsilon \sum_{k:a_k \in A^*} (x_k + y_k)\} \tag{10}$$

$$s.t. \quad \alpha \leq x_k, \quad \forall k \text{ such that } a_k \in A^* \tag{11}$$

$$\alpha \leq y_k, \quad \forall k \text{ such that } a_k \in A^* \tag{12}$$

$$\sum_{j=1}^{m} w_j c_j(a_k, b_{h_k-1}) \ - x_k = \lambda \quad \forall k \text{ such that } a_k \in A^* \tag{13}$$

$$\sum_{j=1}^{m} w_j c_j(a_k, b_{h_k}) \ + y_k = \lambda, \quad \forall k \text{ such that } a_k \in A^* \tag{14}$$

$$\sum_{j=1}^{m} w_j = 1 \quad (15)$$

$$\lambda \in [0.5, 1] \quad (16)$$

$$w_j \geq 0, \quad \forall j \in F \quad (17)$$

As the objective function and all constraints are linear, the above problem is a linear programming problem. It contains $2n+m+1$ variables and $4n+m+2$ constraints. Let us remark that the slack variables x_k and y_k can be eliminated from the problem formulation since they are defined by the constraints (13) and (14). This elimination reduces the number of variables.

5 Experimental Design

This experiment is a laboratory work, i.e., takes its material in a past real world case study to perform a posteriori computations in order to test the operational validity of the optimization model proposed in §4. The data considered comes from the real world application described in [Yu92a] and [Yu92b].

This application considers the problem of assigning a set A of 100 alternatives ($A = \{a_1, a_2, ..., a_{100}\}$ is described in [MFN97]) to three (the initial data specified 5 categories; we grouped the three top categories (C_3, C_4 and C_5) as none of the alternatives were assigned by the ELECTRE TRI model to C_4 and C_5) ordered categories C_1, C_2 and C_3 (two limit profiles b_1 and b_2 define the "frontiers" C_1-C_2 and C_2-C_3) on the basis of 7 criteria (preferences on all criteria are decreasing with the evaluations, i.e., the lower the better).

As no interaction with the DM is possible, we consider the assignment of ELECTRE TRI pessimistic assignment procedure (with the parameters given in [Yu92a]) as assignment examples expressed by a "fictitious" DM. The experimental method consist in using the optimization procedure with different subsets of assignment examples to infer the weights that "best" match with the examples (with the given values for profiles and thresholds).

So as to get consistent results, we generate 80 subsets of A, the cardinality of these subset being either 6, 12, 18, 24, 30, 36, 42 or 48 (10 sets of each size were generated). Each of these subsets is conceived so that the alternatives are assigned uniformly on the three categories. Let us denote A_i^j the j^{th} set of size i. In order to test the ability of the optimization procedure to identify inconsistent information (see §6.3), we consider Err_i^j derived from A_i^j in which an alternative is volontarily assigned to a "wrong" category. The error introduced consist in changing the assignment of an alternative (for example, assigning to C_1 an alternative that should be assigned to C_2). Different types of errors were considered as shown in the table 1. The sets of assignment examples for which computations were performed are described in [MFN97].

Initial Cat.	Error Cat.	Number in each sample
C_1	C_2	2
C_2	C_1	2
C_2	C_3	2
C_3	C_2	2
C_1	C_3	1
C_3	C_1	1

Table 1: type of errors introduced in the sets

The mathematical program corresponding to each set Err_i^j and A_i^j has been solved with different objective functions (see §4.2). The general form of the considered objective function z to be maximized is $z = \min_{k:a_k \in A^*} \{x_k, y_k\} + \epsilon \sum_{k:a_k \in A^*} (x_k + y_k)$ and computations have been performed for $\epsilon = 10^{-3}, 10^{-2}$, $10^{-1}, 1, 10^1, 10^2$. This allows us to check for the variability of the output to the choice of an objective function.

6 Results

The computations have been performed using Cplex on a Sun Sparc 5 workstation with 32 MB memory. Considering the small size of the problem, the computing time never exceeded 0.1 sec.

Before to state the results, it is important to mention that these results are dependent on the data under consideration. The proposed general implications should be understood taking into account this restriction. The reader will find detailled numerical results in [MFN97].

6.1 Is the tool able to increase the "stability" of assignments of alternatives?

Let w^{opt} be the weight vector obtained using the optimization procedure on the basis of the assignment to categories of alternatives from a set A_i^j. Let w^{dm} be the weight vector used to generate the assignment examples. A first validation of the usefullness of the optimization procedure is to check if the assignments of alternatives from A_i^j are more "stable" when using w^{opt} than when considering w^{dm}, i.e., is the tool able to increase the "stability" of assignments of alternatives in a set A_i^j ?

So as to answer this question, we will use the following methodology. In the mathematical program to be solved, the variable α is introduced to transform a MaxMin objective into a Max objective and represents the minimum value among the slack variables x_k and y_k (§4.2). The larger α, the more stable are the assignments of alternatives in A_i^j. The assignments are said to be stable if they are not affected by a modification of the cutting level λ (or of the weights).

Let us denote $\alpha_{dm}(A_i^j)$ the maximum variation on the cutting level λ preserving correct assignment of alternatives from A_i^j with the initial weights, i.e. those given in [Yu92a]. Let us denote $\alpha_{opt}(A_i^j)$ the maximum variation on the cutting level λ preserving correct assignment of alternatives from A_i^j with the weights obtained using the optimization procedure. The improvement of the stability of the assignments provided by the procedure can be evaluated by $\alpha_{opt}(A_i^j) - \alpha_{dm}(A_i^j)$. Table 2 gives the numerical results.

Size: i	$\overline{\alpha}_{opt}(i)$	$\overline{\alpha}_{dm}(i)$	$\overline{\alpha}_{opt}(i) - \overline{\alpha}_{dm}(i)$
6	0.24	0.08	0.16
12	0.23	0.02	0.21
18	0.21	0.08	0.13
24	0.20	0.02	0.18
30	0.19	0.08	0.11
36	0.18	0.03	0.15
42	0.18	0.02	0.16
48	0.10	0.02	0.08
		mean	0.15

Table 2: Improvement of the "stability" of assignments

Considering these results, we can observe:

- Firstly the results show that the larger the set of assignment examples, the less stable the assignments, i.e., the more sensitive are these assignments to a change in weights. This a straightfoward evidence as each assignment example adds two constraints to the program to be solved (see §4.3).
- Secondly, these results show a significant improvement of the stability of the assignments whatever the size of the set of examples (mean value:

0.15). This proves the ability of the optimization procedure to perform "good" weights that enables ELECTRE TRI to reassign the alternatives in a very stable way.

6.2 Which "amount" of information is necessary to infer the weights in a reliable way?

In order to infer in a reliable way a weight vector w^{opt}, the optimization procedure requires information as input, i.e., on the set of assignment examples. What is the amount of information necessary to "calibrate" the model in a satisfactory way? How large should A^* be in order to derive w^{opt} in a reliable manner? This question is essential for practical use to the inference model in real world decision problems. The analyst should have some simple guidelines to manage the interaction with the DM avoiding unnecessary question, but collecting a sufficient information.

In order to determine a "reasonable amount of information" to infer the weights, we use the following experimental scheme: the optimization procedure is performed using different sets of assignment examples, whose size varies from 6 to 48 (10 set for each size, see §5). We observe then the ability of ELECTRE TRI using the infered weights to assign "correctly" the whole set of 100 alternatives. Obviously the ability of ELECTRE TRI using the infered weights to reassign all alternatives correctly increases with the size of the set from which the weights are derived. However, the number of assignment examples expressed by the DM should not be to large.

Let us denote by $\alpha^{100}_{opt}(A^j_i)$ the maximum variation on the cutting level λ perserving correct assignment for all 100 alternatives with the weights inferred from A^j_i. Let $\overline{\alpha}^{100}_{opt}(i)$ be the mean value of the $\alpha^{100}_{opt}(A^j_i)$, for all sets A^j_i of size i.

Let us denote by $err^{100}_{opt}(A^j_i)$ the number of "wrong" assignments among the 100 alternatives with the weights inferred from A^j_i. Let $\overline{err}^{100}_{opt}(i)$ be the mean value of the $err^{100}_{opt}(A^j_i)$, for all sets A^j_i of size i. The results of the computations are grouped in table 3:

As foreseen, the results show that $\overline{\alpha}^{100}_{opt}(i)$ increases and $\overline{err}^{100}_{opt}(i)$ decreases with the size i of the set. Moreover, $\overline{\alpha}^{100}_{opt}(i)$ becomes positive for $12 < i < 18$; such a positive value means that weights inferred from a set of i assignment examples is able (in mean value) to reassign correctly all 100 alternatives.

The number of parameters to be inferred (weights w_j) depend on the number of criteria only. Considering the above results, $2 \times m$ (m being the number of criteria) seems to be a reasonable number of assignment examples to infer the

Size: i	$\overline{\alpha}^{100}_{opt}(i)$	$\overline{err}^{100}_{opt}(i)$
6	-0.093	4.4
12	-0.088	4.0
18	0.076	0.6
24	0.129	0.2
30	0.157	0.0
36	0.112	0.4
42	0.164	0.0

Table 3: Information required to infer weights reliably

weights in a reliable way (as 7 criteria are considered, $12 < 2m < 18$). However, it is important to notice that using a set A^j_i of $2m$ assignment examples does not always infer weights such that $err^{100}_{opt}(A^j_i) = 0$, i.e., some alternatives mights be uncorrectly reassigned. Nevertheless, the $2m$ seems to us a good balance between number of examples required from the DM (necessary limited) and the reliability of the inferred weights. This result needs to be reinforced by a replication of this experiment, particularily in the case where the number of categories exceeds 3.

6.3 Is the tool able to identify the inconsistencies in the DM's assertions?

The optimization procedure that is tested in this experiment is conceived to be integrated in an interactive tool briefly described in §2. In practical decision situations, real DMs do not always provide reliable information. Due to time constraints and cognitive limitations, DM's preferences evolve over time, contains contradictory or inconsistent information. The role of an interacive tool is to help the DM to learn about his/her preferences and their possible representation in a specific aggregation model. Inconsistencies occur when the DM's preferences (in our case a set of assignment examples) can not be expressed through the preference model that is used (ELECTRE TRI in our case). In such cases, it is important to extract from the expressed preferences the inconsistent pieces of information, i.e., the most untypical or contradictory assignment examples. Consequently, a fundamental experimental issue concerns the ability of the tool to identify the inconsistencies in the DM's statements: identifying inconsistencies will help the DM in revising the expressed assertions in order for his/her preference to match the used preference model.

In order to test the ability of the optimization procedure to identify inconsistent information, we consider the sets Err_i^j derived from A_i^j in which an alternative is volontarily assigned to a "wrong" category. The error introduced consist in changing the assignment of an alternative (for example, assigning to C_1 an alternative that should be assigned to C_2).

So as to know if the opimization procedure is able to identify an inconsistency, we will ground on the following idea. Let $a_{k^{err}}$ be the alternative wrongly assigned in Err_i^j, $x_{k^{err}}$ and $y_{k^{err}}$ being the corresponding slack variables (see §4.1). Let us recall that the variable α correspond to the minimum of x_k and y_k for all alternatives in Err_i^j. The alternatives that are the "most difficult" to assign are those (in the interactive process, these alternatives are those which should be proposed to the DM in order to revise the assignments) for which $x_k = \alpha$ or $y_k = \alpha$. Hence, we will consider the error or inconsistency to be "discovered" if $x_{k^{err}} = \alpha$ or $y_{k^{err}} = \alpha$. If $a_{k^{err}}$ is identified as one of the alternative the most diffficult to assign, it might not be the only alternative for wich one of the slack variable equal α. Let $n(Err_i^j)$ denote the number of such alternatives; the lower $n(Err_i^j)$, the more accurate is the identification. We denote $\overline{n}(i)$ the mean value for $n(Err_i^j)$, for all j.

In the result, we observe that the error is always identified ($x_{k^{err}} = \alpha$ or $y_{k^{err}} = \alpha$). Table 4 gives the numerical results:

Size: i	$\overline{\alpha}(i)$	$\overline{n}(i)$	$\frac{\overline{n}(i)}{i}$
6	0.00	4.3	71.7%
12	0.01	6.9	57.5%
18	-0.01	8.8	48.9%
24	-0.01	8.3	34.6%
30	0.01	15.2	50.7%
36	0.01	13.0	36.1%
42	-0.02	12.4	29.5%
48	-0.02	14.1	29.4%

Table 4: Identification of "errors"

Unsurprisingly, we observe a degradation of the value of $\overline{\alpha}(i)$ compared to its value in the case of the initial assignment sets, i.e., without errors (see table 2). Secondly, though the errors are systematically identified, the number of alternatives $\overline{n}(i)$ is increasing with the size i of the sets; however, the proportion of such alternatives is decreasing with i. Finally the results show that the optimization procedure has a good ability to identify suspicious assignments.

6.4 Is the output sensitive to the choice of an objective function?

The output of the optimization phase rely on the choice of an objective function. As different objective functions can be considered, it is important to check the variability of the output to the different functions.

In this study, we investigate a class of objective functions $z(\epsilon)$ to be maximized of the form (see §4.2):

$$z(\epsilon) \ = \min_{k:a_k \in A^*} \{x_k, y_k\} + \epsilon \sum_{k:a_k \in A^*} (x_k + y_k) \tag{18}$$

These objective functions $z(\epsilon)$ aggregate two components:

- a first component which leads to an optimum that account only for the alternative that is the most difficult to assign correctly,
- a second additive component in which a "stable" assignment may be compensated by a less stable one; this second component account for the overall ability of the obtained weight vector to assign the alternatives correctly.

The parameter ϵ enables to tune $z(\epsilon)$ in direction of one of its two components ($\epsilon = 0$ leads to a standard MaxMin criterion while a sufficiently large value for ϵ leads to an additive criterion).

In our experiment, we perform the computations for $\epsilon = 10^{-3}, 10^{-2}, 10^{-1}, 10^0, 10^1, 10^2$ and for each set $A_i^j, i = 6, 12, 24, 30, 36, 42, 48, j = 1, ..., 10$. Firstly, we can observe that different values for ϵ leads to different results. Obviously, we observe that optimal value for α decreases when ϵ increases and $\frac{1}{card(A^*)} \sum_{k:a_k \in A^*} (x_k + y_k)$ increases with ϵ. In other words, the stability of the "worst case" deteriorates while the "mean stability" increases when we emphasize the additive component of the objective function.

A more interesting point deals with the comparative reliability of the weights w_{maxmin} and w_{add} infered using a pure maxmin and a standard additive objective function. More precisely, what is the ability of w_{maxmin} and w_{add} to reassign correctly the whole set of 100 alternatives. Table 5 and 6 presents the values of $\overline{\alpha}_{opt}^{100}(i)$ and $\overline{err}_{opt}^{100}(i)$ (see §6.2).

We observe that:

- when the set of assignment examples is unsufficiently large ($\leq$ 12, see §6.2), w_{maxmin} and w_{add} are equally acurate,

		Size: i $\longrightarrow$						
		6	12	18	24	30	36	42
ϵ	0.001	-0.093	-0.088	0.076	0.129	0.157	0.112	0.164
$\downarrow$	0.01	-0.093	-0.088	0.076	0.129	0.157	0.112	0.164
	0.1	-0.093	-0.088	0.076	0.129	0.157	0.112	0.164
	1	-0.093	-0.088	0.076	0.129	0.138	0.112	0.155
	10	-0.093	-0.088	0.050	0.095	0.114	0.083	0.121
	100	-0.093	-0.088	0.050	0.095	0.114	0.083	0.121

Table 5: $\overline{\alpha}^{100}_{opt}(i)$: ability of w to reassign alternatives

		Size: i $\longrightarrow$						
		6	12	18	24	30	36	42
ϵ	0.001	4.4	4.0	0.6	0.2	0.0	0.4	0.0
$\downarrow$	0.01	4.4	4.0	0.6	0.2	0.0	0.4	0.0
	0.1	4.4	4.0	0.6	0.2	0.0	0.4	0.0
	1	4.4	4.0	0.6	0.2	0.0	0.4	0.0
	10	4.4	4.0	1.2	0.2	0.0	0.4	0.0
	100	4.4	4.0	1.2	0.2	0.0	0.4	0.0

Table 6: $\overline{err}^{100}_{opt}(i)$: number of uncorrect reassignments

- when sufficient information is provided, the maxmin criterion leads to slightly more robust weights,
- the number of uncorrectly reassigned alternatives are almost always equal when using w_{maxmin} or w_{add}.

While both objective functions give good results, a slight advantage is observed in favor of the maxmin criterion in terms of the stability of reassignments.

7 Conclusions

This paper presents an experimental validation of a procedure aiming at inferring the weights of the ELECTRE TRI method on the basis of assignment examples (see [MS98]). The performances of this procedure were tested together with its ability to be integrated in an interactive process based on the aggregation-disaggregation paradigm. In conclusion, we can state the following:

- the results show that the inference procedure derives weights that assign (using ELECTRE TRI) the examples to the "correct" category in a stable way,
- experimental results suggest that a "reasonable" number of assignment examples to infer the weights reliably is $2m$, m being the number of criteria,
- the inference procedure shows a good ability to detect inconsistencies in the user's assertions; this property is particularily important in the perspective of its integration in an interactive process,
- the different objective functions tested did not provided significantly different results in terms of reassignment performances.

Although these results depend on the data under consideration, the empirical results seems robust. These good results concerning the behaviour of the inference procedure must be analysed in relation the use of this inference procedure. The inference phase (formalized by the mathematical program) is not only a simply adjustment process, but is intended to be integrated into an interactive aggregation disaggregation process (see section 2).This interactive process aims at providing the DM a tool for him/her to learn about his/her preferences and their compatibility with the used preference model. In this sense, the presented empirical results are very promising in terms of applicability of the approach proposed in [MS98].

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