Meaningful, Useful and legitimate information

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Outline

1. Motivations
2. The problem
3. Relevance

Why do we care?

- Decisions are based on information.
- We usually call this “evidence”.
- Not any “evidence” is relevant, appropriate or acceptable.
- How do we construct “relevant” evidence for a decision process?

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Under which conditions

such sentences make any sense for the decision process?

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- Binary relations related to comparisons
- Measures performed on well known measurement scales
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We trust our sources and we do not consider unstructured information such as:

- Images/videos/streaming
- Texts/Narratives/Stories
- Maps/Geography

What is the problem?

- From empirical observations to measures.
- From diverse empirical observations performed jointly to measures and thus to a measure.
- From measures to sentences (summarising the observations and/or the measures).
- From summarised sentences to suggested decisions (recommendations).

More formally

If O is a set of empirical observations, M is a set of measures, φ is a sentence and ψ is a suggested decision

- Under which conditions M is consistent with O?
- How to define the notions of $\models_m$, $\models_u$ and $\models_l$ standing for meaningful, useful and legitimate inference?
- How to check questions of the type $O \models_m \varphi$ or $M \models_l \psi$?

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If $\models_i \quad i \in \{m, u, l\}$ stands for the different types of inference previously introduced

How do we define and check that $\varphi \models_i \psi$?

Relevance

Relevant evidence ought be:

- true
- meaningful
- useful
- legitimate

Theorem

The four conditions of truthfulness, meaningfulness, usefulness and legitimacy are independent

Truthness

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Yesterday we had 20C, while today 10C: it is 50% less warm

Is this sentence true?

Meaningfulness

Given a set of measures

a sentence derived from such measures is meaningful if its truthness is independent from any admissible transformation of the measurement scale.

Meaningfulness

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$BMI(x) = \frac{W(x)}{H^2(x)}$. Suppose $W(x) = 50, H(x) = 1.5, W(y) = 120, H(y) = 1.65$, then $BMI(x) = 22, BMI(y) = 44$: therefore the risk of cardiovascular accident is double

This sentence is meaningful, but is not true, since it has never been proven that the risk is proportional.

Meaningfulness 2

Manipulations

Any type of manipulation of numerical information needs to respect the admissible transformations property. In this case we talk about meaningful manipulations.

Example

$x \succ y \succ z \succ w \succ t$

$f(t)$	$f(w)$	$f(z)$	$f(y)$	$f(x)$

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L2	1	2	3	4	10

Example

$x \succ y \succ z \succ w \succ t$

	$f(t)$	$f(w)$	$f(z)$	$f(y)$	$f(x)$
L1	1	2	3	4	5
L2	1	2	3	4	10

The mean of L1 is 3 and the mean object is z. The mean of L2 is 4 and the mean object is y. The median instead is always z independently from any numerical coding of the ordinal scale.

A mean is meaningless in presence of ordinal information, while a median is meaningful.

Usefulness

Can we have meaningful sentences which are useless?

Example:

pollutant	CO_2	SO_2	O_3	dust
t_1	3	5	8	6

The air quality is 8 and therefore we need to launch a warning to the population. This is meaningful and useful for a warning system.

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Example:

pollutant	CO_2	SO_2	O_3	dust
t_1	3	5	8	6
t_2	7	7	7	7
t_1	1	2	8	1

The air quality today is 8; with policy t_2 reduces to 7, while with policy t_3 remains 8. Therefore t_2 is better than t_3. This is meaningful, but useless from a policy comparison perspective.

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An average of the four pollutants is meaningless,

but far more useful from a policy comparison perspective

Legitimacy

- Is a candidate chosen by the electoral system, but not from the majority of the population a legitimate winner?
- Is a racial statistic legitimate as evidence for a public policy?
- Is a gender statistic legitimate if it is binary?
- If the majority of the population (at the polls) prefers introducing the death penalty is this a legitimate reason for passing a bill?

Is majority obvious?

Four candidates and seven examiners with the following preferences.

	a	b	c	d	e	f	g
A	1	2	4	1	2	4	1
B	2	3	1	2	3	1	2
C	3	1	3	3	1	2	3
D	4	4	2	4	4	3	4

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B(x)
15
14
16
25

The Borda count gives B>A>C>D

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If D is not there then A>B>C, instead of B>A>C

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The Condorcet principle gives A>B>C>A !!!!

Legitimacy

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is true, meaningful (if interval scales are used), useful, but not legitimate because manipulable.

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The Condorcet method

is true, meaningful, legitimate, but useless because does not guarantee to provide a winner.

Relevant Evidence

- Meaningfulness needs to be established with respect to some formal properties, axioms, a theory.
- Usefulness needs to be established with respect to a user/client and some purpose.
- Legitimacy needs to be established with respect to a context/culture/law.

Conclusions

- Information needs to be rigorous: the result of some scientific elaboration of empirical observations.
- Don't play with numbers, pay attention to the semantics they carry.
- Always consider the purpose of the decision process and the long term consequences of the information provided.
- Always consider the stakeholders involved in the decision process and their legitimate expectations.