Meaningless but useful information

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Context

- PhD in 1997 in Brussels (Ph. Vincke).
- Postdoc at UC Irvine (D. Luce).
- Frequent collaboration with D. Bouyssou, P. Perny, M. Pirlot, A. Tsoukias and Ph. Vincke.
- Papers about representational measurement in various domains.
  - ❑ MAUT
  - ❑ decision under risk
  - ❑ decision under uncertainty
  - ❑ bibliometrics
  - ❑ fuzzy sets

- Professor in a Psychology department (Ghent University) since 2000.
- Teaching Introduction to Statistics and Research Methods.
- All my colleagues routinely
  - ❑ add ordinal scores (Likert scale),
  - ❑ average ordinal scores,
  - ❑ compare ordinal scores (≠ scales).
- Textbook on Research Methods:

  "The primary advantage of rating-scale questions is that they produce numerical values that can be treated as measurements from an interval scale."

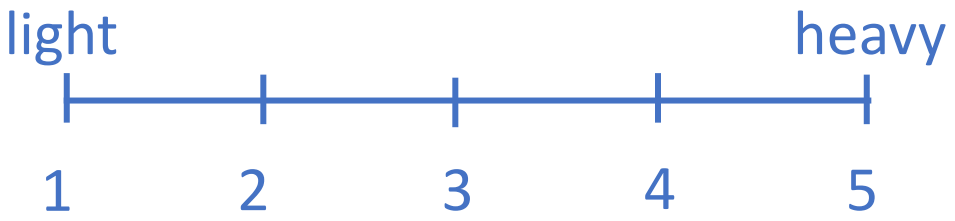
Examples

“One can of corn weighs twice as much as a second.” (Roberts, 1985).

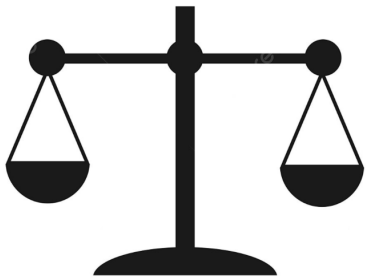
Meaningful?

It depends on the measurement technique and the corresponding admissible transformations.

1. Likert scale:



Meaningless

2. 

Meaningful

“The temperature of one can of corn at closing time yesterday was twice as much as that of a second can.” (Roberts, 1985).

Meaningful?

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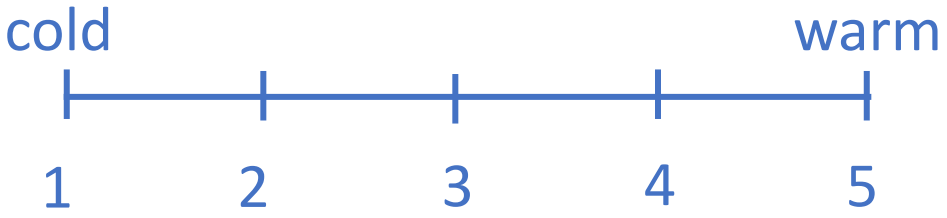

Meaningless

2. 

Meaningless

… but meaningful if we allow **an** absolute zero and use it as a reference.

“One can of corn is twice as tall as a second.” (adapted from Roberts, 1985).

Meaningful?

It depends on the measurement technique and the corresponding admissible transformations.

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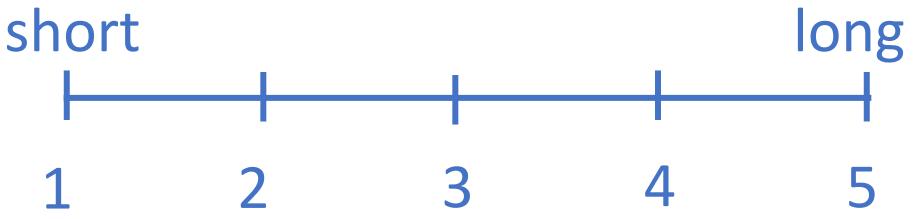
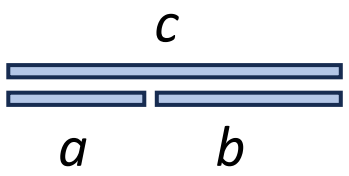


Meaningless

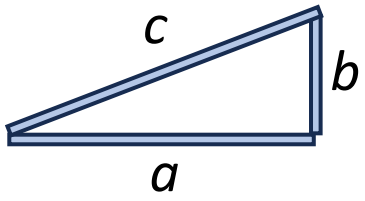
2. Extensive measurement

Meaningless

- ❑ 

$a \oplus b \sim c$

Meaningful

- ❑ 

$a \oplus b \sim c$ (Ellis, 1966)

Meaningful

Conclusion

To assess the meaningfulness of a statement, we need information about

- the class of admissible transformations and
- the measurement technique.

} One cannot be inferred from the other.

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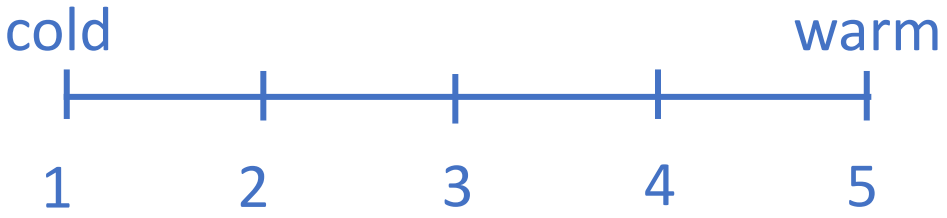

Meaningless

2. 

Meaningless

… but meaningful if we admit an absolute zero and use it as a reference.

3.  in °C

Meaningless

… or meaningful.

… but useful because the truth value of the statement is the same for everyone.

New conclusion

A statement is useful if it is supplemented with enough information to make its truth value independent of the observer.

"In general, to account for such observations, we shall study the uniqueness of the numerical assignment (homomorphism) involved. It is quite possible, given two relational systems $\mathfrak{A}$ and $\mathfrak{B}$ of the same type, for there to be several different functions that map $\mathfrak{A}$ homomorphically into $\mathfrak{B}$. Since this is the case, any statement about measurement should either specify which scale(which homomorphism)is being used or be true independent of scale."

(Roberts, 1985)

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Which statements are useful?

- All meaningful statements.

- The temperature today in Paris is 1.5 times larger than in Brussels (in °C).
- Suleiman the Magnificent was born twice later than Charlemagne (christian time scale).

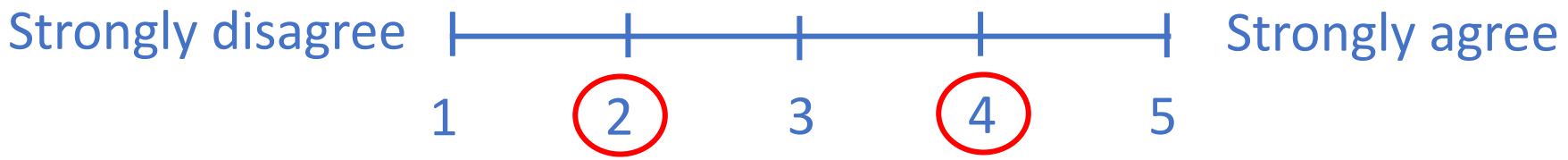
- Diamond is twice harder than apatite (Mohs scale).

  Granite has average hardness 6.8.
- The rank difference between Yates and Alaphilippe is 30 times larger than between Vingegaard and Pogacar (Tour de France, 2023).

  The average rank of Evenepoel in 2023 is 8.3.

Which statements are useful? (ctd)

- Alexis agrees twice more than me with the statement “Greek food is the best” using this Likert scale:



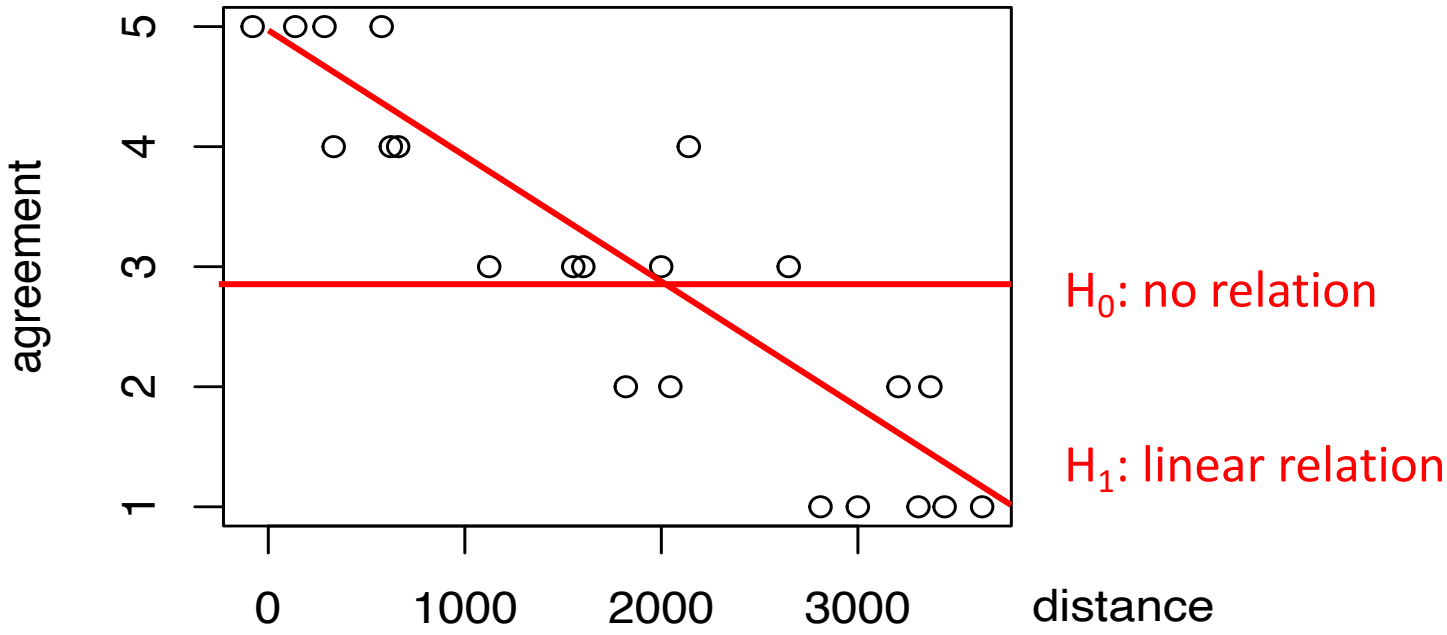
The average agreement in this room with the statement
“Greek food is the best” is 3.2.

The expected agreement with the statement
“Greek food is the best”
is higher among Greek citizens than among foreigners ($t = 1.9$, $p < 0.05$).

The Pearson correlation coefficient between agreement with the statement
“Greek food is the best” and distance from birthplace to Greece is - 0.85.

Does anything go?

- There is a negative linear relationship between agreement with the statement "Greek food is the best" and distance from birthplace to Greece ($ß_0$= 4.9, $ß_1$= -0.001, $p < 0.05$).



H_0: no relation

H_1: linear relation

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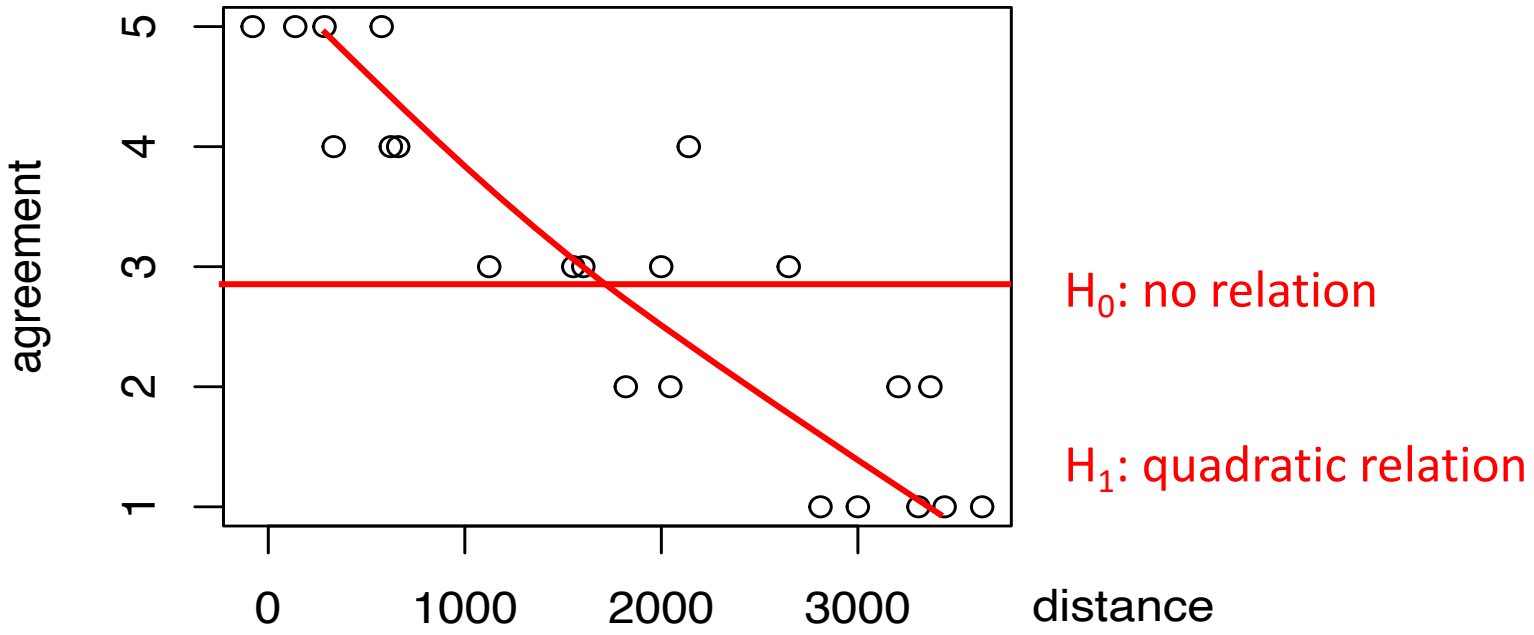
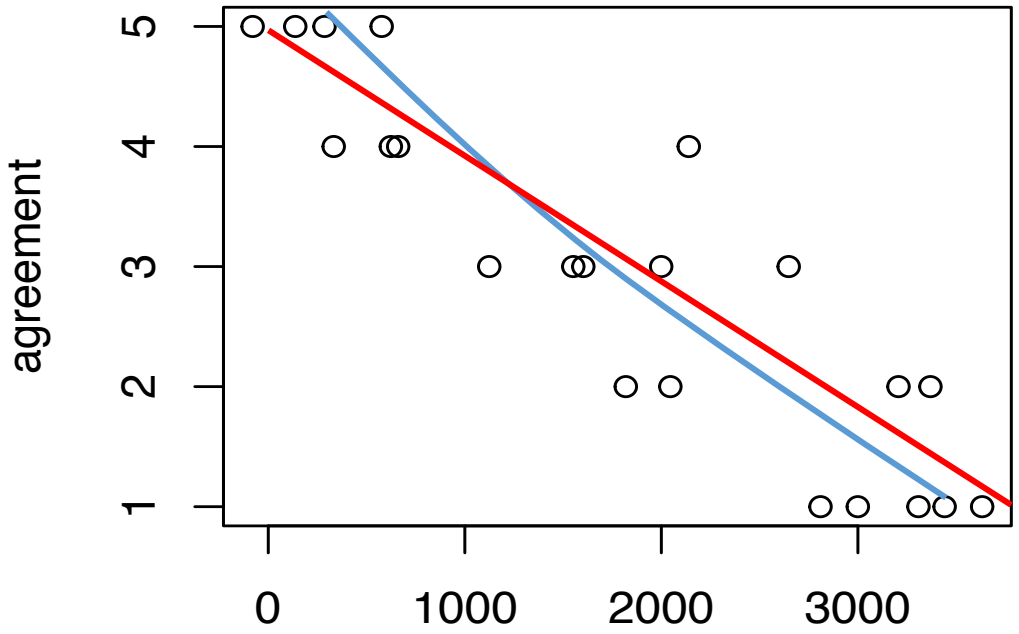




H_1: linear relation

H_2: quadratic relation

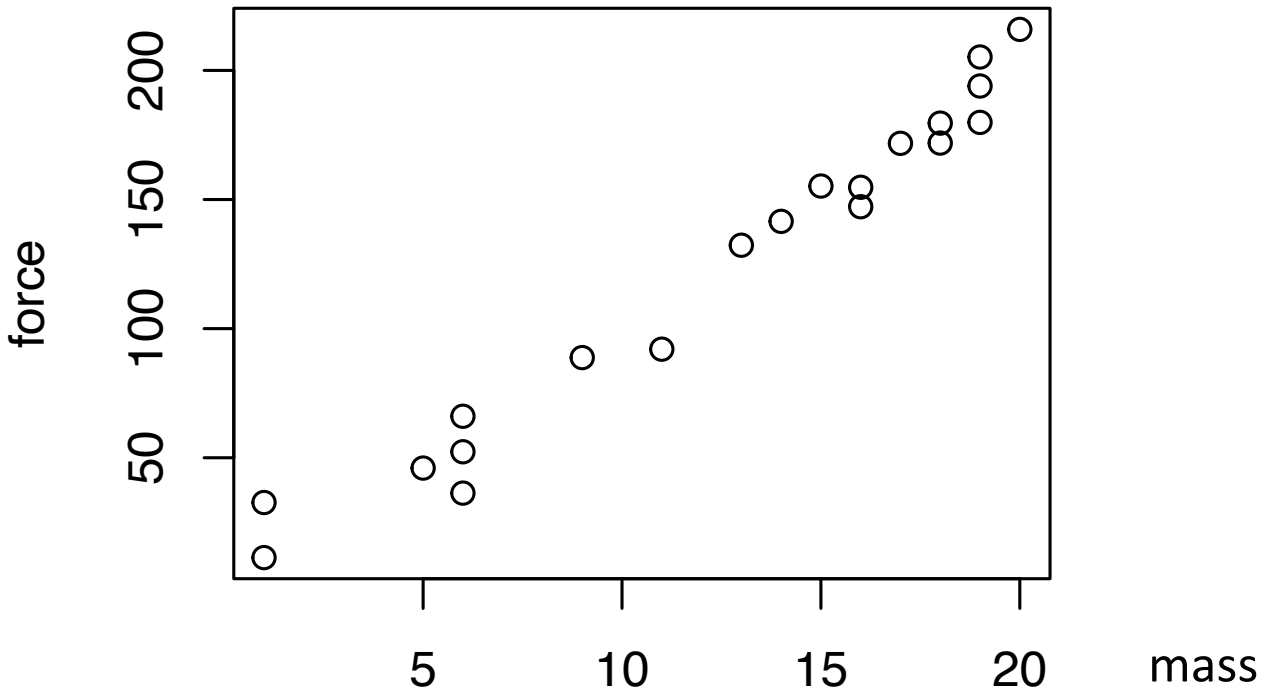
$p > 0.05$

With n very large, discrimination is possible.
The (subjective) probability of a linear relationship is zero.

Yet, the relationship is approximately linear and this is enough for the owner of an international Greek restaurant chain, for choosing the location of new restaurants.

Same problem with ‘stronger’ measurement techniques?

- There is a positive linear relationship between the gravitational force and the mass of an object ($ß_0$= 0.1, $ß_1$= 9.7, $p < 0.05$).



What kind of relationship?
With n very large, discrimination between several relationships is possible.
The (subjective) probability of a linear relationship is high.

Final conclusion

A statement is useful if it is supplemented with enough information to make its truth value independent of the observer.

This is definitely the case if it specifies the homomorphism being used.

The epistemological status of meaningful and useful statements may be different.