An Exact Solution Framework for Multi-Trip Vehicle Routing Problems with Time Windows

Autumn School on Advanced BCP Tools

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Multi-Trip VRP

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Introduction

- Most of the literature on the **Vehicle Routing Problem (VRP)** addresses problems where each vehicle can perform at most one trip per day.
- Many contributions on VRPs where vehicles can perform multiple trips have been published in the last decade.
- These problems are called **Multi-Trip Vehicle Routing Problems (MTVRP)**.
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These problems are called Multi-Trip Vehicle Routing Problems (MTVRP).
**Motivation**

• Such an increasing interest in MTVRPs is due to new practices in, e.g., city logistics and last-mile delivery
• The need of limiting noise and pollution in city centers requires the usage of small vans, electric vehicles, and/or drones and forbids large trucks from entering city centers
• The limited capacity/autonomy of these vehicles forces them to perform multiple trips and to return to the depot to reload multiple times over the day
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Main Research Question to Address in this Talk

What is the best model to solve an MTVRP (with side constraints) to optimality?
Research Question

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What is the best model to solve an MTVRP (with side constraints) to optimality?

Based on the state-of-the-art exact methods for lots of VRPs...

Set Partitioning Models!
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Definition of the Multi-Trip VRP I

Input Data

- $N$: set of customers
- $V$: vertex set, $V = N \cup \{0\}$, where 0 is the depot
- $A$: arc set, $A = \{(i, j) \mid i, j \in V : i \neq j\}$
- $G$: directed graph, $G = (V, A)$
- $t_{ij}$: travel time of arc $(i, j) \in A$
- $K$: fleet of identical capacitated vehicles, $|K| = m$
- $q_i$: demand of customer $i \in N$
- $Q$: vehicle capacity
- $T$: length of the planning horizon
Definition of the Multi-Trip VRP II

Definitions

- A **trip** is a sequence of customers, whose total demand does not exceed $Q$, that can be visited by a vehicle in between two visits at the depot, and that has a fixed departure time from the depot.

- A **journey** is a sequence of non-overlapping trips assigned to a vehicle whose total travel time does not exceed $T$.

The MTVRP aims at defining a set of at most $m$ journeys such that:

1. each customer is visited exactly once
2. the total traveled time is minimized
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**Models with 3- and 4-index Variables**

### 4-index Variables

\[ x_{ij}^{kh} \in \{0, 1\} \] equal to 1 if trip \( h \) of vehicle \( k \in K \) traverses arc \((i, j) \in A\) (0 otherwise)

### 3-index Variables with Vehicle Index (without Trip Index)

\[ x_{ij}^k \in \{0, 1\} \] equal to 1 if vehicle \( k \in K \) traverses arc \((i, j) \in A\) (0 otherwise)

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\[ x_{ij}^h \in \{0, 1\} \] equal to 1 if trip \( h \) traverses arc \((i, j) \in A\) (0 otherwise)
Models with 3- and 4-index Variables

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Models with 3- and 4-index Variables

Pros and Cons

- Polynomial number of variables
- Can be solved with commercial solvers
- Easy to embed additional side constraints
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- High integrality gaps
- BigM constraints
- Symmetries in the vehicles
2-Index Arc-based Model (Koc and Karaoglan (2011)) I

**Variables**

- $x_{ij} \in \{0, 1\}$ equal to 1 if arc $(i, j) \in A$ is traversed (0 otherwise)
- $x'_{ij} \in \{0, 1\}$ equal to 1 if a vehicle visits customers $i, j \in N (i \neq j)$ consecutively with a stop at the depot in between (0 otherwise)
- $\ell_i \in \mathbb{R}_+$ load on board after visiting customer $i \in N$
- $a_i \in \mathbb{R}_+$ arrival time at customer $i \in N$
2-Index Arc-based Model (Koc and Karaoglan (2011)) II

\[
\begin{align*}
\text{min} & \quad \sum_{(i,j) \in A} t_{ij} x_{ij} \quad \text{[Minimize travel times]} \quad (1a) \\
\text{s.t.} & \quad \sum_{(i,j) \in A} x_{ij} = 1 \quad i \in N \quad \text{[Serve each customer]} \quad (1b) \\
& \quad \sum_{(i,j) \in A} x_{ij} = \sum_{(j,i) \in A} x_{ji} \quad i \in V \quad \text{[Flow conservation]} \quad (1c) \\
& \quad \ell_i + q_j \leq \ell_j + Q(1 - x_{ij}) \quad i \in N \ j \in V \quad \text{[Subtour + Load on board]} \quad (1d) \\
& \quad a_i + t_{ij} \leq a_j + T(1 - x_{ij}) \quad i \in V \ j \in N \quad \text{[Subtour + Arrival time]} \quad (1e) \\
& \quad a_i + (t_{i0} + t_{0j}) \leq a_j + T(1 - x'_{ij}) \quad i, j \in N : i \neq j \quad \text{[Arrival time depot visit]} \quad (1f) \\
& \quad t_{0i} \leq a_i \leq T - t_{i0} \quad i \in N \quad \text{[Planning horizon]} \quad (1g) \\
& \quad \sum_{j \in N} x'_{ij} \leq x_{i0} \quad i \in N \quad \text{[Link x with x']} \quad (1h) \\
& \quad \sum_{j \in N} x'_{ij} \leq x_{0j} \quad j \in N \quad \text{[Link x with x']} \quad (1i) \\
& \quad \sum_{(0,j) \in A} x_{0j} - \sum_{i,j \in N : i \neq j} x'_{ij} \leq m \quad \text{[Number of vehicles]} \quad (1j) \\
& \quad x_{ij} \in \{0, 1\} \quad (i, j) \in A \quad (1k) \\
& \quad x'_{ij} \in \{0, 1\} \quad i, j \in N : i \neq j \quad (1l) \\
& \quad q_i \leq \ell_i \leq Q, \quad a_i \in \mathbb{R}_+ \quad i \in N \quad (1m)
\end{align*}
\]
2-Index Arc-based Model (Koc and Karaoglan (2011))

Pros and Cons

- Polynomial number of variables (much fewer than 3- and 4-index models)
- Can be solved with commercial solvers
- Easy to embed side constraints

High integrality gaps

BigM constraints

Instances with 50 customers are already difficult to close
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Trip-based Model (Mingozzi, Roberti, and Toth (2013))

- $\mathcal{H}$ set of all feasible trips
- $c_h$ cost of trip $h \in \mathcal{H}$
- $\alpha_{ih}$ trip $h \in \mathcal{H}$ serves customer $i \in N$ ($\alpha_{ih} = 1$) or not ($\alpha_{ih} = 0$)
- $d_h$ duration of trip $h \in \mathcal{H}$

**Variables**

$x_{hk} \in \{0, 1\}$ trip $h \in \mathcal{H}$ is assigned to vehicle $k \in K$ ($x_{hk} = 1$) or not ($x_{hk} = 0$)

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\begin{align*}
\text{min} & \quad \sum_{h \in \mathcal{H}} \sum_{k \in K} c_h x_{hk} \\
\text{s.t.} & \quad \sum_{h \in \mathcal{H}} \sum_{k \in K} \alpha_{ih} x_{hk} = 1 \quad i \in N \\
& \quad \sum_{h \in \mathcal{H}} d_h x_{hk} \leq T \quad k \in K \\
& \quad x_{hk} \in \{0, 1\} \quad h \in \mathcal{H} \quad k \in K
\end{align*}
\]

[Minimize travel costs] (2a)

[Serve each customer] (2b)

[Planning horizon] (2c)

(2d)
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Minimize

$$\min \sum_{h \in \mathcal{H}} c_h \sum_{k \in K} x_{hk}$$

[Minimize travel costs] (2a)

Subject to

$$\sum_{h \in \mathcal{H}} \sum_{k \in K} \alpha_{ih} x_{hk} = 1 \quad i \in N$$

[Serve each customer] (2b)

$$\sum_{h \in \mathcal{H}} d_h x_{hk} \leq T \quad k \in K$$

[Planning horizon] (2c)

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Trip-based Model (Mingozzi, Roberti, and Toth (2013))

Pros and Cons

- Small integrality gaps
- Instances with 100-120 customers can be closed
- Easy to embed side constraints
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- Exponential number of variables
- Symmetries in the vehicles
- Column generation/branch(-and-cut)-and-price needed
- Additional constraints can make the pricing problem difficult
Journey-based Model (Mingozzi, Roberti, and Toth (2013))

\( \mathcal{R} \) set of all feasible journeys

\( c_r \) cost of journey \( r \in \mathcal{R} \)

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\min \sum_{r \in \mathcal{R}} c_r x_r \quad \text{[Minimize travel costs]} \tag{3a}
\]

\[
\text{s.t.} \sum_{r \in \mathcal{R}} \alpha_{ir} x_r = 1 \quad i \in N \quad \text{[Serve each customer]} \tag{3b}
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\[
\sum_{r \in \mathcal{R}} x_r \leq m \quad \text{[Number of vehicles]} \tag{3c}
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\( x_r \in \{0, 1\} \quad r \in \mathcal{R} \quad \) \[3d]\
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Main Side Constraints and Academic Extensions

- **Time Windows**: each customer $i \in N$ must be visited within a time interval $[a_i, b_i]$
- **Service-Dependent Loading Times**: vehicle loading time at the depot depends on the customers visited in the next trip
- **Limited Trip Duration**: maximum time between the departure from the depot and the arrival time at the last customer of the trip
- **Profits**: a profit $p_i$ is associated with each customer $i \in N$; hierarchical objective function: maximize profit first; minimize routing cost second
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- $c_h$: cost of trip $h \in \mathcal{H}$
- $\alpha_{ih}$: trip $h \in \mathcal{H}$ serves customer $i \in N$ ($\alpha_{ih} = 1$) or not ($\alpha_{ih} = 0$)
- $\tau_{th}$: trip $h \in \mathcal{H}$ is active at time $t \in [a_0, b_0]$ ($\tau_{th} = 1$) or not ($\tau_{th} = 0$)

Variables

- $x_h \in \{0, 1\}$: trip $h \in \mathcal{H}$ is selected ($x_h = 1$) or not ($x_h = 0$)

Mathematical Formulation:

\[
\begin{align*}
\text{min} & \quad \sum_{h \in \mathcal{H}} c_h x_h & \quad \text{[Minimize travel costs]} \quad (4a) \\
\text{s.t.} & \quad \sum_{h \in \mathcal{H}} \alpha_{ih} x_h = 1 & \quad i \in N \quad \text{[Serve each customer]} \quad (4b) \\
& \quad \sum_{h \in \mathcal{H}} \tau_{th} x_h \leq m & \quad t \in [a_0, b_0] \quad \text{[No overlaps]} \quad (4c) \\
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Trip-based Model (Hernandez et al. (2016))

Pros and Cons

- Small integrality gaps
- Easy to embed additional side constraints defining the feasibility of the trips
Trip-based Model (Hernandez et al. (2016))

Pros and Cons

- Small integrality gaps
- Easy to embed additional side constraints defining the feasibility of the trips
- Exponential number of variables
- Column generation/branch(-and-cut)-and-price needed
- Side constraints make the pricing problem difficult
- Constraints (4c) to add in a cutting plane fashion
- Instances with 25 customers can be out of reach
Journey-based Model (Hernandez et al. (2014, 2016))

- \( \mathcal{R} \): set of all feasible journeys
- \( c_r \): cost of journey \( r \in \mathcal{R} \)
- \( \alpha_{ir} \): journey \( r \in \mathcal{R} \) serves customer \( i \in N \) (\( \alpha_{ir} = 1 \)) or not (\( \alpha_{ir} = 0 \))

**Variables**

\[ x_r \in \{0, 1\} \text{ journey } r \in \mathcal{R} \text{ is selected } (x_r = 1) \text{ or not } (x_r = 0) \]

\[
\begin{align*}
\min & \quad \sum_{r \in \mathcal{R}} c_r x_r & & \text{[Minimize travel costs]} \\
\text{s.t.} & \quad \sum_{r \in \mathcal{R}} \alpha_{ir} x_r = 1 & & i \in N \quad \text{[Serve each customer]} \\
& \quad \sum_{r \in \mathcal{R}} x_r \leq m & & \text{[Number of vehicles]} \\
& \quad x_r \in \{0, 1\} & & r \in \mathcal{R} \\
& \quad x_r & & \text{[Variable selection]} \\
\end{align*}
\]
Journey-based Model (Hernandez et al. (2014, 2016))

- $\mathcal{R}$ set of all feasible journeys
- $c_r$ cost of journey $r \in \mathcal{R}$
- $\alpha_{ir}$ journey $r \in \mathcal{R}$ serves customer $i \in \mathcal{N}$ ($\alpha_{ir} = 1$) or not ($\alpha_{ir} = 0$)

**Variables**

- $x_r \in \{0, 1\}$ journey $r \in \mathcal{R}$ is selected ($x_r = 1$) or not ($x_r = 0$)

\[
\begin{align*}
\min & \quad \sum_{r \in \mathcal{R}} c_r x_r \quad [\text{Minimize travel costs}] \\
\text{s.t.} & \quad \sum_{r \in \mathcal{R}} \alpha_{ir} x_r = 1 \quad i \in \mathcal{N} \quad [\text{Serve each customer}] \\
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\begin{align*}
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\text{s.t.} & \quad \sum_{r \in \mathcal{R}} \alpha_{ir} x_r = 1 \quad i \in \mathcal{N} & \quad \text{[Serve each customer]} \\
& \quad \sum_{r \in \mathcal{R}} x_r \leq m & \quad \text{[Number of vehicles]} \\
& \quad x_r \in \{0, 1\} \quad r \in \mathcal{R} & \quad \text{(5d)}
\end{align*}
\]
Journey-based Model (Hernandez et al. (2014, 2016))

Pros and Cons

- Small integrality gaps (smaller than trip-based model)
- Easy to embed additional side constraints both related to trips and journeys
Journey-based Model (Hernandez et al. (2014, 2016))

Pros and Cons

- Small integrality gaps (smaller than trip-based model)
- Easy to embed additional side constraints both related to trips and journeys

- Exponential number of variables
- Column generation/branch(and-cut)-and-price needed
- Pricing problem more difficult than trip-based model
- Instances with 25 customers can be out of reach
The Concept of Structure

Definition of Structure

A **structure** \( s = (0, i_1, i_2, \ldots, i_{\mu_s}, 0) \) is an ordered set of \( \mu_s \) customers that can be visited in between two visits at the depot and can start from the depot within time interval \([e_s, \ell_s]\), such that:

1. capacity constraints are satisfied
2. the duration \( d_s \) and the cost \( c_s \) are constant for each departure time from the depot within \([e_s, \ell_s]\)
3. the duration \( d_s \) is the minimum duration to serve the set of customers in the given order
Structure-based Model (Paradiso et al. (2019))

- $S$ set of all feasible structures
- $c_s$ cost of structure $s \in S$
- $\alpha_{is}$ structure $s \in S$ serves $i \in N$ ($\alpha_{is} = 1$) or not ($\alpha_{is} = 0$)

Variables

- $x_s \in \{0, 1\}$ structure $s \in S$ is selected ($x_s = 1$) or not ($x_s = 0$)

Variables

\[
\begin{align*}
\min & \quad \sum_{s \in S} c_s x_s & \text{[Minimize travel costs]} \\
\text{s.t.} & \quad \sum_{s \in S} \alpha_{is} x_s = 1 & i \in N & \text{[Serve each customer]} \\
& \quad \sum_{s \in \hat{S}} x_s \leq \eta_m(\hat{S}) & \hat{S} \subseteq S & \text{[Structure feasibility constraints]} \\
& \quad x_s \in \{0, 1\} & s \in S & \\
\end{align*}
\]

where $\eta_m(\hat{S})$ is the maximum number of structures of the set $\hat{S}$ that can be simultaneously in a solution given the number of vehicles $m$. 

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Exact Framework for MT-VRP with Time Windows

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Mathematical Models for Variants of the MTVRP

Structure-based Model (Paradiso et al. (2019))

\[ S \] set of all feasible structures
\[ c_s \] cost of structure \( s \in S \)
\[ \alpha_{is} \] structure \( s \in S \) serves \( i \in N \) \((\alpha_{is} = 1)\) or not \((\alpha_{is} = 0)\)

Variables

\[ x_s \in \{0, 1\} \] structure \( s \in S \) is selected \((x_s = 1)\) or not \((x_s = 0)\)

\[ \min \sum_{s \in S} c_s x_s \] [Minimize travel costs] \hspace{1cm} (6a)

s.t. \[ \sum_{s \in S} \alpha_{is} x_s = 1 \hspace{0.5cm} i \in N \] [Serve each customer] \hspace{1cm} (6b)

\[ \sum_{s \in \hat{S}} x_s \leq \eta_m(\hat{S}) \hspace{0.5cm} \hat{S} \subseteq S \] [Structure feasibility constraints] \hspace{1cm} (6c)

\[ x_s \in \{0, 1\} \hspace{0.5cm} s \in S \] \hspace{1cm} (6d)

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\[
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\]

\[
\text{s.t. } \sum_{s \in S} \alpha_{is} x_s = 1 \quad i \in N \quad \text{[Serve each customer]} \tag{6b}
\]

\[
\sum_{s \in \hat{S}} x_s \leq \eta_m(\hat{S}) \quad \hat{S} \subseteq S \quad \text{[Structure feasibility constraints]} \tag{6c}
\]

\[
x_s \in \{0, 1\} \quad s \in S \tag{6d}
\]

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Structure-based Model (Paradiso et al. (2019))
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- Small integrality gaps
- Easy to embed additional side constraints related to trips
- Fewer variables than trip-based (and journey-based) model
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## Trip vs Journey vs Structure (-Based Models)

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<th>Structure</th>
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<td>Complexity of algorithms</td>
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Sketch of an Exact Method based on Structure-based Model
Paradiso et al. (2019)

1. **Compute SP Bound**: solve LP relaxation of (7) without (7c) to compute dual sol. $u^1$ of cost $LB_1$

2. **Enumerate Structures**: enumerate structures ($\tilde{S}$) of red. cost $\leq UB - LB_1$ w.r.t. $u^1$, where UB is a guessed upper bound

3. **Compute SP plus Relaxed SFC**: solve LP relaxation of (7) with relaxed (7c) to compute dual sol. $u^2$ of cost $LB_2$

4. **Reduce set of structures**: remove from $\tilde{S}$ structures of red. cost $> UB - LB_2$ w.r.t. $u^2$

5. **Branch-and-cut**: solve (7) by replacing $S$ with $\tilde{S}$

6. **Optimality check**: if no feasible sol. of cost $\leq UB$ exists, increase UB and go to Step 2

**Structure-based Model**

\[
\begin{align*}
\text{min} & \quad \sum_{s \in S} c_s x_s \quad & (7a) \\
\text{s.t.} & \quad \sum_{s \in S} \alpha_{is} x_s = 1 \quad & i \in N \quad (7b) \\
& \quad \sum_{s \in \hat{S}} x_s \leq \eta_m(\hat{S}) \quad \hat{S} \subseteq S \quad & (7c) \\
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\begin{align*}
\min_{s \in S} & \quad c_s x_s \\
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Mathematical Models for Variants of the MTVRP

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### Computational Results

**MTVRP with Time Windows, Loading Times**

| Group | $|N|$ | Inst | Trip-based | Journey-based | Structure-based |
|-------|-----|------|------------|---------------|-----------------|
|       |     |      | Intel Core i7 2670QM | Intel Core i7 2670QM | Virtual CPU 2.59GHz |
|       |     |      | %Gap | Opt | $T_{tot}$ | %Gap | Opt | $T_{tot}$ | %Gap | Opt | $T_{tot}$ |
| C     | 25  | 8    | 2.24 | 8   | 108   | 2.12 | 7   | 805   | 0.73 | 8   | 19    |
| R     | 25  | 11   | 2.41 | 11  | 646   | 1.19 | 7   | 6,925 | 0.78 | 11  | 115   |
| RC    | 25  | 8    | 5.41 | 6   | 6,671 | 2.86 | 5   | 2,963 | 1.91 | 8   | 880   |
| C     | 40  | 8    |      |     |       | 1.51 | 7   | 2,170 |      |     |       |
| R     | 40  | 11   | 0.41 | 10  | 418   |      |     |       | 0.83 | 8   | 872   |
| RC    | 40  | 8    |      |     |       |      |     |       |      |     |       |
| C     | 50  | 8    |      |     |       | 1.41 | 3   | 3,577 |      |     |       |
| R     | 50  | 11   |      |     |       |      |     |       | 0.59 | 7   | 312   |
| RC    | 50  | 8    |      |     |       |      |     |       |      |     |       |
### Computational Results

#### MTVRP with Time Windows, Loading Times

| Group | \(|N|\) | Inst | %Gap | Opt | \(T_{\text{tot}}\) | %Gap | Opt | \(T_{\text{tot}}\) | %Gap | Opt | \(T_{\text{tot}}\) |
|-------|-------|------|------|-----|-------------------|------|-----|-------------------|------|-----|-------------------|
| C     | 25    | 8    | 2.24 | 8   | 108               | 2.12 | 7   | 805               | 0.73 | 8   | 19                |
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|       | 40    | 8    |      |     |                   |      |     |                   |      |     |                   |
|       |       |      |      |     |                   |      |     |                   |      |     |                   |
|       | 50    | 8    |      |     |                   |      |     |                   |      |     |                   |

**Trip-based**

Hernandez et al. (2016)

Intel Core i7 2670QM

**Journey-based**

Hernandez et al. (2016)

Intel Core i7 2670QM

**Structure-based**

Paradiso et al. (2019)

Virtual CPU 2.59GHz

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R. Roberti

Exact Framework for MT-VRP with Time Windows
## Computational Results

**MTVRP with Time Windows, Loading Times**

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MTVRP with Time Windows, Loading Times

| Group | $|N|$ | Inst | %Gap | Opt | $T_{tot}$ | %Gap | Opt | $T_{tot}$ | %Gap | Opt | $T_{tot}$ |
|-------|-----|------|------|-----|----------|------|-----|----------|------|-----|----------|
|       |     |      |      |     |          |      |     |          |      |     |          |
|       |     |      |      |     |          |      |     |          |      |     |          |
| C     | 25  | 8    | 2.24 | 8   | 108      | 2.12 | 7   | 805      | 0.73 | 8   | 19       |
| R     | 25  | 11   | 2.41 | 11  | 646      | 1.19 | 7   | 6,925    | 0.78 | 11  | 115      |
| RC    | 25  | 8    | 5.41 | 6   | 6,671    | 2.86 | 5   | 2,963    | 1.91 | 8   | 880      |
|       |     |      |      |     |          |      |     |          |      |     |          |
| C     | 40  | 8    |      |     |          |      |     |          | 1.51 | 7   | 2,170    |
| R     | 40  | 11   |      |     |          |      |     |          | 0.41 | 10  | 418      |
| RC    | 40  | 8    |      |     |          |      |     |          | 0.83 | 8   | 872      |
|       |     |      |      |     |          |      |     |          |      |     |          |
| C     | 50  | 8    |      |     |          |      |     |          | 1.41 | 3   | 3,577    |
| R     | 50  | 11   |      |     |          |      |     |          | -    | 0   | -        |
| RC    | 50  | 8    |      |     |          |      |     |          | 0.59 | 7   | 312      |
## Computational Results

### MTVRP with Time Windows, Loading Times

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### Computational Results

MTVRP with Time Windows, Loading Times, Limited Trip Duration

| Group | $|N|$ | Inst | %Gap | Opt | $T_{tot}$ | %Gap | Opt | $T_{tot}$ |
|-------|-----|------|------|-----|----------|------|-----|----------|
|       |     |      | Trip-based | Structure-based |
|       |     |      | Hernandez et al. (2014) | Paradiso et al. (2019) |
|       |     |      | Intel Core 2 Duo 2.10GHz | Virtual CPU 2.59GHz |
| C     | 25  | 16   | 1.91 | 16   | 420      | 0.38 | 16   | 14       |
| R     | 25  | 22   | 0.76 | 22   | 33       | 0.25 | 22   | 2        |
| RC    | 25  | 16   | 2.35 | 11   | 18       | 0.49 | 16   | 2        |
| C     | 40  | 16   | 1.25 | 13   | 511      | 0.48 | 16   | 151      |
| R     | 40  | 19   | 1.43 | 12   | 1,738    | 1.06 | 19   | 220      |
| RC    | 40  | 2    | -    | 0    | -        | 0.67 | 2    | 11       |
| C     | 50  | 16   | -    | 0    | -        | 0.22 | 16   | 62       |
| R     | 50  | 22   | -    | 0    | -        | 0.22 | 22   | 20       |
| RC    | 50  | 16   | -    | 0    | -        | 0.28 | 16   | 11       |
### Computational Results

MTVRP with Time Windows, Loading Times, Limited Trip Duration

| Group | \(|N|\) | Inst | %Gap | Opt | \(T_{\text{tot}}\) | %Gap | Opt | \(T_{\text{tot}}\) |
|-------|--------|------|-------|-----|----------------|-------|-----|----------------|
|       |        |      |       |     |                |       |     |                |
| Trip-based |       |       |       |     |                |       |     |                |
| Hernández et al. (2014) |       |       |       |     |                |       |     |                |
| Intel Core 2 Duo 2.10GHz |       |       |       |     |                |       |     |                |
| Structure-based |       |       |       |     |                |       |     |                |
| Paradiso et al. (2019) |       |       |       |     |                |       |     |                |
| Virtual CPU 2.59GHz |       |       |       |     |                |       |     |                |
| C     | 25     | 16   | 1.91  | 16  | 420            | 0.38  | 16  | 14            |
| R     | 25     | 22   | 0.76  | 22  | 33             | 0.25  | 22  | 2             |
| RC    | 25     | 16   | 2.35  | 11  | 18             | 0.49  | 16  | 2             |
| C     | 40     | 16   | 1.25  | 13  | 511            | 0.48  | 16  | 151           |
| R     | 40     | 19   | 1.43  | 12  | 1,738          | 1.06  | 19  | 220           |
| RC    | 40     | 2    | -     | 0   | -              | 0.67  | 2   | 11            |
| C     | 50     | 16   | 0.22  | 16  | 62             | 0.22  | 16  | 20            |
| R     | 50     | 22   | 0.22  | 22  | 20             | 0.22  | 22  | 20            |
| RC    | 50     | 16   | 0.28  | 16  | 11             | 0.28  | 16  | 11            |
## Computational Results

MTVRP with Time Windows, Loading Times, Limited Trip Duration

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R.Roberti

Exact Framework for MT-VRP with Time Windows
### Computational Results

**MTVRP with Time Windows, Loading Times, Limited Trip Duration**

| Group | $|N|$ | Inst | %Gap | Opt | $T_{tot}$ | %Gap | Opt | $T_{tot}$ |
|-------|-----|------|------|-----|---------|------|-----|---------|
| Trip-based | | | | | | | | |
| Hernandez et al. (2014) | | | Intel Core 2 Duo 2.10GHz | | | | | |
| Structure-based | | | Paradiso et al. (2019) | | | | | Virtual CPU 2.59GHz |
| | | | | | | | | |
| C  | 25  | 16  | 1.91 | 16 | 420 | 0.38 | 16 | 14 |
| R  | 25  | 22  | 0.76 | 22 | 33  | 0.25 | 22 | 2 |
| RC | 25  | 16  | 2.35 | 11 | 18  | 0.49 | 16 | 2 |
| C  | 40  | 16  | 1.25 | 13 | 511 | 0.48 | 16 | 151 |
| R  | 40  | 19  | 1.43 | 12 | 1,738 | 1.06 | 19 | 220 |
| RC | 40  | 2   | -    | 0  | -    | 0.67 | 2  | 11  |
| C  | 50  | 16  | -    | 0  | -    | 0.22 | 16 | 62 |
| R  | 50  | 22  | -    | 0  | -    | 0.22 | 22 | 20 |
| RC | 50  | 16  | -    | 0  | -    | 0.28 | 16 | 11 |
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**MTVRP with Time Windows, Loading Times, Limited Trip Duration**

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### Computational Results

#### Drone Routing Problem

| $|N|$ | Inst | %Gap | Opt | $T_{tot}$ | %Gap | Opt | $T_{tot}$ |
|---|---|---|---|---|---|---|---|
| 10 | 10 | 4.49 | 10 | 0 | 0.40 | 10 | 0 |
| 15 | 10 | 5.47 | 4 | 9 | 1.28 | 10 | 1 |
| 20 | 10 | 3.69 | 5 | 18 | 0.86 | 10 | 2 |
| 25 | 37 | 2.68 | 22 | 59 | 0.62 | 37 | 1 |
| 30 | 10 | 0 | 0 | 0 | 0.52 | 10 | 4 |
| 35 | 10 | 0 | 0 | 0 | 0.44 | 10 | 11 |
| 40 | 37 | 3.83 | 4 | 4,168 | 0.20 | 37 | 5 |
| 45 | 10 | 0 | 0 | 0 | 0.36 | 10 | 13 |
| 50 | 5 | 0 | 0 | 0 | 1.72 | 5 | 275 |

Cheng et al. (2018)
Intel X5650 2.67GHz

Paradiso et al. (2019)
Virtual CPU 2.59GHz
## Computational Results

**Drone Routing Problem**

| $|N|$ | Inst | %Gap | Opt | $T_{tot}$ | %Gap | Opt | $T_{tot}$ |
|-----|------|------|-----|----------|------|-----|----------|
| 10  | 10   | 4.49 | 10  | 0        | 0.40 | 10  | 0        |
| 15  | 10   | 5.47 | 4   | 9        | 1.28 | 10  | 1        |
| 20  | 10   | 3.69 | 5   | 18       | 0.86 | 10  | 2        |
| 25  | 37   | 2.68 | 22  | 59       | 0.62 | 37  | 1        |
| 30  | 10   | 0    | 0   | 0        | 0.52 | 10  | 4        |
| 35  | 10   | 0    | 0   | 0        | 0.44 | 10  | 11       |
| 40  | 37   | 3.83 | 4   | 4,168    | 0.20 | 37  | 5        |
| 45  | 10   | 0    | 0   | 0        | 0.36 | 10  | 13       |
| 50  | 5    | 0    | 0   | 1.72     | 0    | 5   | 275      |

**Arc-based**
- Cheng et al. (2018)
- Intel X5650 2.67GHz

**Structure-based**
- Paradiso et al. (2019)
- Virtual CPU 2.59GHz
### Computational Results

#### Drone Routing Problem

| |N| |Inst| %Gap| Opt| T<sub>tot</sub> | %Gap| Opt| T<sub>tot</sub> |
|---|---|---|---|---|---|---|---|---|
|10|10|4.49|10|0|0.40|10|0|
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|20|10|3.69|5|18|0.86|10|2|
|25|37|2.68|22|59|0.62|37|1|
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|35|10|0|0|0.44|10|11|
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| $|N|$ | Inst | %Gap | Opt | $T_{tot}$ | %Gap | Opt | $T_{tot}$ |
|----|------|------|-----|----------|------|-----|---------|
| 10 | 10   | 4.49 | 10  | 0        | 0.40 | 10  | 0       |
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| 20 | 10   | 3.69 | 5   | 18       | 0.86 | 10  | 2       |
| 25 | 37   | 2.68 | 22  | 59       | 0.62 | 37  | 1       |
| 30 | 10   | 0    | 0   |          | 0.52 | 10  | 4       |
| 35 | 10   | 0    | 0   |          | 0.44 | 10  | 11      |
| 40 | 37   | 3.83 | 4   | 4,168    | 0.20 | 37  | 5       |
| 45 | 10   | 0    | 0   |          | 0.36 | 10  | 13      |
| 50 | 5    | 0    | 0   |          | 1.72 | 5   | 275     |
# Table of Contents

- Introduction
- Multi-Trip VRP
- Mathematical Models for the MTVRP
- Variants of the MTVRP
- Mathematical Models for Variants of the MTVRP
- Computational Results
- Conclusions and Open Questions
Conclusions

- Increasing **interest in MTVRPs**, mainly motivated by city logistics and last-mile delivery
  - **Trip-based** and **journey-based** models are effective to solve the MTVRP
  - **To handle side constraints, structure-based** models seem the better choice, even better than set-partitioning models
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• Increasing interest in MTVRPs, mainly motivated by city logistics and last-mile delivery
• Trip-based and journey-based models are effective to solve the MTVRP
• To handle side constraints, structure-based models seem the better choice, even better than set-partitioning models
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- Research on MTVRPs is scarce and 50-customer instances are already challenging, how can large instances be solved?
- Are there better models (maybe models not based on arcs, structures, trips, or journeys)?
- Can we use models not based on arcs or routes to solve other VRPs?

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References


