# Polyhedral Investigation and Branch-and-Cut for the Constrained-Routing and Spectrum Assignment Problem in Spectrally Flexible Optical Networks*

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**Abstract.** In this work we focus on a variant of the Routing and Spectrum Assignment problem (RSA), namely the Constrained-Routing and Spectrum Assignment (C-RSA). The C-RSA problem can be stated as follows. Consider a spectrally flexible optical network as an undirected, loopless, and connected graph $G$, and an optical spectrum $\mathcal{S}$ of available contiguous frequency slots, and a multiset of traffic demands $\mathcal{K}$. The C-RSA consists in assigning for each traffic demand $k \in \mathcal{K}$ a path in $G$ and an interval of contiguous frequency slots in $\mathcal{S}$ subject to technological constraints, while optimizing some linear objective function(s). To the best of our knowledge, a polyhedral approach to the C-RSA has not been considered before. The main aim of our work is to provide a deep polyhedral investigation, and design a cutting-plane based approach to handle large sized instances of the C-RSA. In order to do that, we propose, first, an integer linear programming edge-node formulation for the C-RSA with polynomial number of variables and exponential number of constraints separable in polynomial time using network flows. We further identify several families of valid inequalities to obtain tighter bounds, and prove that these inequalities are facet-defining under some necessary and sufficient conditions. Moreover, we present separation algorithms based on some heuristics and greedy-algorithms for these inequalities. Based on these results, we devise a branch-and-cut algorithm for the problem, study its behavior considering large sized instances, and increase its effectiveness through some enhancements based on primal-heuristics to obtain tighter bounds.

**Keywords:** Spectrally flexible optical network, network design, constrained-routing, spectrum assignment, complexity, ILP, polyhedron, dimension, pre-processing, valid inequality, facet, separation, branch-and-cut algorithm, conflict-graph, threshold graph, interval graph, perfect graph, intersection graph, heuristic, greedy-algorithm.

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