Cooperative games in open anonymous environments

Øystein Walle,
Helen Kwong

Cooperative games

- **Coalitional Games in Open Anonymous Environments**
  Makoto Yokoo *et.al.*
- **A Compact Representation Scheme for oalitional Games in Open Anonymous Environments**
  Naoki Ohta *et.al.*

Outline

- Traditionally, we've studied *characteristic function games* where $v(C)$ is the value a coalition can obtain
- This is vulnerable to manipulation: An agent may pretend to be two different agents, or several agents maybe appear to be one
- This calls for a more fine-grained representation

- Instead of defining the valuation function over the agents themselves, we define them over their *skills*.

Definition (Skills)

T is the set of all possible skills. Each agent i has one or more skills $S_i \in T$. For simplicity, each skill is unique: $\forall i \neq j, S_i \cap S_j = \emptyset$.

Definition (Valuation functions)

A characteristic function $w : 2^T \rightarrow \mathbb{R}$ gives a value for each set of skills, not agents. Let $v : 2^N \rightarrow \mathbb{R}$ be the usual valuation function.

- Then for a given set of agents C we have $v(C) = w(\bigcup_{i \in C} S_i)$
- $w(S)$ is more fine-grained than $v(C)$, since the latter can be derived from the former, but not the other way around.

- There is a special agent called the *mechanism designer*. He knows the upper bound of T and he knows w.
- If agent i is interested in joining a coalition, he declares his skills to the mechanism designer
- The mechanism designer determines the value division among the participants

Definition (Hiding skills)

If an agent i has a set of skills S_i, for any $S'_i \subseteq S_i$, it can declare that it only has S'_i. We assume that an agent cannot claim a skill it doesnt have, as this lie would easily be exposed.

Definition (False names)

An agent can use multiple identifiers. Such a false-name manipulation corresponds to a partition of S_i into multiple identifiers.

Definition (Collusion)

Multiple agents can collude and pretend to be a single agent. They can declare the skills to be the union of their skills, or a subset thereof.

These manipulations can be combined to produce more complex manipulatons.

Theorem

Let S be the union of a coalition's skills. Any manipulation in which this coalition submits several indentifiers with non-overlapping subsets of S as their skills can be achieved by a combination of the previous three manipulations.

Outline

2 Maniplation of traditional Solution concepts
Definitions
Vulnerability

Definition (Shapley value)

Given an ordering o of the set of agents N in the colation, let $C(o,i)$ be the set of agents in N that appear before i in the ordering o. Then the Shapley value for agent i is

$$Sh_i(N,i) = \frac{1}{|N|!}\sum_{o}(v(C(o,i)\cup\{i\}) - v(C(o,i)))$$

Definition (Core)

Given a set of agents N, a value division x among agents is in the core if the following two conditions hold

1. $\forall C \subset N, \sum_{i \in C} x_i \geq v(C)$
2. $\sum_{i \in N} x_i = v(N)$

Definition (ε-core)

Given a set of agents N, a value division x among agents is in the ε-core if the following two conditions hold

1. $\forall C \in N, \sum_{i \in C} x_i \geq v(C) - \varepsilon$,
2. $\sum_{i \in N} x_i = v(N)$

- These manipulations can be combined to produce more complex manipulatons.

Theorem

Let S be the union of a coalition's skills. Any manipulation in which this coalition submits several indentifiers with non-overlapping subsets of S as their skills can be achieved by a combination of the previous three manipulations.

Definition (Least core)

Given a set of agents N, a value division x among agents is in the ε-core if the following two conditions hold

1. x is in the core,
2. $\forall \varepsilon' < \varepsilon$, the ε'-core is empty.

- Paper contains examples of manipulation due to false names, collusion and hiding of skills
- Manipulation due to false names is presented here

Example (1)

Let $T = \{a, b, c\}$ be the set of skills and

- $w(T) = 1$
- For any proper subset S of T, $w(S) = 0$.

Let $N = \{1, 2\}$ and

- $S_1 = \{a\}$
- $S_2 = \{b, c\}$

then we have

- $v(\{i\}) = 0$
- $v(N) = 1$.

- There are only two possible orderings of the agents and in both the second agent has a marginal contribution of 1. Therefore the Shaple value for each agent is $1/2$
- Any payoff distribution x which satisfies $x_i \geq 0$ and $x_1 + x_2 = 1$ is in the core.
- The least core has only one outcome. which is identical to the Shapley value, hence the nucleolus is identical to the Shapley value.

Example (2)

Let T and w be as before but let $N = \{1, 2, 3\}$ and

- $S_1 = \{a\}$
- $S_2 = \{b\}$
- $S_3 = \{c\}$

then we have

- For any proper subset C of N, $v(C) = 0$
- $v(N) = 1$.

- There are six possible orderings of the agents and in all the last agent has a marginal contribution of 1. Therefore the Shaple value for each agent is $1/3$.
- Any payoff distribution x which satisfies $x_i \geq 0$ and $x_1 + x_2 + x_3 = 1$ is in the core.
- The least core has only one outcome. which is identical to the Shapley value, hence the nucleolus is identical to the Shapley value.

- In example 1, agent 2 could have two false names and split his skills over the two names.
- Then he could increase the value awarded to it from $1/2$ to $2/3$.
- This proves the following:

Theorem

There exists no payoff distribution function that 1) equally rewards the agents that are symmetric with respect to v, 2) distibutes all the value, and 3) is false-name proof.

One can rather apply solution concepts directly to the skills, treating them as players. This is more robust.

Theorem

Applying any solution concept to the skills directly is robust to false names, collusion and any combination thereof.

Proof.

Because solution concepts applied to the skills directly are indifferent to which agent submitted which skills, changing the identifiers under which skills are submitted never changes the payoffs to those skills. □

Vulnerability

- Hiding skills can still manipulate the solution concepts.
- Due to time, we omit this.

Outline

3. Anonymity-proof Core
   - Core for skills
   - Toward anonymity-proof core

Definition (Core for skills)

For a set of skills $S = \{s_1, s_2, \dots\}$ is in Core(S) if it satisfies the following two conditions

1. $\forall S', \sum_{s_j \in S'} x_{s_j}^S \geq w(S')$
2. $\sum_{s_j \in S} x_{s_j}^S = w(S)$

- The mechanism designer must define an outcome function π that decides, for all possible reports by the agents of their skills, how to divide the value generated by these skills.
- The following are axiomatic conditions that π should satisfy. Some notation:
  - $SS_{C'} = \{S_j \mid j \in C'\}, i \notin C'$
  - $k = (k_1, k_2, \dots)$, k_i is the skills agent i *declares*
  - $S_C = \bigcup_{i \in C} k_i$
  - $SS_C = \{k_i \mid i \in C\}$
  - $SS_{\sim i} = \{k_j \mid j \in N \setminus \{i\}\}$

1. The outcome function π is anonymous.

1. The outcome function π is anonymous.
2. π is never blocked by any coalitions, that is, $\forall k, \forall C \subseteq N, \sum_{i \in C} \pi(k_i, SS_{\sim i}) \geq w(S_C)$ holds

Toward anonymity-proof core

1. The outcome function π is anonymous.
2. π is never blocked by any coalitions, that is, $\forall k, \forall C \subseteq N, \sum_{i \in C} \pi(k_i, SS_{\sim i}) \geq w(S_C)$ holds
3. π is always feasible and always distributes all of the value, that is, $\forall k, \sum_{i \in N} \pi(k_i, SS_{\sim i}) = w(S) holds$

1. The outcome function π is anonymous.
2. π is never blocked by any coalitions, that is, $\forall k, \forall C \subseteq N, \sum_{i \in C} \pi(k_i, SS_{\sim i}) \geq w(S_C)$ holds
3. π is always feasible and always distributes all of the value, that is, $\forall k, \sum_{i \in N} \pi(k_i, SS_{\sim i}) = w(S) holds$
4. π is robust against hiding skills, that is, $\forall S', S''$, where $S'' \subseteq S', \forall SS, \pi(S'', SS) \leq \pi(S', SS)$ holds.

Toward anonymity-proof core

1. The outcome function π is anonymous.
2. π is never blocked by any coalitions, that is, $\forall k, \forall C \subseteq N, \sum_{i \in C} \pi(k_i, SS_{\sim i}) \geq w(S_C)$ holds
3. π is always feasible and always distributes all of the value, that is, $\forall k, \sum_{i \in N} \pi(k_i, SS_{\sim i}) = w(S) holds$
4. π is robust against hiding skills, that is, $\forall S', S''$, where $S'' \subseteq S', \forall SS, \pi(S'', SS) \leq \pi(S', SS)$ holds.
5. *pi* is robust against false-name manipulations, that is, $\forall k, \forall C \subseteq N, C' = N \setminus C, \sum_{i \in C} \pi(k_i, SS_{\sim i}) \leq \pi(S_C, SS_{C'})$ holds.

1. The outcome function π is anonymous.
2. π is never blocked by any coalitions, that is, $\forall k, \forall C \subseteq N, \sum_{i \in C} \pi(k_i, SS_{\sim i}) \geq w(S_C)$ holds
3. π is always feasible and always distributes all of the value, that is, $\forall k, \sum_{i \in N} \pi(k_i, SS_{\sim i}) = w(S) holds$
4. π is robust against hiding skills, that is, $\forall S', S''$, where $S'' \subseteq S', \forall SS, \pi(S'', SS) \leq \pi(S', SS)$ holds.
5. *pi* is robust against false-name manipulations, that is, $\forall k, \forall C \subseteq N, C' = N \setminus C, \sum_{i \in C} \pi(k_i, SS_{\sim i}) \leq \pi(S_C, SS_{C'})$ holds.
6. π is robust against collusions, that is, $\forall k, \forall C \subseteq N, C' = N \setminus C, \sum_{i \in C} \pi(k_i, SS_{\sim i}) \geq \pi(S_C, SS_{C'})$ holds.

Definition (Anonymity-proof Core)

The outcome function π_{ap} is in the anonymity-proof core if π_{ap} satisfies the following conditions

1. For any set of skills $S \subseteq T$, there exists a core outcome for S, that is, some $x^S \in Core(S)$, such that for any skill profile $k = (k_1, k_2, \cdots)$ with $\bigcup_i k_i = S$, $\pi_{ap}(k_i, SS_{\sim i}) = \sum_{s_j \in k_i} x^S_{s_j}$ holds
2. $\forall S', S''$, where $S'' \subseteq S', \forall SS, \pi_{ap}(S'', SS) \leq \pi_{ap}(S', SS)$ holds.

Definition (ε-core for skills)

For a given ε, x^S is in ε-Core(S) if it satisfies the following two conditions

1. $\forall S' \in S, \sum_{s_J \in S'} x^S_{s_j} \geq w(S') - \varepsilon$,
2. $\sum_{s_J \in S} x^S_{s_j} = w(S)$

Definition (Least anonymity-proof core)

The outcome function π_{ap} is in the least anonymity-proof core if π_{ap} satisfies the following conditions:

1. π_{ap} is in the ε-anonymity-proof core,
2. $\forall \varepsilon' < \varepsilon$, the ε'-anonymity-proof core is empty.

The following theorem always holds.

Theorem

$\forall T, v$ there always exists an outcome function that is in the least anonymity-proof core.

Outline

4 Compact representation
- Introduction
- Definitions
- Example
- Results

Motivation

- One drawback of outcome functions: Traditional solution concepts need to specify the value division only for the set of skills S in a game, but now we have to specify value divisions for **all subsets** of S. The length of the representation becomes exponential in the number of skills.

Motivation

- One drawback of outcome functions: Traditional solution concepts need to specify the value division only for the set of skills S in a game, but now we have to specify value divisions for **all subsets** of S. The length of the representation becomes exponential in the number of skills.
- Introduce a **compact representation scheme** which can represent any outcome function in the anonymity-proof core, based on synergetic coalitions.

Simplified notation

Let S be the set of skills in a game and $s \in S$. The outcome function $\pi(s, S)$ is the payoff returned to the agent who declares skill s, when the total set of skills declared by all agents is S. If an agent declares $S' \subseteq S$, his payoff is $\Sigma_{s \in S'} \pi(s, S)$.

Simplified notation

Let S be the set of skills in a game and $s \in S$. The outcome function $\pi(s, S)$ is the payoff returned to the agent who declares skill s, when the total set of skills declared by all agents is S. If an agent declares $S' \subseteq S$, his payoff is $\Sigma_{s \in S'} \pi(s, S)$.

Definition (anonymity-proof core)

An outcome function π is in the *anonymity-proof core* if

Simplified notation

Let S be the set of skills in a game and $s \in S$. The outcome function $\pi(s, S)$ is the payoff returned to the agent who declares skill s, when the total set of skills declared by all agents is S. If an agent declares $S' \subseteq S$, his payoff is $\Sigma_{s \in S'} \pi(s, S)$.

Definition (anonymity-proof core)

An outcome function π is in the *anonymity-proof core* if

- (Pareto efficiency): $\forall S, \Sigma_{s \in S} \pi(s, S) = w(S)$,

Simplified notation

Let S be the set of skills in a game and $s \in S$. The outcome function $\pi(s, S)$ is the payoff returned to the agent who declares skill s, when the total set of skills declared by all agents is S. If an agent declares $S' \subseteq S$, his payoff is $\Sigma_{s \in S'} \pi(s, S)$.

Definition (anonymity-proof core)

An outcome function π is in the *anonymity-proof core* if

- (Pareto efficiency): $\forall S, \Sigma_{s \in S} \pi(s, S) = w(S)$,
- (Robust against hiding skills): $\forall S, \forall S', \forall S''$ such that $S'' \subset S' \subseteq S$, $\Sigma_{s \in S''} \pi(s, S \backslash (S' \backslash S'')) \leq \Sigma_{s \in S'} \pi(s, S)$, and

Simplified notation

Let S be the set of skills in a game and $s \in S$. The outcome function $\pi(s, S)$ is the payoff returned to the agent who declares skill s, when the total set of skills declared by all agents is S. If an agent declares $S' \subseteq S$, his payoff is $\Sigma_{s \in S'} \pi(s, S)$.

Definition (anonymity-proof core)

An outcome function π is in the *anonymity-proof core* if

- (Pareto efficiency): $\forall S, \Sigma_{s \in S} \pi(s, S) = w(S)$,
- (Robust against hiding skills): $\forall S, \forall S', \forall S''$ such that $S'' \subset S' \subseteq S$, $\Sigma_{s \in S''} \pi(s, S \backslash (S' \backslash S'')) \leq \Sigma_{s \in S'} \pi(s, S)$, and
- (No blocking): $\forall S, \forall S' \subseteq S, \Sigma_{s \in S'} \pi(s, S) \geq w(S')$.

Main example

Let $T = \{a, b, c, d, e\}$ be the set of all possible skills. The characteristic function over skills is:

- $w(\{a, b, c, d, e\}) = w(\{a, b, d, e\}) = w(\{b, c, d, e\}) = 2$,
- $w(\{a, b, c, d\}) = w(\{a, b, c, e\}) = w(\{a, c, d, e\}) = 1$,
- $w(\{a, b, c\}) = w(\{a, b, d\}) = w(\{a, b, e\}) = w(\{a, d, e\}) = w(\{b, c, d\}) = w(\{b, c, e\}) = w(\{b, d, e\}) = w(\{c, d, e\}) = 1$,
- $w(\{a, b\}) = w(\{b, c\}) = w(\{d, e\}) = 1$,
- for any other $S \subset T$, $w(S) = 0$.

Main example

Let $T = \{a, b, c, d, e\}$ be the set of all possible skills. The characteristic function over skills is:

- $w(\{a,b,c,d,e\}) = w(\{a,b,d,e\}) = w(\{b,c,d,e\}) = 2$,
- $w(\{a,b,c,d\}) = w(\{a,b,c,e\}) = w(\{a,c,d,e\}) = 1$,
- $w(\{a,b,c\}) = w(\{a,b,d\}) = w(\{a,b,e\}) = w(\{a,d,e\}) = w(\{b,c,d\}) = w(\{b,c,e\}) = w(\{b,d,e\}) = w(\{c,d,e\}) = 1$,
- $w(\{a,b\}) = w(\{b,c\}) = w(\{d,e\}) = 1$,
- for any other $S \subset T$, $w(S) = 0$.

The conventional core gives 1 to b, p to d, q to e, and 0 to a and c, for all p and q such that $p, q \geq 0$ and $p + q = 1$. The anonymity-proof core gives π with:

- $\pi(b, \{a, b, ...\}) = \pi(b, \{b, c, ...\}) = 1$,
- $\pi(d, \{d, e, ...\}) = p, \pi(e, \{d, e, ...\}) = q$, and
- for any other other $S \subset T$ and $s \in S$, $\pi(s, S) = 0$.

Synergetic coalitions

Instead of specifying value divisions for all subsets of S, we do it only for certain *synergetic* groups.

Definition (synergy coalition group)

The *synergy coalition group SCG* is a set of coalitions, each of which has some synergy, i.e., $\forall S \in SCG, \forall \{S_1, ..., S_k\}$, where all the S_j are disjoint and $\bigcup_{1 \leq j \leq k} S_j = S$, $w(S) > \Sigma_{S_j} w(S_j)$ holds.

In our example, the SCG is $\{\{a, b\}, \{b, c\}, \{d, e\}\}$.

Definition (generalized synergy coalition group)

The *generalized synergy coalition group GSCG* is the smallest group of coalitions such that

- $\forall S$, if $S \in SCG$, then $S \in GSCG$,
- $\forall S_1, S_2 \in GSCG$, if $S_1 \cap S_2 \neq \emptyset$, then $S_1 \cup S_2 \in GSCG$.

Definition (generalized synergy coalition group)

The *generalized synergy coalition group* $GSCG$ is the smallest group of coalitions such that

- $\forall S$, if $S \in SCG$, then $S \in GSCG$,
- $\forall S_1, S_2 \in GSCG$, if $S_1 \cap S_2 \neq \emptyset$, then $S_1 \cup S_2 \in GSCG$.

Definition (projection onto $GSCG$)

For a set of skills S, the projection P_S of S onto $GSCG$ is

$$P_S = \{G \in GSCG : G \subseteq S, \forall G' \text{ with } G \subset G' \subseteq S, G' \notin GSCG\}.$$

Definition (generalized synergy coalition group)

The *generalized synergy coalition group* $GSCG$ is the smallest group of coalitions such that

- $\forall S$, if $S \in SCG$, then $S \in GSCG$,
- $\forall S_1, S_2 \in GSCG$, if $S_1 \cap S_2 \neq \emptyset$, then $S_1 \cup S_2 \in GSCG$.

Definition (projection onto $GSCG$)

For a set of skills S, the projection P_S of S onto $GSCG$ is

$$P_S = \{G \in GSCG : G \subseteq S, \forall G' \text{ with } G \subset G' \subseteq S, G' \notin GSCG\}.$$

In our example, the $GSCG$ is $\{\{a,b\},\{b,c\},\{d,e\},\{a,b,c\}\}$.
$P_{\{a,b,c,d,e\}} = \{\{a,b,c\},\{d,e\}\}$.

Compact outcome function

Definition (compact representation)

A *compact outcome function* π_C takes a set of skills $G \in GSCG$ and a skill $s \in G$ as arguments, and returns the value division of skill s when skills G exist.

Compact outcome function

Definition (compact representation)

A *compact outcome function* π_C takes a set of skills $G \in GSCG$ and a skill $s \in G$ as arguments, and returns the value division of skill s when skills G exist.
An outcome function π is expressible by a compact outcome function π_C if for all S and $s \in S$,

$$\pi(s, S) = \begin{cases} \pi_C(s, G) & \text{where } s \in G \text{ and } G \in P_S \text{, if such } G \text{ exists} \\ 0 & \text{otherwise} \end{cases}$$

Back to example

The conventional core gives 1 to b, p to d, q to e, and 0 to a and c, for all p and q such that $p,q \geq 0$ and $p+q=1$. The anonymity-proof core gives π with:

- $\pi(b,\{a,b,...\}) = \pi(b,\{b,c,...\}) = 1$,
- $\pi(d,\{d,e,...\}) = p, \pi(e,\{d,e,...\}) = q$, and
- for any other other $S \subset T$ and $s \in S$, $\pi(s,S) = 0$.

Back to example

The conventional core gives 1 to b, p to d, q to e, and 0 to a and c, for all p and q such that $p, q \geq 0$ and $p + q = 1$. The anonymity-proof core gives π with:

- $\pi(b, \{a, b, ...\}) = \pi(b, \{b, c, ...\}) = 1$,
- $\pi(d, \{d, e, ...\}) = p, \pi(e, \{d, e, ...\}) = q$, and
- for any other other $S \subset T$ and $s \in S$, $\pi(s, S) = 0$.

π is compactly expressible by the following compact outcome function π_C:

- $\pi_C(a, \{a, b, c\}) = \pi_C(c, \{a, b, c\}) = 0, \pi_C(b, \{a, b, c\}) = 1$,
- $\pi_C(a, \{a, b\}) = 0, \pi_C(b, \{a, b\}) = 1$,
- $\pi_C(b, \{b, c\}) = 1, \pi_C(c, \{b, c\}) = 0$,
- $\pi_C(d, \{d, e\}) = p, \pi_C(e, \{d, e\}) = q$.

Example

We can rederive the original outcome function. For example, consider the situation when the total set of declared skills is $\{a, b, c, d, e\}$. Then

- $\pi(d, \{a, b, c, d, e\}) = \pi_C(d, \{d, e\}) = p$ $(P_{\{a,b,c,d,e\}} = \{\{a, b, c\}, \{d, e\}\})$.
- If an agent declares skills b and d, then his payoff is $\pi_C(b, \{a, b, c\}) + \pi_C(d, \{d, e\}) = 1 + p$.

Example

We can rederive the original outcome function. For example, consider the situation when the total set of declared skills is $\{a, b, c, d, e\}$. Then

- $\pi(d, \{a, b, c, d, e\}) = \pi_C(d, \{d, e\}) = p$ $(P_{\{a,b,c,d,e\}} = \{\{a, b, c\}, \{d, e\}\})$.
- If an agent declares skills b and d, then his payoff is $\pi_C(b, \{a, b, c\}) + \pi_C(d, \{d, e\}) = 1 + p$.

By how much is representation size reduced? The original outcome function needs to specify values for $\Sigma_{2 \leq j \leq 5} j \cdot \binom{5}{j} = 75$ combinations, the compact outcome function needs to do so for only 9 combinations.

No-hiding and non-blocking conditions

Definition (no-hiding condition)

A compact outcome function π_C satisfies the *no-hiding condition* if $\forall G \in GSCG, \forall S, S'$ where $S' \subset S \subseteq G$, we have

$$\Sigma_{P \in P_{G \setminus (S \setminus S')}} \Sigma_{s \in (S' \cap P)} \pi_C(s, P) \leq \Sigma_{s \in (G \cap S)} \pi_C(s, G).$$

No-hiding and non-blocking conditions

Definition (no-hiding condition)

A compact outcome function π_C satisfies the *no-hiding condition* if $\forall G \in GSCG, \forall S, S'$ where $S' \subset S \subseteq G$, we have

$$\Sigma_{P \in P_{G \setminus (S \setminus S')}} \Sigma_{s \in (S' \cap P)} \pi_C(s, P) \leq \Sigma_{s \in (G \cap S)} \pi_C(s, G).$$

Definition (non-blocking condition for *SCG*)

A compact outcome function π_C satisfies the *non-blocking condition for SCG* if $\forall G \in GSCG, \forall S \in SCG$ such that $S \subseteq G$, $\Sigma_{s \in S} \pi_C(s, G) \geq w(S)$ holds.

Results

Theorem (sufficient conditions for anonymity-proof core)

If an outcome function π is expressible by a compact outcome function π_C, and π_C satisfies the no-hiding condition and the non-blocking condition for SCG, then π is in the anonymity-proof core.

Results

Theorem (sufficient conditions for anonymity-proof core)

If an outcome function π is expressible by a compact outcome function π_C, and π_C satisfies the no-hiding condition and the non-blocking condition for SCG, then π is in the anonymity-proof core.

Theorem (existence of compactly expressible outcome function)

If the anonymity-proof core is nonempty, then there exists an outcome function π in the anonymity-proof core that is compactly expressible by a compact outcome function π_C.

How compact?

- In the worst case, the size of the *GSCG* can be exponential in the size of the *SCG*. But in practice, the size of the *GSCG* can be much smaller.

How compact?

- In the worst case, the size of the *GSCG* can be exponential in the size of the *SCG*. But in practice, the size of the *GSCG* can be much smaller.
- Simulation: assuming there are 15 skills, 100 members of *SCG* are drawn iid from a uniform distribution. The average size of *GSCG* was 3400, or 10% of 2^{15}.

Outline

5. Anonymity-proof nucleolus
   - Definitions
   - Results

Anonymity-proof nucleolus

Definition (excess)

Given a set of skills $G \in GSCG$ and some $S \subseteq G$, the *excess* of S is $w(S) - \Sigma_{s \in S} \pi_C(s, G)$.
Let $\mathscr{S} = \{S \subset T : S \in SCG \text{ or } S \text{ is a singleton}\}$. The *compact excess vector* for a compact outcome function π_C is the vector of excesses for coalitions in $\mathscr{S}$ in descending order.

Anonymity-proof nucleolus

Definition (excess)

Given a set of skills $G \in GSCG$ and some $S \subseteq G$, the *excess* of S is $w(S) - \Sigma_{s \in S} \pi_C(s, G)$.
Let $\mathscr{S} = \{S \subset T : S \in SCG \text{ or } S \text{ is a singleton}\}$. The *compact excess vector* for a compact outcome function π_C is the vector of excesses for coalitions in $\mathscr{S}$ in descending order.

Definition (anonymity-proof nucleolus)

The *anonymity-proof nucleolus* is the compact outcome function that satisfies the no-hiding condition and gives the lexicographically best compact excess vector.

Example

Let's return to our example. The conventional nucleolus gives 1 to *b*, 0.5 to *d* and *e*, and 0 to *a* and *c*. The outcome function in the anonymity-proof nucleolus is expressible by the following compact outcome function π_C:

- $\pi_C(a, \{a,b,c\}) = \pi_C(c, \{a,b,c\}) = 0, \pi_C(b, \{a,b,c\}) = 1,$
- $\pi_C(a, \{a,b\}) = 0, \pi_C(b, \{a,b\}) = 1,$
- $\pi_C(b, \{b,c\}) = 1, \pi_C(c, \{b,c\}) = 0,$
- $\pi_C(d, \{d,e\}) = 0.5, \pi_C(e, \{d,e\}) = 0.5.$

Results

Some desirable properties of the conventional nucleolus carry over to the anonymity-proof nucleolus.

Results

Some desirable properties of the conventional nucleolus carry over to the anonymity-proof nucleolus.

- The anonymity-proof nucleolus is unique.

Results

Some desirable properties of the conventional nucleolus carry over to the anonymity-proof nucleolus.

- The anonymity-proof nucleolus is unique.
- The anonymity-proof nucleolus always exists.

Results

Some desirable properties of the conventional nucleolus carry over to the anonymity-proof nucleolus.

- The anonymity-proof nucleolus is unique.
- The anonymity-proof nucleolus always exists.
- If the anonymity-proof core is nonempty, then the anonymity-proof nucleolus is in the anonymity-proof core.

Results

Some desirable properties of the conventional nucleolus carry over to the anonymity-proof nucleolus.

- The anonymity-proof nucleolus is unique.
- The anonymity-proof nucleolus always exists.
- If the anonymity-proof core is nonempty, then the anonymity-proof nucleolus is in the anonymity-proof core.
- If two skills s, s' are symmetric, i.e., $w(S \cup \{s\}) = w(S \cup \{s'\})$ for all S such that $s, s' \notin S$, then the anonymity-proof nucleolus π_C gives the same value for s and s'.

Results

Some desirable properties of the conventional nucleolus carry over to the anonymity-proof nucleolus.

- The anonymity-proof nucleolus is unique.
- The anonymity-proof nucleolus always exists.
- If the anonymity-proof core is nonempty, then the anonymity-proof nucleolus is in the anonymity-proof core.
- If two skills s, s' are symmetric, i.e., $w(S \cup \{s\}) = w(S \cup \{s'\})$ for all S such that $s, s' \notin S$, then the anonymity-proof nucleolus π_C gives the same value for s and s'.

However, the anonymity-proof nucleolus does not minimize the largest excess. (Though it does minimize the largest excess of the members of the *SCG*).

Outline

Summary

- Anonymity is the hiding of skills; using falsenames , appearing as a larger or smaller coalition than in reality; and this can corrupt traditional solution consepts

Summary

- Anonymity is the hiding of skills; using falsenames , appearing as a larger or smaller coalition than in reality; and this can corrupt traditional solution consepts
- Introduced the anyonymity-proof core.

Summary

- Anonymity is the hiding of skills; using falsenames , appearing as a larger or smaller coalition than in reality; and this can corrupt traditional solution consepts
- Introduced the anyonymity-proof core.
- The least anonymity-proof core always exists

Summary

- The naive representation of anonymity-proof solution concepts has size exponential in the number of skills that agents declare. We presented a compact representation of the outcome function which specifies values only for synergetic coalitions.

Summary

- The naive representation of anonymity-proof solution concepts has size exponential in the number of skills that agents declare. We presented a compact representation of the outcome function which specifies values only for synergetic coalitions.
- Main results: (1) properties of the compact outcome function that guarantee membership in the anonymity-proof core, (2) nonemptiness of anonymity-proof core guarantees that a core outcome function is compactly expressible.

Summary

- The naive representation of anonymity-proof solution concepts has size exponential in the number of skills that agents declare. We presented a compact representation of the outcome function which specifies values only for synergetic coalitions.
- Main results: (1) properties of the compact outcome function that guarantee membership in the anonymity-proof core, (2) nonemptiness of anonymity-proof core guarantees that a core outcome function is compactly expressible.
- Introduced the anonymity-proof nucleolus, which retains some nice properties of the conventional nucleolus.