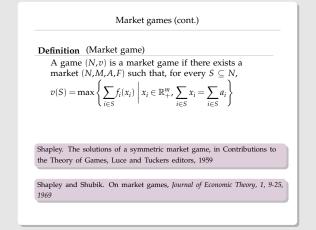


- perform a task.
- Find complementary agents to perform the tasks
  - robots have the ability to move objects in a plant, but multiple robots are required to move a heavy box.

  - transportation domain: agents are trucks, trains, airplanes, ships... a task is a good to be transported.
- Issues:
  - What coalition to form?
  - How to reward each each member when a task is completed?

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	<ul> <li>M is a set of m continuous good</li> </ul>	
	• $A = (a_i)_{i \in N}$ is the initial endowment vector	
	• $F = (f_i)_{i \in N}$ is the valuation function vector	
	Assumptions of the model:	
	<ul> <li>The utility of agent <i>i</i> for possessing <i>x</i> ∈ ℝ<sup>m</sup><sub>+</sub> and an amount of money <i>p</i> ∈ ℝ is <i>u<sub>i</sub>(x,p) = f<sub>i</sub>(x) + p</i>. The money models side payments.</li> </ul>	
	<ul> <li>Initially, agents have no money.</li> </ul>	
	• $p_i$ can be <b>positive</b> or <b>negative</b> (like a bank account).	
	<ul> <li>Agents can increase their utility by trading: after a trade among the members of S, they have an endowment (b<sub>i</sub>)<sub>i∈S</sub> and money (p<sub>i</sub>)<sub>i∈S</sub> such that</li> </ul>	
	$\sum_{i \in S} a_i = \sum_{i \in b} b_i$ and $\sum_{i \in S} p_i = 0$ .	
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	Cost allocation games	

Definition (Cost allocation game) A cost allocation game is a game (N, c) where

- $\circ~N$  represents the potential customers of a public service or a public facility.
- $\circ \ c(S)$  is the cost of serving the members of S

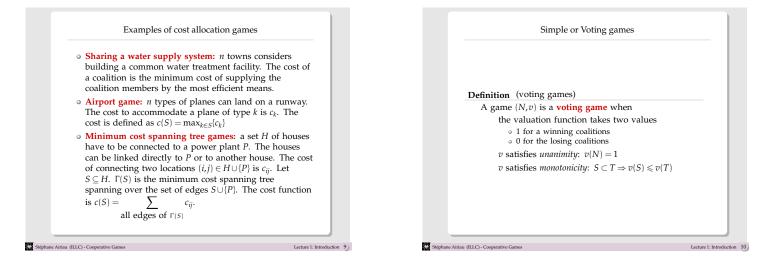
Mathematically speaking, a cost game is a game. The special status comes because of the different intuition (worth of a coalition vs. cost of a coalition).

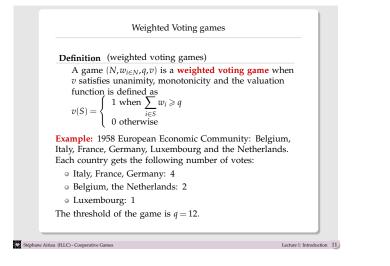
We can associate a cost game with a "traditional game" using the corresponding saving game (N, v) given by

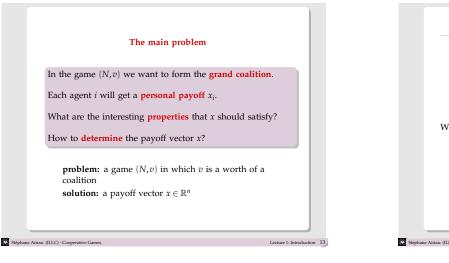
$$v(S) = \sum_{i \in S} c(\{i\}) - c(S).$$

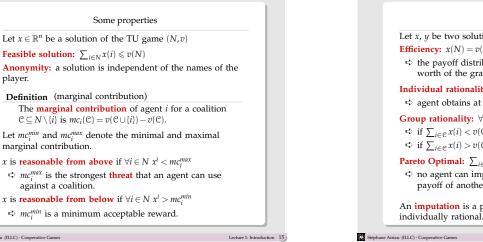
 ${\scriptstyle \odot}$  N is a set of traders

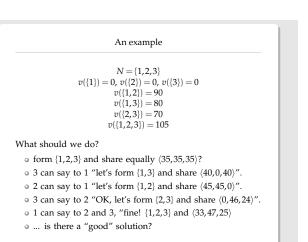
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Some types of TU games

• additive (or inessential):  $v(\mathcal{C}_1 \cup \mathcal{C}_2) = v(\mathcal{C}_1) + v(\mathcal{C}_2)$ 

• superadditive:  $v(\mathcal{C}_1 \cup \mathcal{C}_2) \ge v(\mathcal{C}_1) + v(\mathcal{C}_2)$  satisfied in

many applications: it is better to form larger coalitions.

Convex game appears in some applications in game

trivial from the game theoretic point of view

• weakly superadditive:  $v(\mathcal{C}_1 \cup \{i\}) \ge v(\mathcal{C}_1) + v(\{i\})$ 

• subadditive:  $v(\mathcal{C}_1 \cup \mathcal{C}_2) \leqslant v(\mathcal{C}_1) + v(\mathcal{C}_2)$ 

theory and have nice properties.

• **monotonic:**  $\forall \mathcal{C} \subseteq \mathcal{T} \subseteq N \ v(\mathcal{C}) \leq v(\mathcal{T}).$ 

 $\forall \mathbb{C}_1, \mathbb{C}_2 \subseteq N \mid \mathbb{C}_1 \cap \mathbb{C}_2 = \emptyset, \, i \in N, \, i \notin \mathbb{C}_1$ 

• **convex:**  $\forall \mathcal{C} \subseteq \mathcal{T}$  and  $i \notin \mathcal{T}$ ,  $v(\mathcal{C} \cup \{i\}) - v(\mathcal{C}) \leqslant v(\mathcal{T} \cup \{i\}) - v(\mathcal{T}).$ 

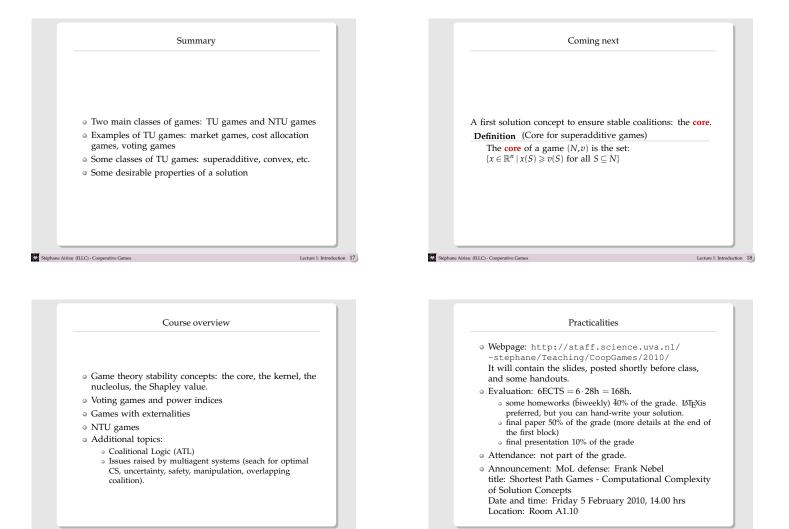
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Let	x, y be two solutions of a TU-game $(N, v)$ .
Effi	ciency: x(N) = v(N)
¢	the payoff distribution is an allocation of the entire worth of the grand coalition to all agents.
Ind	<b>ividual rationality:</b> $\forall i \in N, x(i) \ge v(\{i\})$
¢	agent obtains at least its self-value as payoff.
Gro	<b>oup rationality:</b> $\forall C \subseteq N$ , $\sum_{i \in C} x(i) = v(C)$
¢	if $\sum_{i \in \mathcal{C}} x(i) < v(\mathcal{C})$ some utility is lost
¢	if $\sum_{i \in \mathcal{C}} x(i) > v(\mathcal{C})$ is not possible
Par	eto Optimal: $\sum_{i \in N} x(i) = v(N)$
¢	no agent can improve its payoff without lowering the payoff of another agent.



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