

# KERNEL STABLE COALITION FORMATION

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  - Pareto optimality is insufficient for the evaluation of possible coalitions.

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- ② Generating Proposals
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## 2 Generating Proposals

- 1 If the agent is not leader of the coalition, 4.3.
- 2 For each other coalition, compute a Kernel-stable configuration. Send proposal to strictly dominating coalitional configuration.

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  - ⑤ If grand coalition is formed, or time ends: stop. Else go back to Generating proposals

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  - Speed: efficiency and anytime algorithm



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- Fairly general: Considers both superadditive and non-superadditive games

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- Coalition negotiations are safe with respect to unknown coalition values

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- The coalition negotiation will therefore not be safe if the agent set is changing

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- Other agents might become suspicious because of the delay in the deceiving agent's communication



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- Protocols:
  - Any interaction among agents requires some protocols. As more protocols are enforced on the agents, communication usually decreases. Yet the protocols may be contradictory to the rationality of an individual agent.
  - Any deviation from the protocols must be revealable and penalizable, or the protocols must be self-enforced.
  - Some constrains are needed to avoid an endless loop of rejected proposals for coalition formation.



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- PC relative error:  $\mathbf{e}_r = \frac{\mathbf{e}}{\sum u_i}$

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- In cases where time, communications, and computation are cheap or costless, or in cases where there is a small number of agents, *DEK-CFM* is adequate.

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  - stop iteration if all of the agents have been approached.



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  - ⑥ merge lists of all agents in one list and find the *Pareto optimal PCs*:
    - iteration  $j$ :  $A_i$  s.t.  $z_i \bmod 2^j = 1$  merge its list with  $A_k$  s.t.  $z_k = z_i + 2^{j-1}$
    - find the locally *Pareto optimal PCs* from the merged list and hold by the  $A_i$  with  $z_i \bmod 2^j = 1$
    - stop iteration if all of the agents have been approached.
  - ⑦ choose one of the found *Pareto optimal PCs* by the decision-making method

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- complexity of the computation of coalitional values and configurations is  $O(n^n)$

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- Thus, **DEK-CFM** has  $O(n2^n n^n)$

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- protocols must be agreed on that will direct the agents to a well-defined polynomial set of coalitions.
  - thus, only coalitions of sizes in the ranges  $[K_1; K_2]$  are allowed to be considered for excess calculations.



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- First Stage:
  - ① agents receive proposals as a member of a coalition
  - ② coalitions coordinate their actions either via a representative or by voting (or both)
  - ③ coalitions perform iteratively as follow:
    - transmit a proposal to a target coalition; wait for responses
    - accept  $P_{rp}$  only if  $P_{rp} = P_{pr}$
    - if  $P_{rp}$  was accepted and mutually confirmed, form  $\mathbf{C}_{r+p}$ ; if necessary, choose the representative.
    - send acceptance of  $P_{rp}$  to other coalitions and reject other proposals.





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- thus, the upper limit is of order  $O(n^n)$

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  - These algorithms are enforceable and thus deviations are revealable