S. Ieong and Y. Shoham: Marginal contribution nets: a compact representation scheme for coalitional games

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Evaluation criteria for representation schemes:

- Expressivity
- Conciseness
- Efficiency
- Simplicity

There are special kinds of games that can be represented better using properties of the game, but we want a good general representation scheme.

The basic idea is to use a set of rules to describe the payoffs of coalitions.

$$Pattern \mapsto value$$

We then let the value of a coalition be the sum of all the rules satisfied by that coalition.

A pattern is a conjunction of agents or their negations.

$$\{p_1 \wedge \cdots \wedge p_m \wedge \cdots \neg n_1 \wedge \cdots \wedge \neg n_n\}$$

Where a coalition S satisfies the pattern if all of $p_1, \cdots, p_m \in S$ and $n_1, \cdots, n_n \notin S$.

Let the set of rules be

$$\begin{aligned}\{a \wedge b\} &\mapsto 5\\ \{b\} &\mapsto 2\end{aligned}$$

Then we get that

$$\begin{aligned}v(\{a\}) &= 0\\ v(\{b\}) &= 2\\ v(\{a, b\}) &= 5 + 2 = 7\end{aligned}$$

Let the set of rules be

$$\{a \wedge \neg b\} \mapsto 5$$
$$\{b\} \mapsto 2$$

Then we get that

$$v(\{a\}) = 5$$
$$v(\{b\}) = 2$$
$$v(\{a, b\}) = 2$$

Given a graph game where the value of a coalition is the sum of the values on the edges between them we can represent that game by just taking the edges as rules and their edge values as the rule values.

$$\{a \wedge b\} \mapsto w_{a,b}$$
$$\{c \wedge d\} \mapsto w_{c,d}$$
$$\{e \wedge f\} \mapsto w_{e,f}$$
$$\ldots$$

$v(S) = 1$ iff $S \neq \emptyset$

$$\{a\} \mapsto 1$$
$$\{b \wedge \neg a\} \mapsto 1$$
$$\{c \wedge \neg b \wedge \neg a\} \mapsto 1$$
$$\ldots$$

n rules in total

Marginal contribution networks constitute a fully expressive representation scheme since we can explicitly represent all the 2^n possible coalitions.

$$\begin{aligned} \{p_i\} &\mapsto v(\{p_i\}) \\ &\dots \\ \{p_i \wedge p_j\} &\mapsto v(\{p_i, p_j\}) - v(\{p_i\}) - v(\{p_j\}) \\ &\dots \end{aligned}$$

How do we represent the game such that $v(S) = 1$ iff $|S| > \frac{n}{2}$?

$$\{p_1 \wedge \cdots \wedge p_{\frac{n}{2}+1}\} \mapsto 1$$

$$\cdots \text{ for all subsets of } \frac{n}{2} + 1 \text{ players}$$

$$\{p_1 \wedge \cdots \wedge p_{\frac{n}{2}+2}\} \mapsto -(n+1)$$

$$\cdots \text{ for all subsets of } \frac{n}{2} + 2 \text{ players}$$

- It can sometimes be hard to convert from your original description to the rules needed.
- Lost in translation?
- How do you know that you use as few rules as possible?
- Could it possibly be automatized?
  - Probably computationally hard.
  - And translate from what?

Proposition

The Shapley value of an agent in an MCN is equal to the sum of the Shapley values of the agent over each rule.

Remember that the Shapley value satisfies the Additivity axiom. So we can view each rule as a subgame, and by ADD we can sum them together and get the Shapley value of a player in the whole game.

Only positive literals? Then $\phi_i = \frac{v}{m}$ where m is the number of agents in the rule (by SYM).
Otherwise if i is one of the positive players:

$$\phi_i = \frac{v \times (p-1)!n!}{(p+n)!}$$

Or one of the negative players:

$$\phi_i = \frac{-v \times p!(n-1)!}{(p+n)!}$$

- Linear in the number of rules.
- Embarrassingly parallel.

- Core-membership: Given a coalition game and a payoff vector x, determine if x is in the core.
  - coNP-complete.
- Core-non-emptiness: Given a coalition game and a payoff vector x, determine if x is in the core.
  - coNP-hard.

- Extensions
  - To other types of games.
  - Make the language more concise.
- Rule extraction/generation/optimization.