Overlapping Coalition Formation - Georgios Chalkiadakis, Edith Elkind, Evangelos Markakis and Nicholas R. Jennings

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Content of the presentation

1. Motivation for (this model of) overlapping coalition formation.
2. The model, its core and some results.
3. Reflection.

A model for overlapping coalition formation (OCF)

- In several scenarios it may only be possible to achieve the best outcome if agents can simultaneously belong to more than one coalition.
  - no inherent superadditivity ($v(U \cup T) \geq v(U) + v(T)$) assumption.
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  - no inherent superadditivity ($v(U \cup T) \geq v(U) + v(T)$) assumption.
  - In class we've only discussed coalition structures that consist of disjoint coalitions.
- It may not be possible that an agent not contributing to a coalition receiving payoff from it.
  - not allowing for for cross-coalitional transfers.
  - In class we've mostly discussed TU games.

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- One is better of spending some time to work in the singleton coalition to grand orginality of the work.

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- Furthermore you can imagine that when your not in some coalition, that you won't benefit from the outcomes of that coalition.
  (But I'll come back to this matter.)

Definitions of the OCF model

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- An OCF-game with player set $N = \{1, ..., n\}$ is given by a function $v : [0, 1]^n \rightarrow \mathbb{R}$, where $v(0^n) = 0$.
- A coalition structure $CS_T = (r^1, ..., r^k)$, with $T \subseteq N$ satisfies:
  - $\mathbf{r}^i \in [0, 1]^n$
  - $\{i \in N | r_i \neq 0\} \subseteq T$ for all $i = 1, ..., k$
  - $\sum_{i=1}^{k} r_j^i \leq 1$ for all $j \in T$

Note that there can be infinitely many coalition structures.

Definitions of the OCF model

- Given a coalition structure $|CS| = k$, an *imputation* for CS is a k-tuple $\mathbf{x} = (x^1, ..., x^n)$, where $x^i \in \mathbb{R}^n$, for $i = 1, ..., k$, such that:
  - for all $r^i \in CS$ we have
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    - $r_j^i = 0$ implies $x_j^i = 0$
  - $\sum_{i=1}^{k} x_j^i \geq sup_{CS \in CS_{\{j\}}} v(\{CS\})$ (individual rational)

The core

- An tuple (CS, x) is in the *core* of an OCF-game $G = (N, v)$, if for any set of agents $J \subseteq N$, any coalition structure CS_J on J, and any imputation $\mathbf{y} \in I(CS_J)$, we have $p_j(CS_J, \mathbf{y}) \leq p_j(CS, \mathbf{x})$ for some agent $j \in J$.

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- $\{\{1, 2\}, \{2, 3\}, \{1, 4\}\}$
- The core as we know it:
  $x \in I(CS)$ and $\forall C \subseteq N \sum_{j \in C} x_j \geq v(C)$

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Results

- Theorem 1: An outcome is in the core iff under this outcome the total payments to each subset of agents match the maximum value that can be achieved by this subset.
- Theorem 2: There exists an imputation $\mathbf{x}$ such that $(CS, \mathbf{x})$ belongs to the core iff the game is balanced w.r.t. CS.
- Theorem 3: Convex OCF-games have a non-empty core.

Reflection

- I can think of a lot of real life examples that make use of overlapping coalition structures.
- But I don't really see why not allow for cross-coalition structures.
- At first it looks very intuitively (why would someone profit from a coalition of which he/she is no member?)
- But then when you get a concrete look at the examples it seems not necessarily the case.
- Example: student 1 and 2 are doing better on exercise 2, if 2 had formed a coalition 2 and 3 earlier, since student 2 has therefore become more knowledgable.