

Lecture 1

Introduction and Definition of TU games

1.1 Introduction

Game theory is composed by different fields. Probably the most well known is the field of strategic games that analyse interaction between rational agents: each agent simultaneously takes an action and then receives a payoff that depends on the joint action. The goal of the agent is to maximize the utility they obtain.

Cooperative games is another field that analyses *cooperation* between agents. A *coalition* is simply a set of agents that work together and obtain a payoff for their collective work. It is important to note that the payoff is given to the coalition, not to individual agent. There are two key questions in cooperative games.

- the *selection* problem: which coalitions are going to form?
- the *sharing* problem: once the members have self-organized and achieved their mission, the coalition receives a value. The problem is then how to distribute it to the different members of the coalition.

For some situations, all the agents are intended to work together, and we will assume that there is only one coalition. We will sometimes use this simplification to focus on the sharing problem only. There are many ways to define the payoff distribution, and in this course, we will study different solutions proposed in the literature. Unfortunately, there is no unique and accepted solution to enforce stability, there are different stability criteria, with their own strengths and weaknesses. We will also study some interesting special classes of games. For example, the term coalition is often used in political science: parties may form alliances to obtain more power. Consequently, we will study a class of games that models voting situation. Finally, we will study some different models of cooperative games. For example, in some games, the agents may have preferences over the coalitions, but there is no payoff or values generated by the coalition.

Cooperative games is also a topic of study in Artificial Intelligence. First, the input of the game is by nature exponential: one needs to reason about all possible coalitions,

i.e., all possible subset of the set of agents. Hence, there are some interesting issues in representing the games and computing a solution. There are also some interesting issues to use, in practice, some solution concepts.

The course will mainly focus on the game theoretic aspect of cooperative games, and we will also study AI related issues towards the end of the course. Here is a rough outline of the course.

- The core (2 lectures)
- Games with coalition structure and the bargaining set (1 lecture)
- The nucleolus (1 lecture)
- The kernel (1 lecture)
- The Shapley value (1 lecture)
- Voting games (1 lecture)
- Representation and complexity (1 lecture)
- NTU games and hedonic games (1 lecture)
- Coalition formation and related issues (1 lecture).

There is no textbook for this course. I will provide some lecture notes. The last three chapters of book “A course in game theory” by Osborne and Rubinstein [2] are devoted to cooperative games. I will use some of this material for the lectures on the core, the bargaining set, the kernel, the nucleolus and the Shapley value. The book “An introduction to the theory of cooperative games” by Peleg and Sudhölter [3] contains a rigorous and precise treatment of cooperative games. I used this book for some precision, but it is a more advanced textbook. Whenever appropriate, I will also refer to article from the literature.

1.2 TU games

The game theory community has extensively studied the coalition formation problem [1, 2]. The literature is divided into two main models, depending on whether utility can be transferred between individuals. In a transferable utility game (or TU game), it is assumed that agents can compare their utility and that a common scale of utility exists. In this case, it is possible to define a value for a coalition as the worth the coalition can achieve through cooperation. The agents have to share the value of the coalition, hence utility needs to be transferable. In a so-called non-transferable utility game (or NTU game), inter-personal comparison of utility is not possible, and agents have a preference over the different coalitions of which it is a member. In this section, we introduce the TU games.

1.2.1 Definitions

In the following, we use a utility-based approach and we assume that “everything has a price”: each agent has a utility function that is expressed in currency units. The use of a common currency enables the agents to directly compare alternative outcomes, and it also enables side payments. The definition of a TU game is simple: it involves a set of players and a characteristic function (a map from sets of agents to real numbers) which represents the value that a coalition can achieve. The characteristic function is common knowledge and the value of a coalition depends only on the other players present in its coalition.

Notations

We consider a set N of n agents. A *coalition* is a non-empty subset of N . The set N is also known as the *grand coalition*. The set of all coalitions is 2^N and its cardinality is 2^n . A *coalition structure (CS)* $\mathcal{S} = \{\mathcal{C}_1, \dots, \mathcal{C}_m\}$ is a partition of N : each set \mathcal{C}_i is a coalition with $\cup_{i=1}^m \mathcal{C}_i = N$ and $i \neq j \Rightarrow \mathcal{C}_i \cap \mathcal{C}_j = \emptyset$. We will denote $\mathcal{S}_{\mathcal{C}}$ the set of all partitions of a set of agents $\mathcal{C} \subseteq N$. The set of all CSs is then denoted as \mathcal{S}_N , its size is of the order $O(n^n)$ and $\omega(n^{\frac{n}{2}})$ [4]. The *characteristic function* (or *valuation function*) $v : 2^N \rightarrow \mathbb{R}$ provides the worth or utility of a coalition. Note that this definition assumes that the valuation of a coalition \mathcal{C} does not depend on the other coalitions present in the population.

TU games

1.2.1. DEFINITION. A transferable utility game (TU game) is defined as a pair (N, v) where N is the set of agents, and $v : 2^N \rightarrow \mathbb{R}$ is a characteristic function.

A first example of a TU game is the *majority game*. Assume that the number of agents n is odd and that the agents decide between two alternatives using a majority vote. Also assume that no agent is indifferent, i.e., an agent always strictly prefers one alternative over the other. We model this by assigning to a “winning coalition” the value 1 and to the other ones the value 0, i.e.,

$$v(\mathcal{C}) = \begin{cases} 1 & \text{when } |\mathcal{C}| > \frac{n}{2} \\ 0 & \text{otherwise} \end{cases}$$

Some types of TU games

We now describes some types of valuation functions. First, we introduce a notion that will be useful on many occasion: the notion of marginal contribution. It represent the contribution of an agent when it joins a coalition.

1.2.2. DEFINITION. The *marginal contribution* of agent $i \in N$ for a coalition $\mathcal{C} \subseteq N \setminus \{i\}$ is $mc_i(\mathcal{C}) = v(\mathcal{C} \cup \{i\}) - v(\mathcal{C})$.

The maximal marginal contribution $mc_i^{max} = \max_{C \subseteq N \setminus \{i\}} mc_i(C)$ can be seen as a threat that an agent can use against a coalition: the agent can threaten to leave its current coalition to join the coalition that produces mc_i^{max} , arguing that it is able to generate mc_i^{max} utils. The minimal marginal contribution $mc_i^{min} = \min_{C \subseteq N \setminus \{i\}} mc_i(C)$ is a minimum acceptable payoff: if the agent joins any coalition, the coalition will benefit by at most mc_i^{min} , hence agent i should get at least this amount.

Additive (or inessential): $\forall C_1, C_2 \subseteq N \mid C_1 \cap C_2 = \emptyset, v(C_1 \cup C_2) = v(C_1) + v(C_2)$.

When a TU game is additive, $v(C) = \sum_{i \in C} v(i)$, i.e., the worth of each coalition is the same whether its members cooperate or not: there is no gain in cooperation or any synergies between coalitions, which explains the alternative name (inessential) used for such games.

Superadditive: $\forall C_1, C_2 \subseteq N \mid C_1 \cap C_2 = \emptyset, v(C_1 \cup C_2) \geq v(C_1) + v(C_2)$, in other words, any pair of coalitions is best off by merging into one. In such environments, social welfare is maximised by forming the grand coalition.

Subadditive: $\forall C_1, C_2 \subseteq N \mid C_1 \cap C_2 = \emptyset, v(C_1 \cup C_2) \leq v(C_1) + v(C_2)$: the agents are best off when they are on their own, i.e., cooperation not desirable.

Convex games: A valuation is *convex* if for all $C \subseteq T$ and $i \notin T$ $v(C \cup \{i\}) - v(C) \leq v(T \cup \{i\}) - v(T)$. So a valuation function is convex when the marginal contribution of each player increases with the size of the coalition he joins. Convex valuation functions are superadditive.

Monotonic A function is monotonic when $\forall C_1 \subseteq C_2 \subseteq N, v(C_1) \leq v(C_2)$. In other words, when more agents join a coalition, the value of the larger coalition is at least the value of the smaller one. For example, the valuation function of the majority game is monotonic: when more agents join a coalition, they cannot turn the coalition from a winning to a losing one.

Unconstrained. The valuation function can be superadditive for some coalitions, and subadditive for others: some coalitions should merge when others should remain separated. This is the most difficult and interesting environment.

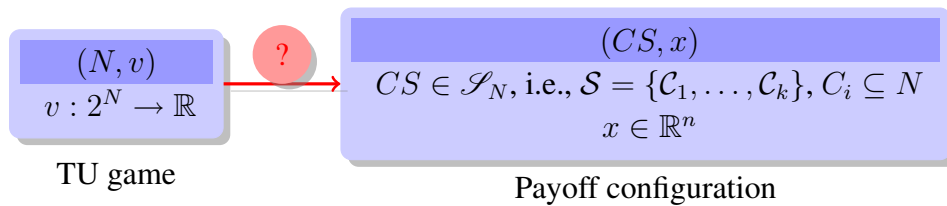


Figure 1.1: What is solving TU games?

The valuation function provides a value to a set of agents, not to individual agents. The *payoff distribution* $x = \{x_1, \dots, x_n\}$ describes how the worth of the coalition is shared between the agents, where x_i is the payoff of agent i . We also use the notation $x(\mathcal{C}) = \sum_{i \in \mathcal{C}} x(i)$. A *payoff configuration (PC)* is a pair (\mathcal{S}, x) where $\mathcal{S} \in \mathcal{S}_N$ is a CS and x is a payoff distribution. Given a TU game (N, v) as an input, the *fundamental question* is what PC will form: what are the coalitions that will form and how to distribute the worth of the coalition (see Figure 1.1).

$$\begin{aligned} N &= \{1, 2, 3\} \\ v(\{1\}) &= 0, v(\{2\}) = 0, v(\{3\}) = 0 \\ v(\{1, 2\}) &= 90 \\ v(\{1, 3\}) &= 80 \\ v(\{2, 3\}) &= 70 \\ v(\{1, 2, 3\}) &= 105 \end{aligned}$$

Table 1.1: An example of a TU game

Let us go over the TU game in Table 1.1. In this example, there are three agents named 1, 2 and 3. There are 7 possible coalitions and the value of each coalition is given in the table. There are 5 CSs which are the following: $\{\{1\}, \{2\}, \{3\}\}$, $\{\{1, 2\}, \{3\}\}$, $\{\{1\}, \{2, 3\}\}$, $\{\{2\}, \{1, 3\}\}$, $\{\{1, 2, 3\}\}$. What PC should be chosen? Should the agents form the grand coalition and share equally the value? The choice of the coalition can be justified by arguing that it is the coalition that generates the most utility for the society. However, is an equal share justified? Agent 3 could propose to agent 1 to form $\{1, 3\}$ and to share equally the value of this coalition (hence, 40 for each agent). Actually, agent 2 can make a better offer to agent 1 by proposing an equal share of 45 if they form $\{1, 2\}$. Agent 3 could then propose to agent 1 to form $\{1, 3\}$ and to let it get 46 (agent 3 would then have 34). Is there a PC that would be preferred by all agents at the same time?

1.2.2 Rationality concepts

In this section, we discuss some desirable properties that link the coalition values to the agents' individual payoff.

Feasible solution: First, one should not distribute more utility than is available. A payoff x is feasible when $\sum_{i \in N} x_i \leq v(N)$.

Anonymity: A solution is independent of the names of the agents. This is a pretty mild solution that will always be satisfied.

Efficiency: $x(N) = v(N)$ the payoff distribution is an allocation of the whole worth of the grand coalition to all the players. In other words, no utility is lost at the level of the population.

Individual rationality: An agent i will be a member of a coalition only when $x_i \geq v(\{i\})$, i.e., to be part of a coalition, a player must be better off than when it is on its own.

Group rationality: $\forall \mathcal{C} \subseteq N, x(\mathcal{C}) \geq v(\mathcal{C})$, i.e., the sum of the payoff of a coalition should be at least the value of the coalition (there should not be any loss at the level of a coalition).

Pareto optimal payoff distribution: It may be desirable to have a payoff distribution where no agent can improve its payoff without lowering the payoff of another agent. More formally, a payoff distribution x is Pareto optimal iff $\nexists y \in \mathbb{R}^n \mid \exists i \in N \mid \{y_i > x_i \text{ and } \forall j \neq i, y_j \geq x_j\}$.

Reasonable from above: an agent should get at most its maximal threat, i.e., $x_i < mc_i^{max}$.

Reasonable from below: the agent should get at least its minimum acceptable reward $x_i > mc_i^{min}$.

Some more notions will be helpful to discuss some solution concepts. The first is the notion of *imputation*, which is a payoff distribution with the minimal acceptable constraints.

1.2.3. DEFINITION. An *imputation* is a payoff distribution that is efficient and individually rational for all agents.

An imputation is a solution candidate for a payoff distribution, and can also be used to object a payoff distribution.

The second notion is the *excess* which can be seen as an amount of complaint or as a potential strength depending on the view point.

1.2.4. DEFINITION. The *excess* related to a coalition \mathcal{C} given a payoff distribution x is $e(\mathcal{C}, x) = v(\mathcal{C}) - x(\mathcal{C})$.

When $e(\mathcal{C}, x) > 0$, the excess can be seen as an amount of complaint for the current members of \mathcal{C} as some part of the value of the coalition is lost. When \mathcal{C} is not actually formed, some agent $i \in \mathcal{C}$ can also see the excess as a potential increase of its payoff if \mathcal{C} was to be formed. Some stability concepts (the kernel and the nucleolus, see below) are based on the excess of coalitions. Another stability concept can also be defined in terms of the excess.

Bibliography

- [1] James P. Kahan and Amnon Rapoport. *Theories of Coalition Formation*. Lawrence Erlbaum Associates, Publishers, 1984.
- [2] Martin J. Osborne and Ariel Rubinstein. *A Course in Game Theory*. The MIT Press, 1994.
- [3] Bezalel Peleg and Peter Sudhölter. *Introduction to the theory of cooperative cooperative games*. Springer, 2nd edition, 2007.
- [4] Tuomas W. Sandholm, Kate S. Larson, Martin Andersson, Onn Shehory, and Fernando Tohmé. Coalition structure generation with worst case guarantees. *Artificial Intelligence*, 111(1–2):209–238, 1999.

Cooperative Games

Lecture 1: Introduction

Stéphane Airiau

ILLC - University of Amsterdam



Why study coalitional games?

Coalitional (or Cooperative) games are a branch of game theory in which **cooperation** or collaboration between agents can be modeled. Coalitional games can also be studied from a computational point of view (e.g., the problem of succinct representation and reasoning).

A coalition may represent a set of:

- persons or group of persons (labor unions, towns)
- objectives of an economic project
- artificial agents

We have a population N of n agents.

Definition (Coalition)

A **coalition** \mathcal{C} is a set of agents: $\mathcal{C} \in 2^N$.

The main problem

- N is the set of all agents (or players)
- $v: 2^N \rightarrow \mathbb{R}$ is the valuation function. For $\mathcal{C} \subseteq N$, $v(\mathcal{C})$ is the value obtained by the coalition \mathcal{C}

problem: a game (N, v) , and we assume agents in N want to cooperate.

solution: a payoff vector $x \in \mathbb{R}^n$ that provides a value to individual agents.

What are the interesting **properties** that x should satisfy?

How to **determine** the payoff vector x ?

An example

$$\begin{aligned} N &= \{1, 2, 3\} \\ v(\{1\}) &= 0, v(\{2\}) = 0, v(\{3\}) = 0 \\ v(\{1, 2\}) &= 90 \\ v(\{1, 3\}) &= 80 \\ v(\{2, 3\}) &= 70 \\ v(\{1, 2, 3\}) &= 105 \end{aligned}$$

What should we do?

- form $\{1, 2, 3\}$ and share equally $\langle 35, 35, 35 \rangle$?
- 3 can say to 1 “let’s form $\{1, 3\}$ and share $\langle 40, 0, 40 \rangle$ ”.
- 2 can say to 1 “let’s form $\{1, 2\}$ and share $\langle 45, 45, 0 \rangle$ ”.
- 3 can say to 2 “OK, let’s form $\{2, 3\}$ and share $\langle 0, 46, 24 \rangle$ ”.
- 1 can say to 2 and 3, “fine! $\{1, 2, 3\}$ and $\langle 33, 47, 25 \rangle$ ”
- ... is there a “good” solution?

Two main classes of games

1- Games with Transferable Utility (TU games)

- Two agents can **compare** their utility
- Two agents can **transfer** some utility

Definition (valuation or characteristic function)

A *valuation function* v associates a real number $v(\mathcal{C})$ to any subset $\mathcal{C} \subseteq N$, i.e., $v: 2^N \rightarrow \mathbb{R}$

Definition (TU game)

A TU game is a pair (N, v) where N is a set of agents and where v is a valuation function.

2- Games with Non Transferable Utility (NTU games)

It is **not** always possible to compare the utility of two agents or to transfer utility (e.g., no price tags). Agents have preference over coalitions.

Today

We provide some examples of TU games.

We discuss some desirable solution properties.

We end with a quick overview of the course and practicalities

Informal example: a task allocation problem

- A set of tasks requiring different expertises needs to be performed, tasks may be decomposed.
- Agents do not have enough resource on their own to perform a task.
- Find complementary agents to perform the tasks
 - robots have the ability to move objects in a plant, but multiple robots are required to move a heavy box.
 - transportation domain: agents are trucks, trains, airplanes, ships... a task is a good to be transported.
- **Issues:**
 - What coalition to form?
 - How to reward each member when a task is completed?

Market games

A **market** is a quadruple (N, M, A, F) where

- N is a set of traders
- M is a set of m continuous good
- $A = (a_i)_{i \in N}$ is the initial endowment vector
- $F = (f_i)_{i \in N}$ is the valuation function vector

Assumptions of the model:

- The **utility** of agent i for possessing $x \in \mathbb{R}_+^m$ and an amount of money $p \in \mathbb{R}$ is $u_i(x, p) = f_i(x) + p$. The money models side payments.
- Initially, agents have **no money**.
- p_i can be **positive** or **negative** (like a bank account).
- Agents can increase their utility by **trading**: after a trade among the members of S , they have an endowment $(b_i)_{i \in S}$ and money $(p_i)_{i \in S}$ such that $\sum_{i \in S} a_i = \sum_{i \in S} b_i$ and $\sum_{i \in S} p_i = 0$.

Market games (cont.)

Definition (Market game)

A game (N, v) is a market game if there exists a market (N, M, A, F) such that, for every $S \subseteq N$,

$$v(S) = \max \left\{ \sum_{i \in S} f_i(x_i) \mid x_i \in \mathbb{R}_+^m, \sum_{i \in S} x_i = \sum_{i \in S} a_i \right\}$$

Shapley. The solutions of a symmetric market game, in *Contributions to the Theory of Games*, Luce and Tuckers editors, 1959

Shapley and Shubik. On market games, *Journal of Economic Theory*, 1, 9-25, 1969

Cost allocation games

Definition (Cost allocation game)

A cost allocation game is a game (N, c) where

- N represents the potential customers of a public service or a public facility.
- $c(S)$ is the cost of serving the members of S

Mathematically speaking, a cost game is a game. The special status comes because of the different intuition (worth of a coalition vs. cost of a coalition).

We can associate a cost game with a “traditional game” using the corresponding saving game (N, v) given by

$$v(S) = \sum_{i \in S} c(\{i\}) - c(S).$$

Examples of cost allocation games

- **Sharing a water supply system:** n towns consider building a common water treatment facility. The cost of a coalition is the minimum cost of supplying the coalition members by the most efficient means.
- **Airport game:** n types of planes can land on a runway. The cost to accommodate a plane of type k is c_k . The cost is defined as $c(S) = \max_{k \in S} \{c_k\}$
- **Minimum cost spanning tree games:** a set H of houses have to be connected to a power plant P . The houses can be linked directly to P or to another house. The cost of connecting two locations $(i, j) \in H \cup \{P\}$ is c_{ij} . Let $S \subseteq H$. $\Gamma(S)$ is the minimum cost spanning tree spanning over the set of edges $S \cup \{P\}$. The cost function is
$$c(S) = \sum_{\text{all edges of } \Gamma(S)} c_{ij}.$$

Simple or Voting games

Definition (voting games)

A game (N, v) is a **voting game** when

the valuation function takes two values

- 1 for a winning coalition
- 0 for the losing coalition

v satisfies *unanimity*: $v(N) = 1$

v satisfies *monotonicity*: $S \subset T \Rightarrow v(S) \leq v(T)$

Weighted Voting games

Definition (weighted voting games)

A game $(N, w_{i \in N}, q, v)$ is a **weighted voting game** when v satisfies unanimity, monotonicity and the valuation function is defined as

$$v(S) = \begin{cases} 1 & \text{when } \sum_{i \in S} w_i \geq q \\ 0 & \text{otherwise} \end{cases}$$

Example: 1958 European Economic Community: Belgium, Italy, France, Germany, Luxembourg and the Netherlands. Each country gets the following number of votes:

- Italy, France, Germany: 4
- Belgium, the Netherlands: 2
- Luxembourg: 1

The threshold of the game is $q = 12$.

Some types of TU games

$$\forall \mathcal{C}_1, \mathcal{C}_2 \subseteq N \mid \mathcal{C}_1 \cap \mathcal{C}_2 = \emptyset, i \in N, i \notin \mathcal{C}_1$$

- **additive (or inessential):** $v(\mathcal{C}_1 \cup \mathcal{C}_2) = v(\mathcal{C}_1) + v(\mathcal{C}_2)$
trivial from the game theoretic point of view
- **superadditive:** $v(\mathcal{C}_1 \cup \mathcal{C}_2) \geq v(\mathcal{C}_1) + v(\mathcal{C}_2)$ satisfied in many applications: it is better to form larger coalitions.
- **weakly superadditive:** $v(\mathcal{C}_1 \cup \{i\}) \geq v(\mathcal{C}_1) + v(\{i\})$
- **subadditive:** $v(\mathcal{C}_1 \cup \mathcal{C}_2) \leq v(\mathcal{C}_1) + v(\mathcal{C}_2)$
- **convex:** $\forall \mathcal{C} \subseteq \mathcal{T}$ and $i \notin \mathcal{T}$,
 $v(\mathcal{C} \cup \{i\}) - v(\mathcal{C}) \leq v(\mathcal{T} \cup \{i\}) - v(\mathcal{T})$.
Convex game appears in some applications in game theory and have nice properties.
- **monotonic:** $\forall \mathcal{C} \subseteq \mathcal{T} \subseteq N$ $v(\mathcal{C}) \leq v(\mathcal{T})$.

Some properties

Let $x \in \mathbb{R}^n$ be a solution of the TU game (N, v)

Feasible solution: $\sum_{i \in N} x(i) \leq v(N)$

Anonymity: a solution is independent of the names of the player.

Definition (marginal contribution)

The **marginal contribution** of agent i for a coalition $\mathcal{C} \subseteq N \setminus \{i\}$ is $mc_i(\mathcal{C}) = v(\mathcal{C} \cup \{i\}) - v(\mathcal{C})$.

Let mc_i^{min} and mc_i^{max} denote the minimal and maximal marginal contribution.

x is **reasonable from above** if $\forall i \in N \ x^i < mc_i^{max}$

$\Leftrightarrow mc_i^{max}$ is the strongest **threat** that an agent can use against a coalition.

x is **reasonable from below** if $\forall i \in N \ x^i > mc_i^{min}$

$\Leftrightarrow mc_i^{min}$ is a minimum acceptable reward.

Some properties

Let x, y be two solutions of a TU-game (N, v) .

Efficiency: $x(N) = v(N)$

\Leftrightarrow the payoff distribution is an allocation of the entire worth of the grand coalition to all agents.

Individual rationality: $\forall i \in N, x(i) \geq v(\{i\})$

\Leftrightarrow agent obtains at least its self-value as payoff.

Group rationality: $\forall \mathcal{C} \subseteq N, \sum_{i \in \mathcal{C}} x(i) = v(\mathcal{C})$

\Leftrightarrow if $\sum_{i \in \mathcal{C}} x(i) < v(\mathcal{C})$ some utility is lost

\Leftrightarrow if $\sum_{i \in \mathcal{C}} x(i) > v(\mathcal{C})$ is not possible

Pareto Optimal: $\sum_{i \in N} x(i) = v(N)$

\Leftrightarrow no agent can improve its payoff without lowering the payoff of another agent.

An **imputation** is a payoff distribution x that is efficient and individually rational.

Summary

- Two main classes of games: TU games and NTU games
- Examples of TU games: market games, cost allocation games, voting games
- Some classes of TU games: superadditive, convex, etc.
- Some desirable properties of a solution

Coming next

A first solution concept to ensure stable coalitions: the **core**.

Definition (Core for superadditive games)

The **core** of a game (N, v) is the set:
 $\{x \in \mathbb{R}^n \mid x(S) \geq v(S) \text{ for all } S \subseteq N\}$

Course overview

- Game theory stability concepts: the core, the nucleolus, the kernel
- A fair solution concept: the Shapley value
- Special types of games: Voting games
- Representation and complexity
- Other model of cooperation: NTU games and hedonic games.
- Issues raised by practical approaches (search for optimal CS, uncertainty, overlapping coalition, etc).

Practicalities

- **Webpage:** <http://staff.science.uva.nl/~stephane/Teaching/CoopGames/2010/>
It will contain the lecture notes and the slides, posted shortly before class.
- **Evaluation:** $6\text{ECTS} = 6 \cdot 28\text{h} = 168\text{h}$.
 - some homeworks (every two or three weeks) 40% of the grade. \LaTeX is preferred, but you can hand-write your solution.
 - final paper 50% of the grade (more details at the end of the first block)
 - final presentation 10% of the grade
 - **no** exam.
- **Attendance:** not part of the grade.