Lecture 10
Non-Transferable Utility Games (NTU games)

An underlying assumption behind a TU game is that agents have a common scale to measure the worth of a coalition. Such a scale may not exist in every situations, which leads to the study of games where the utility is non-transferable (NTU games). We start by introducing a particular type of NTU games called *Hedonic games*. We chose this type of games for the simplicity of the formalism. Next, we provide the classical definition of a NTU game, which can represent a lot more situations.

10.1 Hedonic Games

In an hedonic game, agents have preferences over coalitions: each agent knows whether it prefers to be in company of some agents rather than others. An agent may enjoy more the company of members of $\mathcal{C}_1$ over members of $\mathcal{C}_2$, but it cannot tell by how much it prefers $\mathcal{C}_1$ over C_2. Consequently, it does not make sense to talk about any kind of compensation when an agent is not part of its favorite coalition. The question that each agent must answer is "which coalition to form?".

More formally, let N be a set of agents and $\mathcal{N}_i$ be the set of coalitions that contain agent i, i.e., $\mathcal{N}_i = \{\mathcal{C} \cup \{i\} \mid \mathcal{C} \subseteq N \setminus \{i\}\}$. For a CS $\mathcal{S}$, we will note $\mathcal{S}(i)$ the coalition in $\mathcal{S}$ containing agent i.

10.1.1. DEFINITION. [Hedonic games] An *Hedonic game* is a tuple $(N, (\succeq_i)_{i \in N})$ where

- N is the set of agents
- $\succeq_i \subseteq 2^{\mathcal{N}_i} \times 2^{\mathcal{N}_i}$ is a complete, reflexive and transitive preference relation for agent i, with the interpretation that if $S \succeq_i T$, agent i prefers coalition T at most as much as coalition S.

In a hedonic game, each agent has a preference over each coalition it can join. The solution of a hedonic game is a coalition structure (CS), i.e., a partition of the set of agents into coalition. A first desirable property of a solution is to be Pareto optimal: it would not be possible to find a different solution that is weakly preferred by all agents.

The notion of core can be easily extended for this type of games. Given a current CS, no group of agents should have an incentive to leave the current CS. As it is the case for TU games, the core of an NTU game may be empty, and it is possible to define weaker versions of stability. We now give the definition of stability concepts adapted from [10]. For the core, it is a group of agents that leave their corresponding coalitions to form a new one they all prefer. In the weaker stability solution concepts, the possible deviations feature a single agent i that leaves its current coalition $\mathcal{C}_1 \cup \{i\}$ to join a different existing coalition $\mathcal{C}_2 \in N \setminus \{i\}$ or to form a singleton coalition $\{i\}$. There are few scenarios one can consider, depending on the behavior of the members of $\mathcal{C}_1$ and $\mathcal{C}_2$. For a *Nash stable* $\mathcal{S}$, the behavior of $\mathcal{C}_1$ and $\mathcal{C}_2$ are not considered at all: if i prefers to join $\mathcal{C}_2$, it is a valid deviation. This assumes that the agents in $\mathcal{C}_2$ will accept agent i, which is quite optimistic (it may very well be the case that some or all agents in $\mathcal{C}_2$ do not like agent i). For *individual stability*, the deviation is valid if no agent in $\mathcal{C}2$ is against accepting agent i, in other words the agents in $\mathcal{C}2$ are happy or indifferent about i joining them. Finally, for *contractual individual stability*, the preference of the members of $\mathcal{C}1$ – the coalition that i leaves – are taken into account. Agents in $\mathcal{C}1$ should prefer to be without i than with i. The three stability concepts have the following inclusion: Nash stability is included in Individual stability, which is included in contractual individual stability[1]. We now provide the corresponding formal definitions.

Core stability: A CS $\mathcal{S}$ is $\boldsymbol{core\text{-}stable}$ iff $\nexists \mathcal{C} \subseteq N \mid \forall i \in \mathcal{C}, \mathcal{C} \succ_i \mathcal{S}(i)$.

Nash stability: A CS $\mathcal{S}$ is $\boldsymbol{Nash\text{-}stable}$ iff $(\forall i \in N)$ $(\forall \mathcal{C} \in \mathcal{S} \cup \{\emptyset\})$ $\mathcal{S}(i) \succsim_i \mathcal{C} \cup \{i\}$. No player would like to join any other coalition in $\mathcal{S}$ assuming the other coalitions did not change.

Individual stability A CS $\mathcal{S}$ is $\boldsymbol{individually\ stable}$ iff $(\nexists i \in N)$ $(\nexists \mathcal{C} \in \mathcal{S} \cup \{\emptyset\}) \mid (\mathcal{C} \cup \{i\} \succ_i \mathcal{S}(i))$ and $(\forall j \in \mathcal{C}, \mathcal{C} \cup \{i\} \succsim_j \mathcal{C})$. No player can move to another coalition that it prefers without making some members of that coalition unhappy.

Contractually individual stability: A CS $\mathcal{S}$ is $\boldsymbol{contractually\ individually\ stable}$ iff $(\nexists i \in N)$ $(\nexists \mathcal{C} \in \mathcal{S} \cup \{\emptyset\}) \mid (\mathcal{C} \cup \{i\} \succ_i \mathcal{S}(i))$ and $(\forall j \in \mathcal{C}, \mathcal{C} \cup \{i\} \succsim_j \mathcal{C})$ and $(\forall j \in \mathcal{S}(i) \setminus \{i\}, \mathcal{S}(i) \setminus \{i\} \succsim_j \mathcal{S}(i))$. No player can move to a coalition it prefers so that the members of the coalition it leaves and it joins are better off.

Let us see some examples.

[1]This ordering may appear counter-intuitive at first, but note that the conditions for being a valid deviation are more difficult to meet from Nash stability to Contractual individual stability, and the stability concepts are defined as CS such that no deviation exists

Example 1

$$\boxed{\begin{array}{l}\{1,2\} \succ_1 \{1\} \succ_1 \{1,2,3\} \succ_1 \{1,3\} \\ \{1,2\} \succ_2 \{2\} \succ_2 \{1,2,3\} \succ_2 \{2,3\} \\ \{1,2,3\} \succ_3 \{2,3\} \succ_3 \{1,3\} \succ_3 \{3\}\end{array}}$$

Let us consider each CS one by one.

$\{\{1\},\{2\},\{3\}\}$	$\{1,2\}$ is preferred by both agent 1 and 2, hence it is not Nash stable.
$\{\{1,2\},\{3\}\}$	$\{1,2,3\}$ is preferred by agent 3, so it is not Nash stable, as agents 1 and 3 are worse off, it is not a possible move for individual stability. no other move is possible for individual stability.
$\{\{1,3\},\{2\}\}$	agent 1 prefers to be on its own (hence the CS is not Nash stable, and then, neither individually stable).
$\{\{2,3\},\{1\}\}$	agent 2 prefers to join agent 1, and agent 1 is better off, hence the CS is neither Nash stable nor Individually stable.
$\{\{1,2,3\}\}$	agents 1 and 2 have an incentive to form a singleton, hence the CS is neither Nash stable, nor Individually stable.

As a conclusion:

- $\{\{1,2\},\{3\}\}$ is in the core and is individually stable.
- There is no Nash stable partitions.

Example 2

$$\boxed{\begin{array}{l}\{1,2\} \succ_1 \{1,3\} \succ_1 \{1,2,3\} \succ_1 \{1\} \\ \{2,3\} \succ_2 \{1,2\} \succ_2 \{1,2,3\} \succ_2 \{2\} \\ \{1,3\} \succ_3 \{2,3\} \succ_3 \{1,2,3\} \succ_3 \{3\}\end{array}}$$

Again, let us consider all the CSs.

$\{\{1\},\{2\},\{3\}\}$	$\{1,2\}$, $\{1,3\}$, $\{2,3\}$ and $\{1,2,3\}$ are blocking. As a result, the CS is not Nash stable
$\{\{1,2\},\{3\}\}$	$\{2,3\}$ is blocking, and since 2 can leave 1 to form $\{1,2\}$, it follows that the CS is not Nash stable
$\{\{1,3\},\{2\}\}$	$\{1,2\}$ is blocking, and since 1 can leave 3 to form $\{1,2\}$, it follows that the CS is not Nash stable
$\{\{2,3\},\{1\}\}$	$\{1,3\}$ is blocking, and since 3 can leave 2 to form $\{1,3\}$,it follows that the CS is not Nash stable
$\{\{1,2,3\}\}$	$\{1,2\}$, $\{1,3\}$, $\{2,3\}$ are blocking, but it is contractually individually stable

As a conclusion, we obtain that

- The core is empty.

- $\{\{1,2,3\}\}$ is the unique Nash stable partition, unique individually stable partition (no agent has any incentive to leave the grand coalition).

Example 3

$$\boxed{\begin{array}{l}\{1,2\} \succ_1 \{1,3\} \succ_1 \{1\} \succ_1 \{1,2,3\}\\ \{2,3\} \succ_2 \{1,2\} \succ_2 \{2\} \succ_2 \{1,2,3\}\\ \{1,3\} \succ_3 \{2,3\} \succ_3 \{3\} \succ_3 \{1,2,3\}\end{array}}$$

For this game we can show that

- The core is empty (similar argument as for example 2).
- There is no Nash stable partition or individually stable partition. But there are three contractually individually stable CSs: $\{\{1,2\},\{3\}\}$, $\{\{1,3\},\{2\}\}$,$\{\{2,3\},\{1\}\}$.

For example, we can consider all the possible changes for the CS $\{\{1,2\},\{3\}\}$:

- agent 2 joins agent 3:

  In this case, both agents 2 and 3 benefit from forming $\{\{1\},\{2,3\}\}$, hence $\{\{1,2\},\{3\}\}$ is not Nash or individually stable. As agent 1 is worse off it is not a legal deviation for contractual individual stability.
- agent 1 joins agent 3, but agent 1 has no incentive to join agent 3, hence this is not a deviation.
- Agent 1 or 2 forms a singleton coalition, but neither agent has any incentive to form a singleton coalition. Hence, there is no deviation.

In Examples 2 and 3, we see that the core may be empty. The literature in game theory focuses on finding conditions for the existence of the core. In the AI literature, Elkind and Wooldridge have proposed a succinct representation of Hedonic games [20] and Brânzei and Larson considered a subclass of hedonic games called the affinity games [12].

10.2 NTU games

We now turn to the most general definition of an NTU game. This definition is more general than the definition of hedonic gamess. The idea is that each coalition has the ability to achieve a set of outcomes. The preference of the agents are about the outcomes that can be brought about by the different coalitions. The formal definition is the following:

10.2.1. DEFINITION. [NTU Game] A non-transferable utility game (NTU Game) is defined by a tuple $(N, X, V, (\succ_i)_{i\in N})$ such that

- N is a set of agents;
- X is a set of outcomes;
- $V : 2^N \to 2^X$ is a function that describes the outcomes $V(\mathcal{C}) \subseteq X$ that can be brought about by coalition $\mathcal{C}$;
- $\succ_i$ is the preference relation of agent i over the set of outcomes. The relation is assumed to be transitive and complete.

Intuitively, $V(\mathcal{C})$ is the set of outcomes that the members of $\mathcal{C}$ can bring about by means of their joint-actions. The agents have a preference relation over the outcomes, which makes a lot of sense. This type of games is more general that the class of hedonic games or even TU games, as we can represent these games using a NTU game.

10.2.2. PROPOSITION. *A hedonic game can be represented by an NTU games.*

Proof. Let $(N, (\succeq_i^H)_{i \in N})$ be a hedonic game.

- For each coalition $\mathcal{C} \subseteq N$, create a unique outcome $x_{\mathcal{C}}$.
- For any two outcomes x_S and x_T corresponding to coalitions S and T that contains agent i, We define $\succeq_i$ as follows: $x_S \succeq_i x_T$ iff $S \succeq_i^H T$.
- For each coalition $\mathcal{C} \subseteq N$, we define $V(\mathcal{C})$ as $V(\mathcal{C}) = \{x_C\}$.

□

10.2.3. PROPOSITION. *A TU game can be represented as an NTU game.*

Proof. Let (N, v) be a TU game.

- We define X to be the set of all allocations, i.e., $X = \mathbb{R}^n$.
- For any two allocations $(x, y) \in X^2$, we define $\succeq_i$ as follows: $x \succeq_i y$ iff $x_i \geq y_i$.
- For each coalition $\mathcal{C} \subseteq N$, we define $V(\mathcal{C})$ as $V(\mathcal{C}) = \{x \in \mathbb{R}^n \mid \sum_{i \in N} x_i \leq v(\mathcal{C})\}$. $V(\mathcal{C})$ lists all the feasible allocation for the coalition $\mathcal{C}$.

□

First, we can note that the definition of the core can easily be modified in the case of NTU games.

10.2.4. DEFINITION. $core(V) = \{x \in V(N) \mid \nexists \mathcal{C} \subset N,\, \nexists y \in V(\mathcal{C}), \forall i \in \mathcal{C}\ y \succ_i x\}$

An outcome $x \in X$ is blocked by a coalition $\mathcal{C}$ when there is another outcome $y \in X$ that is preferred by all the members of $\mathcal{C}$. An outcome is then in the core when it can be achieved by the grand coalition and it is not blocked by any coalition. As is the case for TU game, it is possible that the core of an NTU game is empty.

As for TU games, we can define a balanced game and show that the core of a balanced game is non-empty.

10.2.5. DEFINITION. [Balanced game]A game is *balanced* iff for every balanced collection B, we have $\bigcap_{\mathcal{C}\subseteq B} V(\mathcal{C}) \subset V(N)$

10.2.6. THEOREM (THE SCARF THEOREM). *The core of a balanced game is non-empty.*

10.2.1 An application: Exchange Economy

For TU games, we studied *market games* and proved such games have a non-empty core. We now consider a similar game without transfer of utility. There is a set of continuous goods that can be exchanged between the agents. Each agent has a preference relation over the bundle of goods and tries to obtain the best bundle possible.

Definition of the game

The main difference between the exchange economy and a market game is that the preference is ordinal in the exchange econmomy whereas it is cardinal in a market game.

10.2.7. DEFINITION. An *exchange economy* is a tuple $(N, M, A, (\succeq_i)_{i\in N})$ where

- N is the set of n agents
- M is the set of k continuous goods
- $A = (a_i)_{i\in N}$ is the initial endowment vector
- $(\succeq_i)_{i\in N}$ is the preference profile, in which $\succeq_i$ is a preference relation over bundles of goods.

Given an exchange economy $(N, M, A, (\succeq_i)_{i\in N})$, we define the associated *exchange economy game* as the following NTU game $(N, X, V, (\succeq_i)_{i\in N})$ where:

- The set of outcomes X is defined as $X = \big\{(x_1, \ldots, x_n) \,|\, x_i \in \mathbb{R}^k_+ \text{ for } i \in N\big\}$. Note that $x_i = \langle x_{i1}, \ldots, x_{ik}\rangle$ represents the quantity of each good that agent i possesses in an outcome x.
- The preference relation for an agent i is defined as: for $(x, y) \in X^2$ $x \succeq_i y \Leftrightarrow x_i \succeq_i y_i$. Each player is concerned by its own bundle only.

- The value sets are defined as $\forall \mathcal{C} \subseteq N$,

$$V(\mathcal{C}) = \left\{ x \in X \,\middle|\, \sum_{i \in \mathcal{C}} x_i = \sum_{i \in \mathcal{C}} a_i \ \wedge \ x_j = a_j \text{ for } j \in N \setminus \mathcal{C} \right\}.$$

  The players outside $\mathcal{C}$ do not participate in any trading and hold on their initial endowments. When all agents participate in the trading, we have $V(N) = \left\{ x \in X \mid \sum_{i \in N} x_i = \sum_{i \in N} a_i \right\}$.

Solving an exchange economy

Let us assume that we can define a price p_r for a unit of good r. The idea would be to exchange the goods at a *constant price* during the negotiation.

Let us define a price vector $p \in \mathbb{R}^k_+$. The amount of each good that agent i possesses is $x_i \in R^k_+$. The total cost of agent i's bundle is $p \cdot x_i = \sum_{r=1}^{k} p_r x_{i,r}$. Since the initial endowment of agent i is a_i, the agent has at his disposal an amount $p \cdot a_i$, and i can afford to obtain a bundle y_i such that $p \cdot y_i \leq p \cdot a_i$.

We can wonder about what an ideal situation would be. Given the existence of the price vector, we can define a *competitive equilibrium*, the idea is to make believe to each player that it possesses the best outcome.

10.2.8. DEFINITION. [Competitive equilibrium] The *competitive equilibrium* of an exchange economy $(N, X, V, (\succeq_i)_{i \in N})$ is a *pair* (p, x) where $p \in \mathbb{R}^k_+$ is a price vector and $x \in \left\{ (x_1, \ldots, x_n) \mid x_i \in \mathbb{R}^k_+ \text{ for } i \in N \right\}$ such that

- $\sum_{i \in N} x_i = \sum_{i \in N} a_i$ (the allocation results from trading)
- $\forall i \in N$, $p \cdot x_i \leq p \cdot a_i$ (each agent can afford its allocation)
- $\forall i \in N \ \forall y_i \in \mathbb{R}^k_+ \ (p \cdot y_i \leq p \cdot a_i) \Rightarrow x_i \succeq_i y_i$
  Among all the allocations that an agent can afford, it obtains one of its most favorites outcomes.

This competitive equilibrium seems like an ideal situation, and surprisingly, Arrow and Debreu [1] proved such equilibrium is guaranteed to exist. This is a deep theorem, and we will not study the proof here.

10.2.9. THEOREM (ARROW & DEBREU, 1954). *Let $(N, M, A, (\succeq_i)_{i \in N})$ be an exchange economy. If each preference relation $\succeq_i$ is continuous and strictly convex, then a competitive equilibrium exists.*

The proof of the theorem is not constructive, i.e., it guarantees the existence of the equilibrium, but not how to obtain the price vector or the allocation. The following theorem links the allocation with the core:

10.2.10. THEOREM. *If (p, x) is a competitive equilibrium of an exchange economy, then x belongs to the core of the corresponding exchange economy game.*

Proof. Let us assume x is not in the core of the associated exchange economy game. Then, there is at least one coalition $\mathcal{C}$ and an allocation y such that $\forall i \in \mathcal{C}\ y \succ_i x$. By definition of the competitive equilibrium, we must have $p \cdot y_i > p \cdot a_i$. Summing over all the agents in $\mathcal{C}$, we have $p \cdot \sum_{i \in \mathcal{C}} y_i > p \cdot \sum_{i \in \mathcal{C}} a_i$. Since the prices are positive, we deduce that $\sum_{i \in \mathcal{C}} y_i > \sum_{i \in \mathcal{C}} a_i$, which is a contradiction. □

It then follows that if each preference relation is continuous and strictly convex, then the core of an exchange economy game is non-empty. In an economy, the outcomes that are immune to manipulations by groups of agent are competitive equilibrium allocation.

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