Cooperative games: homework 2

deadline: Friday March 23rd, 5pm

Your answers must be explained and justified. It is your responsability to submit your work on time. Passed the deadline, I will accept the homework up to a week late and your grade will be reduced by 50% (the same consequence applies whether you are 15min or 7days late). Submit the homework or place it in my pigeonhole at ILLC. Finally, submit your *own personal* work, the homeworks are about cooperation, but not an exercise of cooperation. If you need clarification, you can contact me by email or you can stop by my office (Science Park 904, C3.125).

Excercise 1. *A game* $(N, w_{i\in N}, q, v)$ *is a* **weighted voting game** *when* v *satisfies unanimity, monotonicity and the valuation function is defined as:*

$$v(S) = \begin{cases} 1 \text{ when } \sum_{i\in S} w_i \geq q \\ 0 \text{ otherwise} \end{cases}$$

We note such a game by $(q : w_1, \ldots, w_n)$

Let (N, v) *be the game associated with the 6-player weighted majority game (3:1,1,1,1,1,0). Prove that* $x = \langle \frac{1}{7}, \ldots, \frac{1}{7}, \frac{2}{7} \rangle \in BS(N, v)$.

Excercise 2. *Let* (N, v) *be a TU game with a non-empty core. Prove that* $Nu(N, v) \subseteq Core(N, v)$

Excercise 3. *Consider the following three-player game:* $N = \{1, 2, 3\}$, $v(\{1\}) = 0$, $v(\{2\}) = 0$, $v(\{3\}) = 0$, $v(\{1, 2\}) = 90$, $v(\{1, 3\}) = 80$, $v(\{2, 3\}) = 70$, $v(\{1, 2, 3\}) = 120$. *Determine the nucleolus* x *of this game.*

Excercise 4. *What can you say about the set*

$$\left\{ x \in X_{N,v,\mathcal{S}} | \forall y \in X_{N,v,\mathcal{S}} \;\; e(y)^{\blacktriangleright} \geq_{lex} e(x)^{\blacktriangleright} \right\},$$

where $X_{N,v,\mathcal{S}}$ *is the set of feasible payoff vectors for the game* $(N, v, \mathcal{S})$*?*