Centralised and decentralised protocols

Allocating goods

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Multiagent Systems and Economics

- Game Theory
  - non-cooperative games
  - cooperative games
- Mechanism Design
- Social Choice Theory

Today

- Utility functions : know what you handle
- Centralised mechanisms for allocating goods
  - auctions for single goods
  - multi-unit auctions
  - combinatorial auctions
    - winner determination
    - representation language
- Decentralised mechanisms

Preferences and Utility functions

Ordinal Preferences

The **preference relation** of agent i over alternative agreements is a **binary relation** $\succeq_i$:
$x \succeq_i y \Leftrightarrow$ agreement x is not better than y for agent i.

A preference relation $\succeq_i$ is usually required to be

- **transitive** : if you prefer x over y and y over z, you should also prefer x over z
- **complete** for any two agreements x and y, agent i "knows" which one she prefers.

From this "weak" preference, one can define a "strict" preference $\succ$ and indifference relation $\sim$.

Discussion : useful model, but not without problems
(e.g. transitivity (Sorites Paradox), completeness (I may not know enough to he a preference over all pairs of agreements), humans do not always have rational preferences...)

Using numbers to represent preferences

To start with : there is **no objective way** to measure utility.

- **ordinal scale** : a scale is *ordinal* if its admissible transformations are all striclty increasing transformations
  - ➾ qualitative comparison
  - ➾ no information about differences or ratio

  ex : statements such as this agreement is "two times better" or "the difference between agreement a and b is more than the difference between c and d" are *meaningless*.
- **interval scale** : a scale is an *interval* scale if its admissible transformations are all positive affine transformations
  - ➾ quantitative comparisons
  - ➾ differences are *meaningful* (difference between 20°C and 15°C is the same as the difference between 5°C and 0°C, but is different from the difference between 75°F and 70°F)
  - ➾ ratio between two agreements are *meaningless*
- **ratio scale** : a scale is a ratio scale if its admissible transformations are all homothetic transformations

Obtaining utilities : preference elicitation

asking pairwise comparisons ex : do you prefer X over Y ?

Theorem

For a transitive and complete relation $\succeq$ over a finite set X of agreements, there exists a function $u : X \to \mathbb{R}$ such that

$$u_i(x) \geqslant u_i(y) \text{ iff } x \succeq y$$

➪ ordinal scale.

asking lotteries ex : do you prefer obtaining X for sure over obtaining Y or Z with equal probability ?

Theorem (von Neumann-Morgenstern EU theorem)

A preference relation over lotteries satisfies completeness, transitivity, continuity and independence iff it can be represented by a function that has the expected utility form
i.e. for p and p' two probability distributions,

$$p \succeq p' \text{ iff } \sum_i p_j u_j \geqslant \sum_i p'_j u_j.$$

➪ interval scale

Utility and money

Utility function for money is a nonlinear function, usually assumed to be non-symmetric, bounded and concave.

- non-symmetric : losing or making money is treated differently
- bounded : beyond a certain point, money is not useful
- concave : an increase of 1EUR is more important for someone poor than someone rich

As the relation of utility and money is not necessarily linear, we may not be able to use money as a measure of utility.

(One can model an attitude towards risk : is the agent risk-seeker (convex relation), risk-neutral (linear relation) or risk-averse (concave relation) ?)

If one assumes that the relation between utility and money is linear and if the slope is the same for all agents, the agents have **transferable utilities**.

A lot more can be said about preferences and utility functions (cf textbooks on decision theory)!

- ordinal preferences cannot represent intensities
- cardinal preferences cannot handle incomparabilities (ordinal preferences can)

This is for **single agent**! For multiagent, is it meaningful to compare the utility of two agents ?

Key points :

- When you manipulate numbers, only use meaningful operations! We are used to reason in terms of numbers : measures, percentages, ratios... We may be tempted by adding or making ratios when it is not meaningful
- sometimes, it is also not meaningful to compare the utility of two agents
- be aware that it is not easy to obtain utility functions

For the rest of the lecture, we **assume** agents have a utility function over the agreements.

A bit of social Choice

What is a good agreement for collective choice ?

Unanimity Principle

An agreement x is **Pareto-dominated** by an agreement y iff :

- $x \succeq_i y$ for all members i of society and
- $x \succ_j y$ for at least one member j of society

An agreement is **Pareto optimal** iff it is not Pareto-dominated by any other feasible agreements

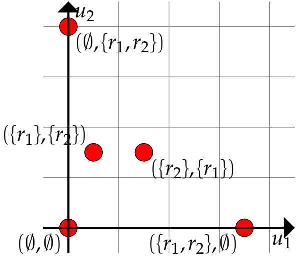
The **unanimity principle** states that society should not select an agreement that is Pareto dominated by another agreement.

(named after Vilfredo Pareto, Italian economist, 1848–1923)

An agreement should be Pareto optimal, but there are usually **many** Pareto optimal agreements.

ex : two resources r_1 and r_2 needs to be allocated between two agents.

	u_1	u_2
$\emptyset$	0	0
$\{r_1\}$	1	3
$\{r_2\}$	3	3
$\{r_1, r_2\}$	7	8



There are some **other** measure of social welfare

- **utilitarian** : maximises $\sum_i u_i$
- **egalitarian** : maximises $\min_i u_i$
- **Nash product** : maximises $\prod_i u_i$
- etc...

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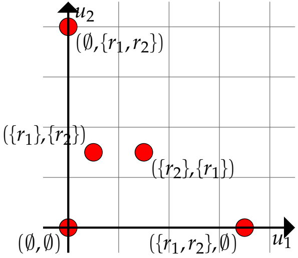

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Centralised protocol for multiagent resource allocation

- single-good auctions
- multi-units auctions
- combinatorial auctions

Decentralised protocols

Centralised protocol for multiagent resource allocation :

Auctions

- Single-good auctions
- multi-unit auctions
- combinatorial auctions

How to allocate scarce resources ?

Many agents desire to obtain the use of a scarce resource

- if it is not scarce, maybe all agents can use it
- otherwise, what is a reasonable way for allocating that resource ?
- ⇨ allocate the resources to those that value them the most : **efficiency**

Terminology :

- the agents that want the use of a resource are the **bidders**
- an **auctioneer** is an agent that runs the auction and who is in charge of allocating a resource to one of the bidders
- Bidders want to obtain the resource for a *minimum* price
- An auctioneer wants a *maximum* price
- An auctioneer chooses the *type of auctions*
- Bidders choose their *strategy* for participating to the auction.

Dimensions of an auction protocol

- Are bids made by the agents known to each other ?
  - **open cry** : bids are common knowledge
  - **sealed-bid** : bids are only known by the auctioneer
- one shot vs multiple bids
  - **one shot** : each agent makes a single bid and auctioneer announces the winner and the price
  - **ascending auctions** : multiple bids that are increasing
  - **descending auctions** : multiple bids that are decreasing
- Who is the winner ? (winner determination problem)
  - trivial question for the single-item auction,
  - not so trivial for combinatorial auctions
- What price does the winner pays ?

We assume transferable utilities. Agent i has

- a utility $u_i \in \mathbb{R}$ for the resource auctioned in a single-item auction
- a utility function $u_i : 2^R \to \mathbb{R}$ where R is the set of all resources. $u_i(C)$ is the utility for a *bundle* of resources (a subset) $C \subseteq R$

Single-item auctions

English Auctions

- open cry
- ascending auction : agents can place a bid higher than the current highest bid. When no more bids are placed, the auction terminates.
- the winner is the highest bidder
- the winner pays the amount of her bid

The auctioneer can set a **reservation price** : if no bidder is willing to bid that price, the auctioneer keep the resource.

Dominant strategy : bid a small amount more than the current bid the price reaches the agent's utility.

Dutch Auctions

- open cry
- descending auction : auctioneer starts announcing a very high value and then continuously lowers the offer price until an agent makes a bid. The auction then terminates.
- the winner is the highest bidder
- the winner pays the amount of her bid (i.e. the price announced by the auctioneer right before the bidder made the bid)

First-price sealed-bid auction

- sealed-bid
- one shot
- the winner is the highest bidder
- the winner pays the amount of her bid.

➯ Bidding slightly below the utility is a *dominant strategy*.

Vickrey Auctions

- William Vickrey (Nobel Prize in Economics 1996)
- sealed-bid
- one shot
- the winner is the highest bidder
- The winner pays the amount of the second highest bid.

➯ Telling the truth is a *dominant strategy*.

Choosing the auction type

It depends on the risk attitude of the bidders/auctioneer

- actually First price and Dutch auctions are strategically equivalent
- risk neutral bidders : expected revenue is equivalent for the four types of auctions we presented if valuations are independently drawn from a uniform distribution. (Revenue-equivalence Theorem (Vickrey 1961))
- risk averse bidders : Dutch and First-price sealed bid yield higher revenue
- risk averse auctioneer : Vickrey or English auctions are better

some variants :

- with entry cost
- with uncertain number of bidders

Lies and Collusion

- all four auctions are susceptible to collusion :
  groups of agent can form and decide on a low bid and share revenue later.
  ➩ Optimal when the grand coalition is formed
  ➩ Auctioneer can try to avoid that bidders identify other bidders
- auctioneer may cheat for Vickrey auction.
- auctioneer may place “fake” bids (known as shills)

Values

- Independent private values : each agent has a private valuation for the resource
- Common value : all agents have the same valuation, but they do not know it (they have different signals)
  For exemple, auction about land that may contain oil. Different experts have different beliefs. Once the auction ends :
  - Should the winner be happy that she paid less than her valuation of the resource ? ⇨ the winner realises she had the most optimistic signal, and then may reduce her belief → she may then believes she lost utility ⇨winner's curse

  or
  - Should the winner be worried that nobody else valued the resource so highly ?
- value with resale

Multi unit auctions

Multi unit auctions

Instead of selling a single resource, an action may sell n copies of that resource.

ex1 : 10 resources

- bidder A : 5 copies, 20 per copy or nothing
- bidder B : 3 copies or less for 15 per copy
- bidder C : 5 copies for 15 per copy or nothing
- bidder D : 1 copy for 15

How to allocate the 10 copies ?

ex2 : n resources, bidders want only one item.

Bidder	A	B	C	D
Bid	25	20	15	8

- what price for each item ? (different or same ?)
- how many should the auctioneer actually sells ? (maybe an auctioneer is better off by selling less resources)

Examples

for scenario 2 with n resources for sale :

- best n bidders paying the price of the first loser
- best n bidders paying the price of the last winner
- run a sequence of single-resource auctions

For scenario 2, there is also a revenue equivalence theorem.

For scenario 1 :
we can determine the winners by choosing the social-welfare-maximising allocation if we know the valuation function of each agent i (for each number of copies, the function returns a value) $v_i : \{1...n\} \to \mathbb{R}$

➩ it works by asking a lot of information to the bidders

➩ one needs to specify a *language* for describing the valuation function

Combinatorial auctions

the problem

An auctioneer wants to sell a set $R = \{r_1, \ldots, r_n\}$ of different resources. For example :

- set of paths (shipping rights, bandwidth in a network)
- electromagnetic spectrum (e.g. for cell phone signal)

Each bidder has a valuation function v_i of the form

$$v_i : 2^R \to \mathbb{R}$$

for each set of resources (also called bundle) $S \subseteq R$, $v_i(S)$ is the value of having the bundle S (no externality).

how should an auctioneer sell the goods ?

Complements and substitutes

- **Complements** : the value assigne to a set is greater than the sum of the values assigned to its elements.
  standard example for complements would be a pair of shoes (left and right one)
- **subsitutes** : the value assigned to a set is lower than the sum of the values assigned to its elements.
  standard example : tickets to a football match and to the opera the same night

Solutions

First solution : organize a single-unit auction for each resource in R.

- easy for the auctioneer
- difficult for the bidders if there are some complements or subsitute

  ex : I want to buy a sofa and a small table, or two recliners. If I get one seat and a sofa, that is not good as it will not be practical in my studio!

✘

Another solution : organize a single auction in which bidders can bid for sets of resources ⇨ **combinatorial auctions**

- one seller (auctioneer) and several potential buyers (bidders), many goods to be sold
- Bidding : bidders bid by submitting their valuation (not necessarily truthful)
- Clearing : auctioneers announces a number of winning bids (i.e. who obtains the bundle and for what price)

Winner determination Problem

The **winner determination problem** (WDP) is the problem of finding a set of winning bids

- that is feasible
- that will maximise the sum of the price offered

The sum of the prices can be given two interpretations :

- if the simple pricing rule is used where bidders pay what they offered, the sum is the revenue of the auctioneer
- if the prices offered are interpreted as individual utilities, then the sum is the utilitarian social welfare of the chosen allocation

Example

Each bidder submits a bid describing their valuation. Each bid (B_i, p_i) specifies which price p_i the bidder is ready to pay for obtaining the bundle $B_i \subseteq R$.

The auctioneer may accept at most one atomic bid per bidder (we'll discuss other bidding languages)

Agent 1	$(\{a,b\},5)$, $(\{b,c\},7)$, $(\{c,d\},6)$
Agent 2	$(\{a,d\},7)$, $(\{a,c,d\},8)$, $(\{c,d\},6)$
Agent 3	$(\{b\},5)$, $(\{a,b,c,d\},12)$, $(\{c,d\},6)$

What would be the optimal solution ?

The decision problem underlying the WDP is NP-complete

Theorem

Let $K \in \mathbb{Z}$. The problem of checking whether there exists a solution to a given combinatorial auction instance generating a revenue exceeding K is NP-complete.

Proof

This problem is equivalent to WELFARE OPTIMISATION

- checking NP-membership is easy
- NP-hardness follows from SET PACKING

□

Solving WDP

In practice, it is often possible to solve enven large WDP.
Two main approaches :

- integer programming formulation and resolution with CPLEX
- AI search algorithms with heuristics (depth-first branch and bound, $A^\star$)

Bidding Languages

Let $\mathcal{R}$ be the set of goods. The valuation function of an agent is a function $u : 2^{\mathcal{R}} \to \mathbb{R}$

- when defining a language, there are two main questions :
  - how expressive is ? can I represent all valuation functions or only a family ?
  - how succinct is it ? do I need long expression to represent a valuation function

We assume that the valuations are normalised and monotonic :

- v normalised iff $v(\emptyset) = 0$
- v is monotonic iff $v(X) \leqslant v(Y)$ whenever $X \subseteq Y$.

Atomic Bids

An **atomic bid** is a pair (B, p) where $B \subseteq \mathcal{R}$ is a bundle of goods and and $p \in \mathbb{R}^+$ is price. Intuitively, it means the agent is ready to pay p for obtaining the set B.
The atomic bid (B, p) defines the valuation function

$$v(X) = \begin{cases} p \text{ if } B \subseteq X \\ 0 \text{ otherwise} \end{cases}$$

ex : my atomic bid is $(\{a,b\},4)$ and you offere the bundle $\{a,b,c\}$, then your offer satisfies my bid. Had you proposed $\{b,d\}$, it would not have satisfied my bid

Atomic bids alone cannot express very interesting valuation functions.

➩ how can we combine atomic bids ?

The XOR language

we specify a number of atomic bids, but we will pay for **at most one** of these to be satisfied. If more than one bid are satisfied, we pay for the largest atomic bid satisfied.

ex : $(\{a,b\},3)$ XOR $(\{c,d\},5)$: I would pay 3 for a bundle containing a and b but not c and d, I would pay 5 for a bundle containing c and d but not a and b, and I would pay 5 for a bundle containing a,b,c and d
➾ it is fully expressive
➾ but it is not so compact as one may need to use a number of atomic bid that is exponential in the number of goods.

The OR language

We do as in the XOR but this time, we are willing to pay for more than one bundle.
suppose I make the bid $\{(Z_1, p_1) \text{OR} \ldots \text{OR} (Z_k, p_k)\}$
Suppose I get goods in the set Z'. To determine my valuations, I need to find the set W of atomic bids such that

- every bid in W is satisfied by Z'
- each pair of bids in W has mutually disjoint sets of goods
- there is not other set of bids satisfying the above two properties and which sum of prices is higher than the sum of the prices in W

➯ it is **not** fully expressive
➯ but it is more compact than XOR

Other bidding languages

- based on extension of OR languages
- based on combinations of OR and XOR
- based on logics

Quick word on distributed resource allocation

Contract-net protocol approaches

We take inspiration from Contract-Net protocols :

- negotiation starts with an initial allocation
- agents asynchronously negotiate resources
- deals to move from one allocation to another, ie $\delta = (A, A')$
- deals can involve payments (utility transfer) ;
- we may have a neighbourhood relation between agents defining a negotiation topology (here fully connected unless stated otherwise)
- agents accept deals on the basis of a rationality criterion, we assume myopic IR : $v_i(A') - v_i(A) > p(i)$

- Types of goals : Efficient/fair allocation
  - Pareto optimal allocation, utilitarian social welfare
  - egalitarian, envy-freeness
- Types of deals
  - exchange of a single resource : 1-deal
  - exchange between two agents : bilateral deal
  - otherwise : complex deal
- domain restrictions
  - monotonicity
  - modularity

Earlier results (mainly due to Sandholm) :

- a deal is IR (with money) iff it increases utilitarian social welfare (thus generates a surplus).
- allows to show that any sequence of IR deals converges to an allocation maximizing utilitarian social welfare
- however, may require very complex deals to be implemented during the negotiation (in fact, for any conceivable deal we may construct a scenario requiring exactly that deal).

Some references

- **general** :
  - *An Introduction to MultiAgent Systems* by M. Wooldridge (John Wiley & Sons, 2009) (11.1 utilities and preferences, 11.3 Pareto efficiency, social welfare, 12.1 social welfare functions, 14 Allocating scarce resources)
  - *Multiagent Systems Algorithmic, Game-Theoretic, and Logical Foundations* by Y. Shoham and K. Leyton-Brown (Cambridge University Press, 2009) (3.1.2 preferences and utility, 3.2.1 Pareto optimality, 9.4 Existence of social functions, 11 Protocols for multiagent resource allocation)
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