Exact Algorithms Via Monotone Local Search

$c^k \Rightarrow (2 - \frac{1}{c})^n$

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Should someone mainly interested in exact algorithms care about parameterized complexity?

- Well yes, parameterized algorithms *are* exact algorithms...
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- More interestingly, when $c < 4$,

$$\max_{0 \leqslant \alpha \leqslant 1} \{min\{c^\alpha, 2^{H(\alpha)}\}\} < 2$$

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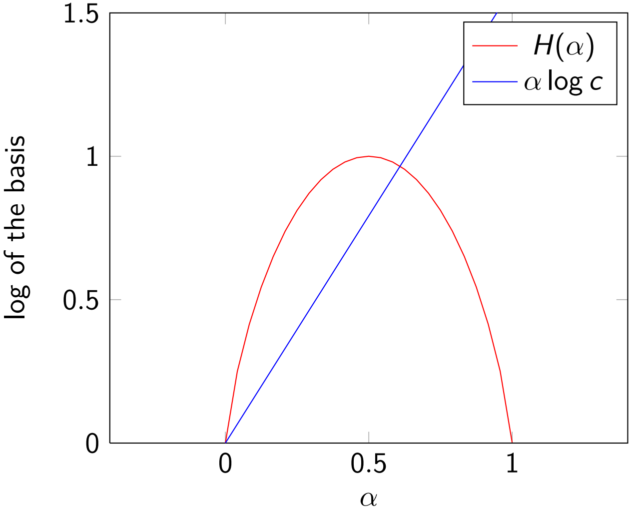
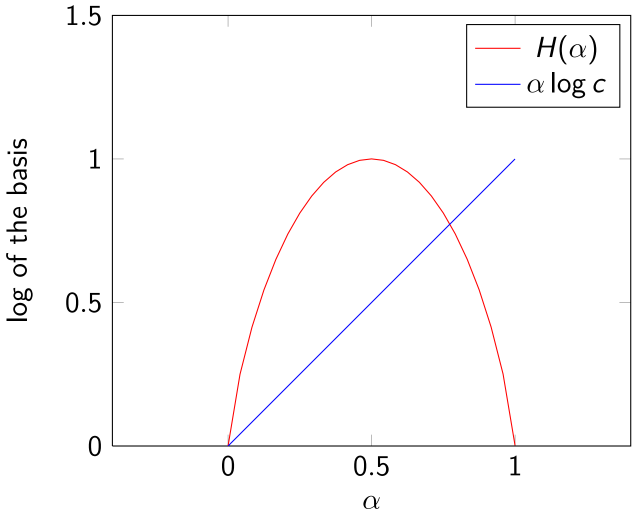
$$\max_{0 \leqslant k \leqslant n} \{min\{c^k, \binom{n}{k}\}\} = \max_{0 \leqslant \alpha \leqslant 1} \{min\{2^{\alpha \log c}, 2^{H(\alpha)}\}\}^n$$

where $H(x) = -x \log x - (1-x) \log(1-x)$

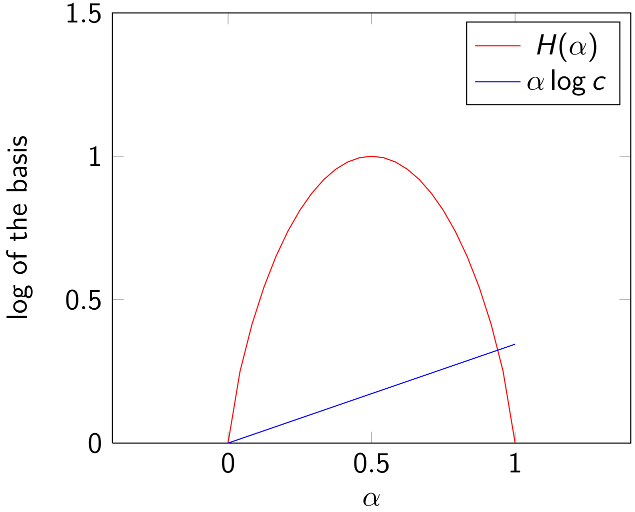
since $\binom{n}{\alpha n} \leqslant 2^{H(\alpha)n}$



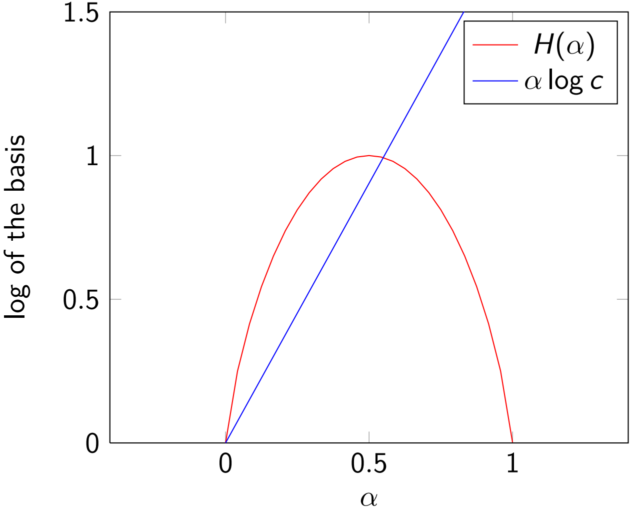
3^k implies 1.953^n



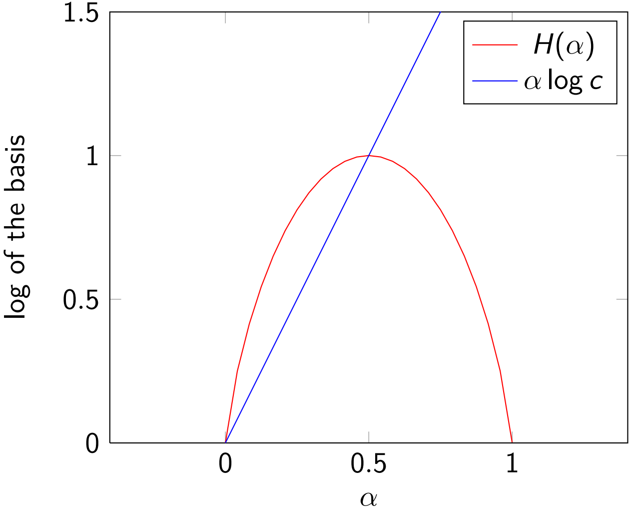
2^k implies 1.709^n



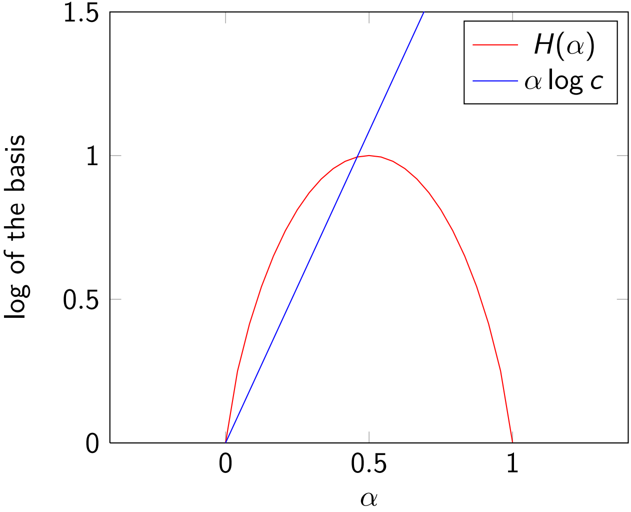
1.27^k implies 1.253^n



3.5^k implies 1.991^n



4^k implies 2^n



4.5^k implies nothing

A very simple algorithm inspired by local search

Let Π be a *subset* problem.

- Guess the size of the optimal solution k.
- Select $t \leqslant k$ elements uniformly at random.
- Complete the solution with $k - t$ elements in FPT time c^{k-t}.

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Slight caveat: FPT algorithm for the *extension version* of Π.

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$\max\limits_{0 \leqslant k \leqslant n} \min\limits_{0 \leqslant t \leqslant k} \frac{\binom{n}{t}}{\binom{k}{t}} c^{k-t} \leqslant (2 - \frac{1}{c})^n$ (whiteboard)

Breakthrough!

Problem Name	Parameterized		New bound	Previous Bound	
Feedback Vertex Set	3^k (r)	[13]	1.6667^n (r)		
Feedback Vertex Set	3.592^k	[28]	1.7217^n	1.7347^n	[21]
Subset Feedback Vertex Set	4^k	[42]	1.7500^n	1.8638^n	[19]
Feedback Vertex Set in Tournaments	1.6181^k	[30]	1.3820^n	1.4656^n	[30]
Group Feedback Vertex Set	4^k	[42]	1.7500^n	NPR	
Node Unique Label Cover	$\|\Sigma\|^{2k}$	[42]	$(2 - \frac{1}{\|\Sigma\|^2})^n$	NPR	
Vertex (r, ℓ)-Partization $(r, \ell \leq 2)$	3.3146^k	[3, 29]	1.6984^n	NPR	
Interval Vertex Deletion	8^k	[8]	1.8750^n	$(2 - \varepsilon)^n$ for $\varepsilon < 10^{-20}$	[4]
Proper Interval Vertex Deletion	6^k	[40, 7]	1.8334^n	$(2 - \varepsilon)^n$ for $\varepsilon < 10^{-20}$	[4]
Block Graph Vertex Deletion	4^k	[1]	1.7500^n	$(2 - \varepsilon)^n$ for $\varepsilon < 10^{-20}$	[4]
Cluster Vertex Deletion	1.9102^k	[5]	1.4765^n	1.6181^n	[17]
Thread Graph Vertex Deletion	8^k	[27]	1.8750^n	NPR	
Multicut on Trees	1.5538^k	[26]	1.3565^n	NPR	
3-Hitting Set	2.0755^k	[41]	1.5182^n	1.6278^n	[41]
4-Hitting Set	3.0755^k	[17]	1.6750^n	1.8704^n	[17]
d-Hitting Set $(d \geq 3)$	$(d - 0.9245)^k$	[17]	$(2 - \frac{1}{(d-0.9245)})^n$		[11, 17]
Min-Ones 3-SAT	2.562^k	[31]	1.6097^n	NPR	
Min-Ones d-SAT $(d \geq 4)$	d^k		$(2 - \frac{1}{d})^n$	NPR	
Weighted d-SAT $(d \geq 3)$	d^k		$(2 - \frac{1}{d})^n$	NPR	
Weighted Feedback Vertex Set	3.6181^k	[1]	1.7237^n	1.8638^n	[18]
Weighted 3-Hitting Set	2.168^k	[39]	1.5388^n	1.6755^n	[11]
Weighted d-Hitting Set $(d \geq 4)$	$(d - 0.832)^k$	[17, 39]	$(2 - \frac{1}{d-0.932})^n$		[11]

What we did not talk about

- derandomization with so-called *set-inclusion-families*.
- extension to permissive FPT algorithms.
- can be used for enumeration.