Parameterized Complexity of Cardinality-Constraint Problems in Bipartite Graphs

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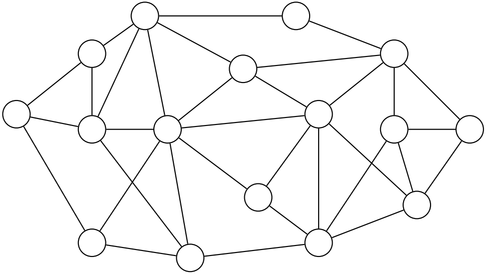
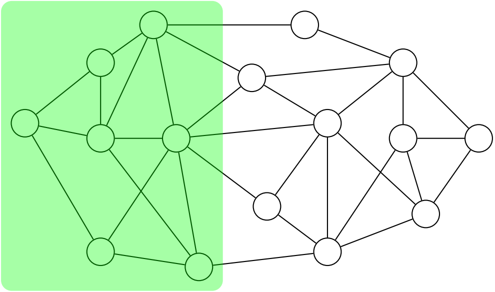
June 6, 2014



$G, k = 7(, p)$



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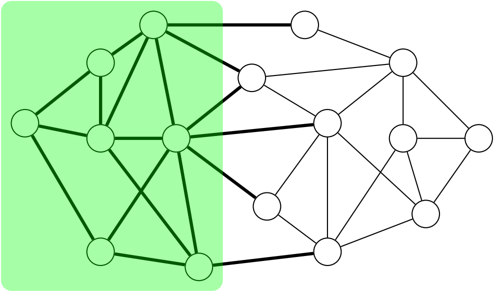
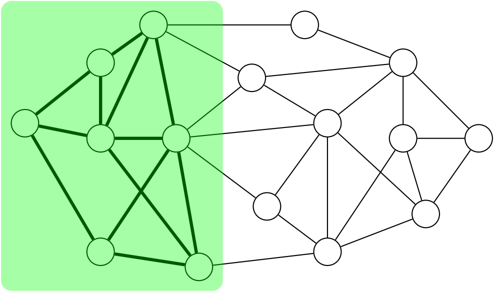
Cardinality-Constraint Graph Problems

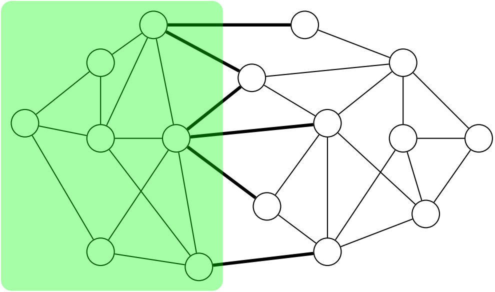


Max/Min k-Vertex Cover

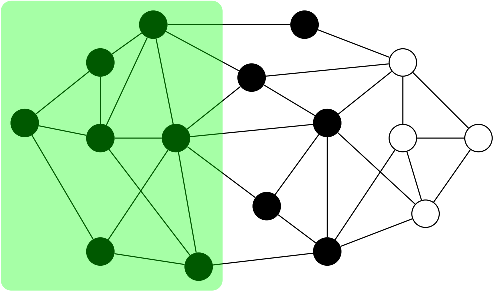
Cardinality-Constraint Graph Problems



k-Densest/k-Sparsest



Max/Min $(k, n - k)$-Cut



Max/Min k-Dominating Set

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Theorem (Joret and Vetta '12, Caskurlu and Subramani '13, Apollonio and Simeone '14)

Max k-Vertex Cover (hence, k-Sparsest) is NP-complete in bipartite graphs.

Parameterized Complexity

Theorem (Cai '08)

Max/Min k-Vertex Cover, k-Densest, k-Sparsest, Max/Min $(k, n-k)$-Cut are W[1]-complete.

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Max/Min k-Vertex Cover, k-Densest, k-Sparsest, Max/Min $(k, n-k)$-Cut are W[1]-complete.

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Max k-Densest is W[1]-complete in bipartite graphs.

Theorem (Raman and Saurabh '08)

Max k-Dominating Set is W[2]-complete in bipartite graphs.

Parameterized Complexity (2)

Theorem (Bläser '03)

Max k-Vertex Cover is FPT parameterized by p.

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Max k-Vertex Cover is FPT parameterized by p.

Theorem (Cygan, Lokshtanov, Pilipczuk, Pilipczuk and Saurabh '14)

Min $(k, n-k)$-cut is FPT parameterized by p.

Parameterized Complexity (3)

Theorem (Cai, Chan and Chan '06)

LGPPs are solvable in time $O^*(2^{(\Delta+1)k}((\Delta+1)k)^{O(\log((\Delta+1)k))})$.

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Theorem (Shachnai and Zehavi '14)

Non degrading LGPPs are solvable in time $O^*(4^{k+o(k)}\Delta^k)$.

We will show the following result:

Theorem

Generalized Max k-Vertex Cover is FPT in bipartite graphs.

where the k vertices should be picked in $V' \subseteq V$.

We order the vertices by decreasing degrees: $v_1, v_2, \ldots v_n$.

Lemma

If $d(v) \leqslant d(v_k) - k$, v is not in an optimal solution.

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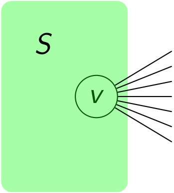

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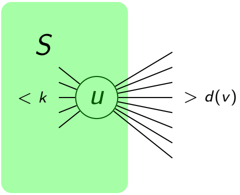

$u \in \{v_1, \ldots, v_k\} \setminus S$

Lemma

If $d(v_k) \leqslant d(v_1) - k$, any optimal solution intersects $\{v_1, \ldots, v_{k-1}\}$.

If $S \cap \{v_1, \ldots, v_{k-1}\} = \emptyset$, we replace any vertex in S by v_1.

Lemma

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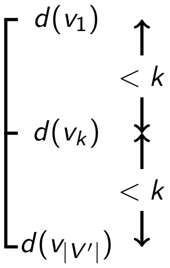
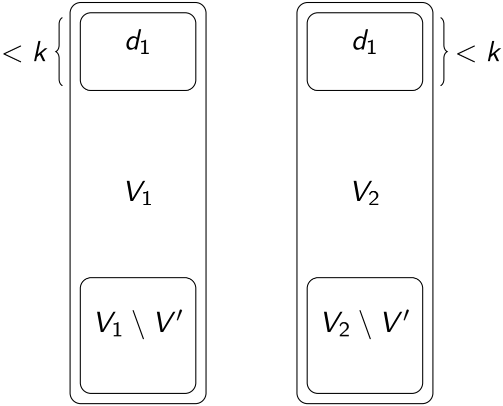
If $S \cap \{v_1, \ldots, v_{k-1}\} = \emptyset$, we replace any vertex in S by v_1.

Necessary sets or *intersectivity* $\rightsquigarrow$ bounded branching tree $O(k^k)$.

Conclusion

Vertices in V' have degree in $[d(v_1) - 2k, d(v_1)]$.




$< k$
d_1
V_1
$V_1 \setminus V'$
d_1
$< k$
V_2
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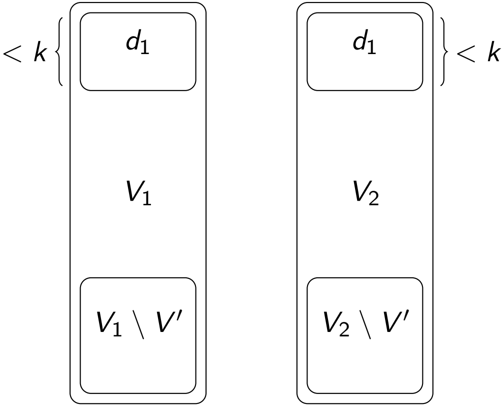
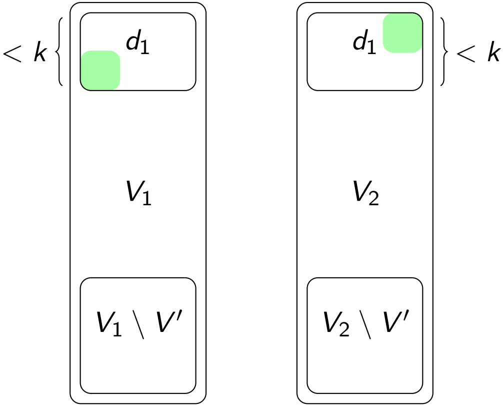
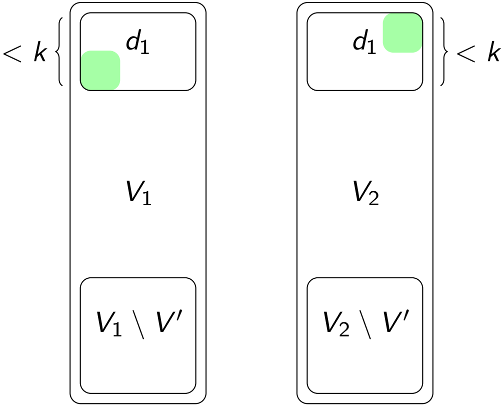
Otherwise, we take k vertices in one part and gets kd_1 edges.



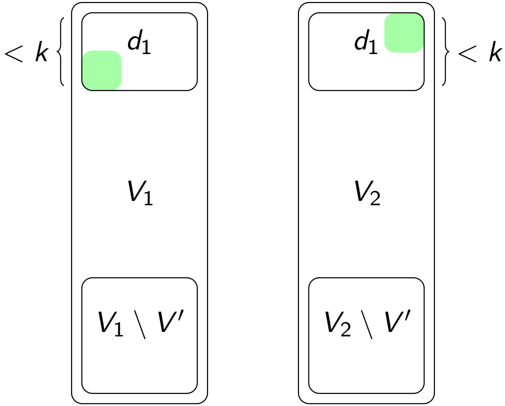
We guess the part of an optimal solution in 2^{2k}.



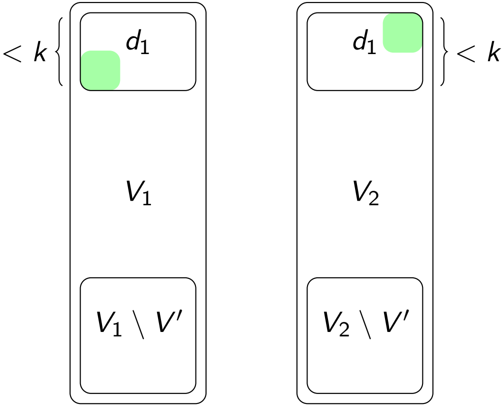
This is done at most $2k$ times.



We find an optimal solution in $O^*(2^{(2k)^2})$.



We find an optimal solution in $O^*((2k)^{3k})$.



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Max $(k, n-k)$-Cut is FPT in bipartite graphs.

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Conclusion

Vertices in V' have degree in $[d(v_1) - 4k, d(v_1)]$.

We find an optimal solution in $O^*((4k)^{5k})$.

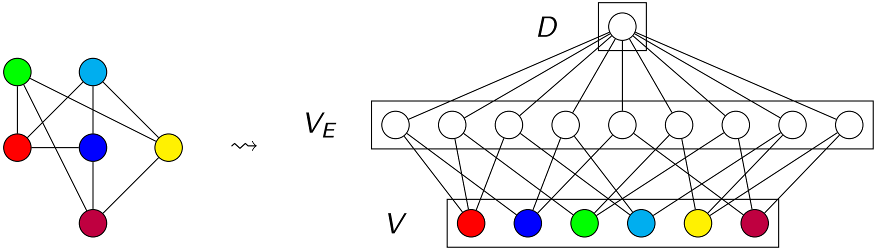
Theorem

Min k-Vertex Cover is W[1]-complete in bipartite graphs.

Theorem

Min $(k, n-k)$-Cut is W[1]-complete in bipartite graphs.

We reduce from k-Clique in regular graphs.



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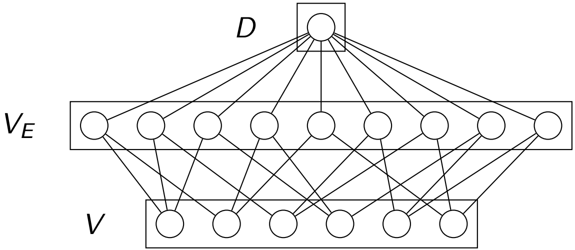
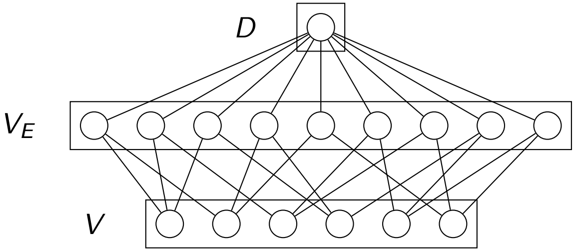
$(k + \binom{k}{2})$-Vertex Cover touching less than $\Delta k - 2\binom{k}{2}$ edges?

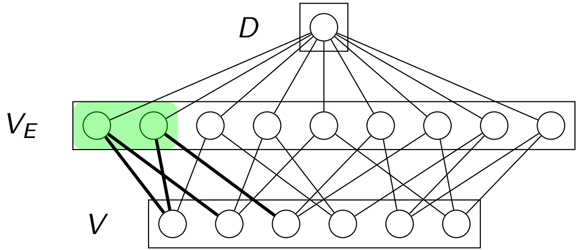


$(k + \binom{k}{2})$-Densest with $2\binom{k}{2}$ edges?



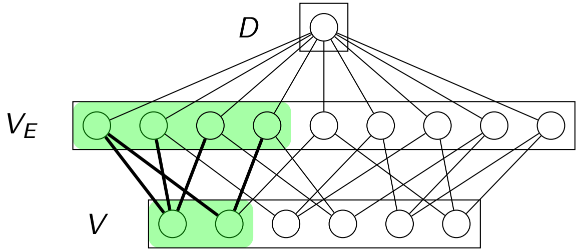
$$(k + \binom{k}{2})\text{-Densest with } 2\binom{k}{2} \text{ edges?}$$

$$\text{If } |S \cap V_E| < \binom{k}{2}, \text{ then } |E(S)| < 2\binom{k}{2}.$$



$(k + \binom{k}{2})$-Densest with $2\binom{k}{2}$ edges?

If $|S \cap V| < k$, then $|E(S)| < 2\binom{k-r}{2} + k - r \leqslant 2\binom{k}{2}$.



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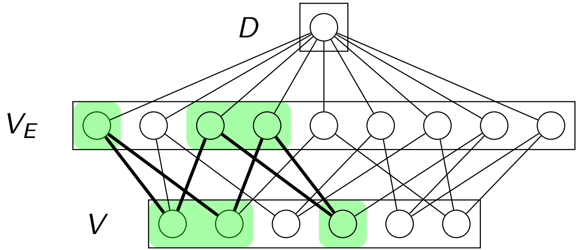

Perspectives

- Polynomial kernel for Max k-Vertex Cover?
- Can we rule out an algorithm in $O^*(c^k)$ with c constant?
- Parameterized complexity of Max k-Vertex Cover in chordals?

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- Parameterized complexity of Max k-Vertex Cover in chordals?
- Complexity of k-Densest in interval graphs?
- Complexity of k-Densest in planar graphs?