Modelling inaccurate determination, uncertainty, imprecision using multiple criteria

Denis Bouyssou

Abstract

The purpose of this paper is to study how the consideration of several criteria, as opposed to a more traditional mono-criterion approach, helps the modelling of imprecision, uncertainty and inaccurate determination (I.U.I.D.) in a decision-aid study. After a brief review of the main sources of inaccurate determination, uncertainty and imprecision that arises in a decision-aid situation, we show that the use of multiple criteria allows to build partial preference structures, to discuss in a powerful way the precision of the evaluation on each criteria and to create a clear language between the actors via the use of the evaluation tableau. We argue that this proves useful in dealing with I.U.I.D.

1- Introduction.

The use of multiple criteria in decision-aid models is often justified (see *e.g.* Zeleny (1982) or Sch rlig (1985)) by the fact that the world is governed by multiple objectives and that any decision implies to balance "pros" and "cons". This widely-shared point of view can however be criticized (see Bouyssou (1987) and Roy (1988b)). Using a mono-criterion approach to decision-aid does not imply that one considers that "reality" is governed by a single criterion. It is well-known that, in this kind of models, multiple objectives are often taken into account *e.g.* via the use of constraints, sensivity analysis and "prices" allowing to convert heterogeneous consequences into a single unit. As emphasized by Roy (1988b), the use of multiple criteria does not simply appear as a generalization of traditionnal approaches but constitutes a new paradigm for analysing and helping decisions.

In this paper we wish to outline what we consider as an important justification for entering this new paradigm : the management of imprecision, uncertainty and inaccurate determination (I.U.I.D) that is part of most decision situations. Our analysis follows that of Roy (1988a). He distinguished four main sources of I.U.I.D. that the analyst has to deal with. We briefly present them in section 2. In section 3, we recall the main originalities of models explicitely using several criteria. In section 4 we try to show how the consideration of several criteria helps the modelling of I.U.I.D. and leads to models that are significantly different from those deriving from the consideration of a unique criterion in this respect.

2- The four main sources of uncertainty, imprecision and inaccurate determination in decision models (Roy (1988a)).

a) The "map" is not the "territory".

Locating a plant, chosing an equipment, investing in new activities are crucial decisions for a firm. The purpose of decision-aid is to compare such complex alternatives. If one wants

to use a formal model of decision-aid, the complexity of these alternatives and their consequences makes it often impossible to compare them directly. This comparison is made possible through the use of "maps" of these complex "territories". For an alternative, a map consists of a model of the consequences of its implementation (in order to describe an alternative it is possible to use several maps of different "scale" using *e.g.* a hierarchical model). These maps create a tractable language that allows an effective communication between the various actors of the decision process and provides an adequate basis for the comparison of the alternatives. However the establishment of the maps inevitably involves many simplifications, omissions and distorsions which introduce in the model an important source of arbitrariness. Indeed, there are often several and equally valid ways of building these maps. While forced to use maps in order to compare territories, the analyst has to make a tradeoff between the richness and the readability of the maps : the "richer" is one map the closer it is to the territory, but the more difficult it may be to compare it to other maps.

b) The "future" is not a "present" to come.

The alternatives that are to be compared will only be implemented in a more or less distant future. Thus, at the time of the study, the consequences of the implementation of an alternative are very often unpredictable for they depend on environmental factors and/or the strategy of other actors that are still unknown and may well be influenced by the implementation of that alternative. This is the most classical source of I.U.I.D. that is mentionned in every textbook on decision models. Many efforts have been devoted to cope with this unpredictability using, *e.g.*, probability distributions, plausibility measures, scenarios, etc.

As Roy (1988a) mentionned, the unpredictability of the consequences of implementing an alternative also stems from the fact that the alternatives are not completely specified at the time of the study. When a firm tries to compare several sites for locating a new plant, the precise characteristics of each site may not have been completely investigated yet. Furthermore, the precise draft of the plant to be built may not be available and may well depend on the site chosen. Thus, even if one could predict with a very high precision the consequences of an alternative, an element of inaccurate determination would remain since the alternatives are still "projects".

c) The data are not the result of exact measurement.

The establishment of a map usually involves the consideration of two types of data. Data of type I are closely linked to the territory that the analyst wishes to describe. The modelling of uncertainty mentionned in the above paragraph will apply to this first type of data. For instance, suppose that an analyst has to evaluate the human consequences of building a polluting plant on a given site. He will have to cope with uncertainty since he will be forced to envisage various scenarios for the growth of the population in that area. He will also have to deal with

imprecision since the present number of people living close to the projected plant is far from being perfectly known : counting houses on a map or on-site studies do not lead to precise evaluations. Thus, it is important to realize that many figures used in decision-aid models are only "order of magnitudes". This imprecision is often seen as stemming from the measurement techniques that are used. It also comes from the fact that, in many situations, the very definition of what "should" be measured is very imprecise. Using the same example as above, it is not clear how the analyst should take into account schools, hospitals, second homes, etc. This inaccurate determination of what is to be measured is certainly at least as important as the imprecision inherent to any kind of measurement.

Data of type II concern the way the first type of data is used in the construction of the map. Parameters like discounting rates or utility functions designed to capture an attitude toward risk are examples of this second type of data. They are more linked to a particular value system than to an alternative. In our siting example such data could consist of the weight assigned to each inhabitant that is function of the distance between his residence and the projected plant, the attitude toward risk of the firm concerning the amount of nuisance created for the riparians, etc. Though techniques have been created to assess these data, it is important to keep in mind that they are very often "created" as well as "measured" (see *e.g.* the work of McCord and de Neufville (1983) concerning utility functions).

d) The model is not the description of a real entity independent of the model.

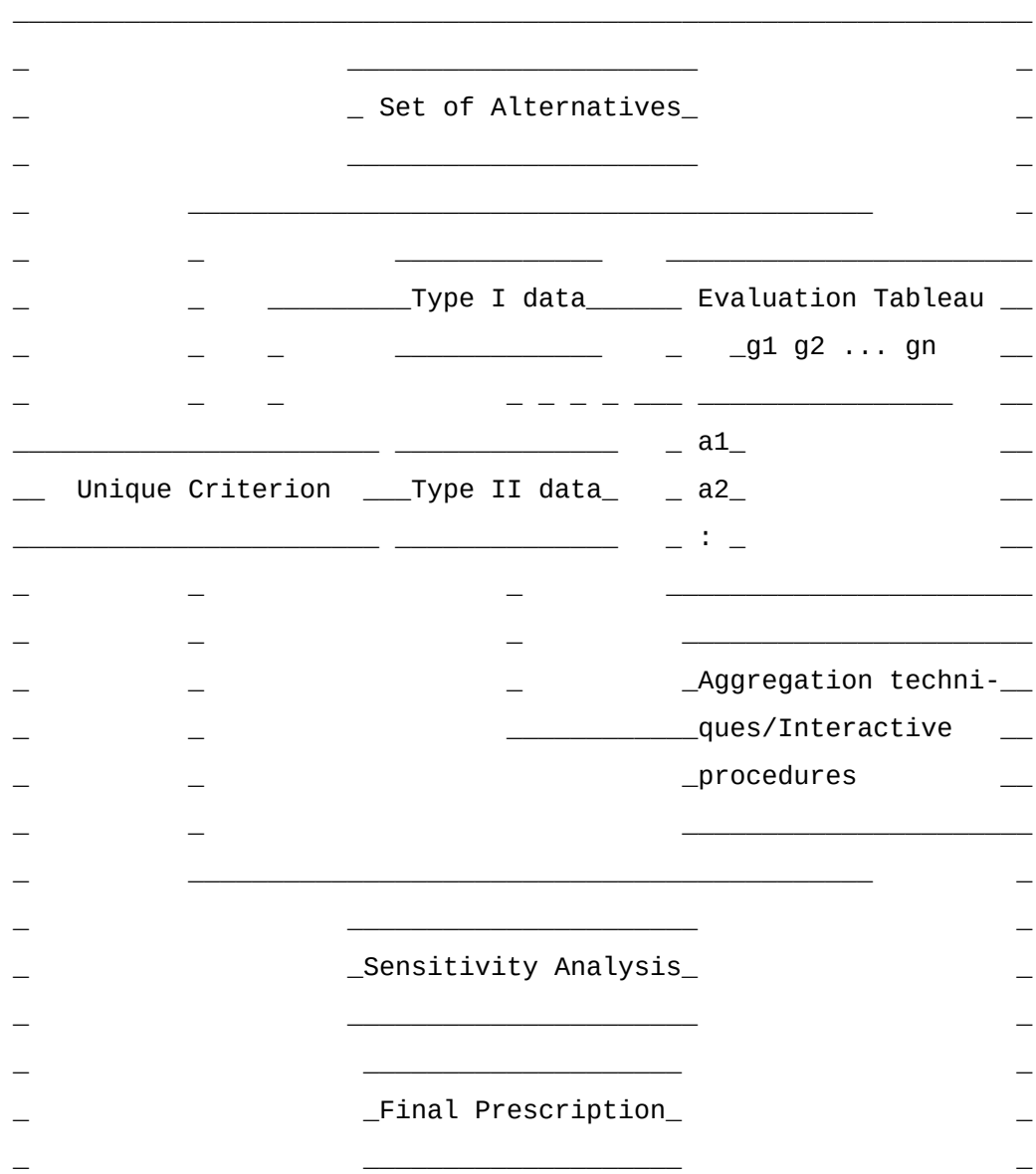
Data of the second type are connected with certain aspects of the preference system(s) of the actor(s) involved in the decision process. It is well known that the questionning process used by the analyst in order to obtain these data may significantly influence the answers (see Bouyssou (1984)). This is all the more true since the preference system of an actor may not be completely structured at the time of the study : areas of firm conviction may well coexist with areas of hesitation and ambiguity in which the influence of the model on what is to be "captured" is overwhelming. Furthermore, the various actors may well disagree and, as a result of a discussion, some actors may change their mind on some point thus creating some "inconsistencies" with previously stated judgements. In such cases the management of these hesitations, contradictions and conflicts seems a prerequisite to any convincing decision-aid model. This is linked to what Roy and Bouyssou (1986) called a *constructive* attitude towards decision-aid, as opposed to a *descriptive* one, in which the role of the analyst is not to describe as accurately as possible supposedly pre-existing preferences but to provide information and tools that are useful for justifying, building and arguing preferences.

3 - The multiple criteria approach to decision-aid.

From the point of view of the management of I.U.I.D., the main feature of an approach using multiple criteria is to break down the modelling process into two different phases : the construction of the criteria (which gives rise to the evaluation tableau) and the aggregation of

these criteria (see Fig. 1). As advocated by Roy (1985), the analyst should use the smallest possible amount of type II data in the construction of the various criteria. We noted in section 2, that some data of type II such as utility functions have to be taken into account in order to build the criteria. However, sensitive information such as the tradeoffs between the various criteria are only introduced in the aggregation phase, contrary to what is usually done in a mono-criterion approach in which the construction of the unique criterion involves at the same time data of both types.

This approach is based on what could be called an "*act of faith*", *i.e.*, the belief that the explicit construction of several criteria will have a "positive role" in the modelling process. It rests on an underlying *assumption* stating that in most decision-aid studies it is possible to identify a small number of "points of view" (usually between three and no more than ten, at least at the upper level if a hierarchical model is used) around which it is possible to build a familly of criteria that is exhaustive and simple enough to be accepted as a basis of discussion by all the actors of the decision process.



_ Figure 1 : Mono-criterion and multiple criteria approaches _
_ to decision-aid. _

4- The management of imprecision, uncertainty and inaccurate determination.

We will center our discussion in this section around what we consider to be the three main originalities of the multiple criteria approach in the management of I.U.I.D.

a) The establishment of partial preference structures.

In a multiple criteria approach, a criterion is used to "sum up" evaluations on consequences related to a same point of view (see Roy (1985)), *e.g.* cost, safety, environment, etc. Formally a criterion can be defined as a function associating a real number $g_i(a)$ to each alternative "a", such that every actor in the decision process admits that if $g_i(a) \geq g_i(b)$ then alternative "a" is at least as good as alternative "b" on the point of view that underlies the definition of criterion g_i.

The interest of such partial preference relations for the management of I.U.I.D. is tied to the fourth source we mentionned in section 2. These partial preference relations can be seen as the stable part of the preference structure of the actors and thus as a possible basis for discussion between the actors. The criteria are designed in order to aggregate consequences that are very "close" to one another for they relate to the same point of view and are as free as possible from potentially highly conflictual type II data such as tradeoffs or weights. In locating a plant it seems reasonable to admit that environmentalists could agree with technicians on the definition of an index of safety of an installation whereas, in most countries, they will definitely disagree on the possible tradeoffs between safety and costs.

As opposed to mono-criterion models which use "sensitive" information from the beginning, the multiple criteria approach aims at modelling what appears to be sufficienly stable in the perception of the actors in order to obtain a first basis for consensus. This may prove useful in conflictual decision or when the perception of the problem by the various actors is still very poor. Partial preference relations are often useful tools for the management of hesitation and conflicts. In this respect the "analytical" side of models using several criteria may be seen more as a volontarist attitude in order to obtain decomposable and, thus, stable models than as a philosophical commitment.

b) The discussion of the "precision" of the evaluations on the criteria.

In a mono-criterion approach it is not unusual to consider that if g(a), the evaluation of alternative "a" on the criterion, is greater than g(b) then "a" is considered as being strictly preferred to "b", even if the difference between the evaluations is very small. Given the first three sources of I.U.I.D. we mentionned, this mode of comparison may lead to

"unconvincing" preference situations. Small differences in somewhat arbitrary maps do not imply that the territories really differ. Furthermore, the way the analyst has dealt with the imprecision and/or uncertainty affecting most data in order to obtain the evaluations on the unique criterion is certainly not the only sensible one. Other reasonable ways of doing could have lead to different evaluations and, possibly, to a reversal of the comparison of "a" and "b". In this type of models, the only way to test the significance of a conclusion is to perform a thorough sensitivity analysis. A sensitivity analysis should ideally combine all plausible values for the parameters. However, given the number of these parameters and the complexity of the calculations leading to the map g(a), such a thorough sensitivity analysis can rarely be performed. It very often appears as a "one-dimensional" sensivity analysis testing the robustness of the conclusions by varying only one parameter at a time.

These difficulties are still present in a multiple criteria approach. Yet, in this framework, the analyst may try to deal with these difficulties separately on each criterion. Since the model leading to the definition of each criterion is usually far less complex and uses much fewer parameters than the one that would lead to unique criterion this may be seen as an advantage. Let us note however that the analyst has often to introduce mostly "volontarist" hypotheses (such as probabilistic independence of some probability distribution for instance) in order to be able to cope with uncertainty and imprecision separately on each criterion.

Dealing with uncertainty and imprecision separately on each criterion, may prove useful from the point of view of sensitivity analyses. Furthermore, as argued by Roy (1985), the analyst may also try to define on each criterion a preference structure taking into account the fact that small differences may not be significant. This can be done using thresholds, a difference between two evaluations being significant only if it is "sufficiently" large. The evaluation of these thresholds is not an easy task. As long as the model leading to definition of the criterion is not too complex, simple reasonings may be used to give them a (not unreasonable) numerical value (see Bouyssou and Roy (1987)). Because of the complexity of models leading to a unique criterion such reasonings that rely on intuitive considerations and the evaluation of experts can rarely be used in this approach so that the "precision" of the evaluations on that criterion can only be questionned through a sensitivity analysis. However, in order for the criteria to be accepted by all actors, it is sometimes essential to discuss the "precision" of the evaluations of the alternatives before the last stage of the study : a criterion showing non-convincing preference situations is likely to be rejected by some actors (even if they are told that a sensitivity analysis will be performed).

c) The use of an evaluation tableau and the aggregation phase.

In many decision-aid contexts (see Roy (1985)), the evaluation tableau has an interest in itself, independently of the application of any formal technique leading to the establishment of a prescription. In the discrete case this evaluation tableau is the classical alternatives *vs* criteria matrix. In the continuous case, *i.e.*, when the set of alternatives is either infinite or very large,

subsets of the evaluation tableau, like the so-called payoff matrix, are used by most procedures. The use of an evaluation tableau may prove useful in dealing with the sources of U.I.U.D. we mentionned.

This evaluation tableau first constitutes a language that is common to all the actors of the decision process. This map shared by all actors is undoubtedly richer than the one created by a unique criterion and, thus, closer to the territories that are to be compared. However, in spite of this richness, we are not aware of real-world situations in which this map has been rejected because of its complexity. In fact it seems than in most real-world studies, it is possible to reach a consensus concerning the various points of view to be taken into account.

The map created by the evaluation tableau is operational in many respects. The establishment of a prescription implies the assessment of many data of type II. This assessment is usually based on "imaginary" alternatives (see Keeney and Raiffa (1976)). Using a mono-criterion approach one is bound in order to speak of such alternatives either to describe each of their consequences and characteristics, which is often very difficult, or to give their evaluations on the unique criterion in which case the alternatives have very little intuitive appeal. For instance in large linear programs, an alternative can either be described using a vector of hundreds of decision variables or by giving the value of the objective function. The use of multiple criteria often allows to reach a convenient compromise for speaking of imaginary alternatives. This gives the analyst a sound framework for assessing data of type II and discussing their "precision".

The evaluation tableau allows the actors to implement simple reasonnings such as dominance, the use of aspiration levels or a simple lexicographic method in order to justify and elaborate preferences. In face of an evaluation tableau, most actors will recognize that a decision will inevitably be the result of a compromise between several conflictual objectives. It is then difficult for the analyst working into this framework to convince people to accept his recommandations just because of the sophistication of the methods he uses. This may allow the analyst to avoid some difficulties that are frequently encountered when using a unique criterion (see Roy (1981) and also Ackoff (1979)) and, thus, gives him tools for the management of hesitation and conflicts. In this approach, the analyst often proposes an "optimal" solution. Confronted to that optimal solution, an actor is likely either to accept it without restriction because of its "scientificity" or to reject it because the many simplifications, ommissions and distorsions contained in the model are incompatible with his value system (on these aspects, see GRETU (1980)). The use of multiple criteria allows and, sometimes, forces the analyst to regard its model as support to reflexion, negotiation and creativity tolerating hesitations, ambiguities and iterations.

If the analysis goes further than the construction of an evaluation tableau then it is clear that highly sensitive data of type II will be needed either through the assessment of weights and/or tradeoffs or through a dialogue driven by an interactive procedure. By separating as much as possible this information from the rest of the data, an approach using multiple criteria

allows to clearly locate conflicts between actors and perform thorough sensitivity analysis. This is true even if the aggregation method aims at building a unique criterion on the basis of the familly of criteria contained in the evaluation tableau (which is done for instance in Multiattribute Utility Theory, see Keeney and Raiffa (1976)). This particular form of aggregation should not be confounded with an approach directly aiming at building a unique criterion. In the latter case, data linked to the description of the alternatives are often inextricably mixed with data linked to a particular preference system.

Using multiple criteria can be seen as a "diplomacy of small steps", trying to model what can be modelled in spite of the presence of hesitations, conflicts and ambiguities. Many other intermediate steps could be envisaged apart from the evaluation tableau. It seems however that the use of multiple criteria gives to the analyst clear and sound tools to deal with I.U.I.D. Central to the the management of I.U.I.D. into this framework is what could be called a "volontarist analytical" approach which is only very partially covered by the classical "divide and conquer" approach. Division appears here less as a cartesian device than as a pragmatic way of building a convincing prescription for decision-aid.

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