Decision-aid : an elementary introduction with emphasis on multiple criteria

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Abstract

The purpose of this paper is to present an elementary introduction to the concepts and methods used in decision-aid. After having defined the notions of action, consequence and criterion we show the interest of multicriteria analysis and outline the main methods in that field: aggregation into a unique criterion, aggregation into an outranking relation, interactive approach. We conclude by some remarks on the scope of a scientific approach towards aiding decision.

1. Introduction

Since ancient times man has sought support in abstractions and hypothetico-deductive reasoning to guide and legitimate his deeds. As early as the Pythagorians, abstract knowledge, the mystery of numbers and the harmony of spheres were seen to confer power over matter. In the XVIIth century, one of society's best-known gamblers, the Chevalier de Méré, had enumerated 21 different possible combinations for throwing dice. He thought he could safely bet on getting a double-six if he threw the dice 21 times. Yet, he lost this bet more often than he won. He subsequently asked his friend Blaise Pascal to explain this to him. Thus began the latter's famous work on probability. In the XIXth century, Auguste Comte dreamed of a government guided by science.

With the end of World War II a growing number of research organizations devoted to the analysis and preparation of decisions began to appear. Shortly thereafter institutions of many kinds built up decision-aid units which brought together mathematicians, statisticians, computer scientists, economists and specialists in Operational Research. Out of this activity grew a number of journals and other publications, often highly technical in nature.

The objective of this paper is to present an elementary introduction to the field of decision-aid with special emphasis on multiple criteria decision-aid. The paper is organized as follows. In section 2, a number of examples will help clarify the scope and limits of decision-aid, which we attempt to define in section 3. The main stages of the decision-aid process are described in sections 4-6. The next three sections describe the three major approaches underlying most of existing decision-aid procedures. We conclude this paper with some remarks on the validity and usefulness of decision-aid techniques.

2. Some examples

Decision-aid, as we understand the term here (see section 3), is concerned with :

- defining a production plan *e.g.* adapting a refinery's operations to a variable demand for fuel of different types according to the grades of crude oil available;

- scheduling tasks in workshops or on construction sites;
- defining an inventory management policy;
- determining the frequency of maintenance operations for various kinds of equipment;
- allocating limited resources *e.g.* airplane crew rotations;
- selecting one of many variants of the same project *e.g.* the route of high-voltage lines or highways, the location of a factory or a child-care complex;
- arbitrating between competing projects of different types *e.g.* R & D projects or new products ready to be launched on the market;

We borrow this list, which is far from exhaustive, from a number of real-world studies. More strategic decisions such as:

- restructuring a group, abandoning an activity or buying out a company,
- deciding whether or not to begin work on a supersonic airplane such as the Concorde,
- deciding whether or not to institute currency exchange controls,
- choosing, among possible responses to the Cuban crisis of 1962, a blockade of the Island, a massive attack which would annihilate the regime, or a limited attack which would destroy the Soviet missile bases but spare the urban civilian population (an example from history which has given rise to many studies),

can also benefit from, but in a much more limited way, the concepts, ways of reasoning and procedures which we shall present here. Sfez (1973) has underlined the limitations involved in problems of these dimensions: the variety and complexity of the logic used by each actor involved in the decision process make it virtually impossible to submit such processes to any form of control. In what follows, therefore, the reader should keep in mind our first set of examples.

3. Purpose of decision-aid

3.1. Definitions (see Roy (1985))

Decision-aid is the activity of one who uses explicit — but not necessarily completely formalized — models to obtain elements of answers to questions raised by an actor involved in a decision process. These elements tend to clarify the decision and, usually, to prescribe[1] or simply to encourage behavior that will increase the coherence between the evolution of the process and the objectives supported by this actor.

Thus defined, decision-aid is only very partially concerned with a search for truth. The theories or, more simply, the methodologies, models and techniques on which it is based and which we will discuss below, usually have a different aim: to reason out the change prepared by a

[1] The word is used here with the same connotation it has in medicine: it is clear to the analyst, as well as to the decision-maker, that the latter is not bound to observe the prescribed behavior.

decision in such a way as to make it more consistent with the goals and system of values of the one for whom or in whose name decision-aid is to be performed.

This conception of decision-aid necessitates an initial attempt at clarification: how can we most appropriately define the decision ? In what terms should we differentiate and individuate the various possibilities for action available to us ? Where should we draw the line between what is feasible and what is not ? In a siting problem, it may happen that all points in a given territory deserve consideration as potential sites. In that case, the various possible courses of action may be differentiated according to their geographical coordinates x_1, x_2 (*e.g.*, latitude and longitude). In other instances, only those sites which meet a minimum number of requirements (*e.g.*, availability of usable land, proximity to railways or roads) will be considered. In such cases, the possible courses of action can be enumerated in a list a_1, a_2, ..., a_m.

We use the term action (or "alternative" , "possibility") to designate anything which appears to be a possible contribution to making a decision and which can either be represented by a set of coordinates or as an item in a list. Depending on the situation (see section 2) an action may appear either as a plan or a program (production planning, scheduling, ...) or as a variant of a project (siting, launching of a new product, ...).

The set A of actions should be seen as a frame of reference at a given stage in the decision-aid process. This frame is likely to evolve at later stages of the process. This is why the actions which belong to the set A are called potential actions. As we mentioned earlier, such a set A can be defined:

- as a subset of Re^m when the standard action a can be characterized by a series of numbers x_1, x_2, …, x_m (geographical coordinates, time, production, …);
- as a list a_1, a_2, …, a_m, each element in this list corresponding to a well-defined course of action.

Finally, let us mention that, for the purposes of reasoning, it is sometimes interesting to envisage, at least tentatively, fictitious or ideal actions, *i.e.* actions which do not stem directly from a real possibility.

3.2. What terms can be used to formulate the problem ?

In many decision contexts, those in charge often speak in terms of an "optimal solution". It should be emphasized that, in order to speak of an optimum, the problem must be formulated in such a way that:

- the solutions envisaged are all mutually exclusive;
- the set of solutions is well-defined and fixed;
- the solutions can be ranked in an incontestable way from the worst to the best.

The set of potential actions A may not always be regarded as a set of solutions possessing these characteristics. This is why it is often preferable, in the field of decision-aid, to attempt, at least initially, to formulate the problem in less restrictive terms. This is the *raison d'être* for the four problem formulations presented in table 1.

Table 1: Four Basic Problem Formulations

	Objective	Result
P.α	clarify the decision through the choice of a subset, as restricted as possible, for the final choice of a single action. This subset should contain the "best" actions ("optimums") or, failing that, "satisfac-tory" actions.	a choice or a selection procedure
P.β	clarify the decision through a sorting consisting in an affectation of each action to a category, these categories being defined a priori (e.g. accepted, rejected, sent back for more information).	a sorting or an assignment procedure
P.γ	clarify the decision through a sorting consisting in an affectation of each action to a category, these categories being defined a priori (e.g. accepted, rejected, sent back for more information).	a ranking or a classifying procedure.
P.δ	clarify the decision through a description of the actions and their consequences.	a description or a cognitive procedure.

The first (P.α) consists of formulating the problem in terms of choosing a good action. This choice is not necessarily optimal in the above sense (optimization is thus a particular case of this problem formulation). The second (P.β) corresponds to the common practice of examinations (medical examinations, academic examinations). Problem formulation P.γ corresponds to a competitive examination which results in a ranking (not necessarily complete). Problem formulation P.δ is worth isolating since, even though it is part of the first three formulations, in some situations, it constitutes an end in itself.

The spirit in which the prescription is to be made is also an important option. The prescription will be different if the study is of a tactical nature, *i.e.*, concerned only by the present set of potential actions, or more strategic, aiming at implementing a methodology or a procedure designed for repeated and/or automated use.

4. One or several criteria ?

In any problem formulation, it is necessary to take into account the consequences of implementing the potential actions. Such consequences are generally numerous. They are perceived and expressed in a variety of terms (*e.g.*, monetary, temporal, spatial, visual). Through the evaluation of these consequences, we can compare actions in terms of preferences.

Faced with a number of generally vague and complex consequences, the preferences of an actor involved in a decision process are not always stable and well-defined. The perception of some consequences can be more or less substantiated; others may be hypothetical or described in terms of probability. Their relative importance may not be clear to the actor. Comparing two actions in terms of preferences seems to be the outcome of conflicting aspects which are encountered as much in the mind of a given actor as among different actors within the decision process. Decision-aid is, above all, help in clarifying how preferences are formed, transformed and argued. At this stage, the key concept is that of criterion.

4.1. What is a criterion ?

Essentially, a criterion is a function that associates each action with a number indicating its desirability according to consequences related to the same "point of view". Hence one could try to define a criterion "damages to the environment" taking into account such consequences as, *e.g.*, impact on fauna, on flora, on air purity,... In formal terms, criterion g is a real-valued function defined on the set A of potential actions so that the comparison of the two numbers g(a) and g(b) allows us to describe and/or argue the result of the comparison of a and b relative to the point of view underlying the definition of g. More precisely, criterion g is a model whereby:

$$g(b) \geq g(a) \Rightarrow b\ S_g\ a$$

where S_g is a binary relation that reads "is at least as good as, relative to the evaluations of the consequences accounted for in the definition of g" (this definition assumes a notion of preferential independence *vis-à-vis* the consequences left out of the model g).

Because of the semantic content of the relation S_g, b S_g a covers situations ranging from indifference between a and b (b I_g a) to the strict preference for b over a (b P_g a).

These two situations are traditionally separated in the following way:

$$g(b) = g(a) \Leftrightarrow b\ I_g\ a,$$
$$g(b) > g(a) \Leftrightarrow b\ P_g\ a.$$

Considering the inevitable arbitrariness entering both the evaluation of the actions on the consequences and the definition of g, this model is not always very realistic in practice: a small positive difference g(b) - g(a) may not be indicative of a strict preference.

A more sophisticated preference modelling is obtained by introducing two thresholds p_g and q_g, with $p_g \geq q_g$, so that when $g(b) \geq g(a)$ we have:

$$g(b) - g(a) \leq q_g \Leftrightarrow b\ I_g\ a,$$
$$p_g < g(b) - g(a) \Leftrightarrow b\ P_g\ a.$$

The situation not covered by these two intervals, namely:

$$q_g < g(b) - g(a) \leq p_g$$

corresponds to a case of hesitation (indetermination) between indifference and strict preference, called weak preferences and denoted Q_g. The functions q_g and p_g are respectively called indifference and preference thresholds[2].

4.2. Building criteria

[2] These thresholds are not necessarily constant and may vary along the scale of the criterion.

Defining a criterion g is, first of all, choosing a point of view along which comparisons are to be made. The choice of a particular point of view allows us to give a concrete meaning to the numbers g(a), given the nature of the various consequences taken into account. In practice, the definition of the number g(a) implies the introduction either of a unit connected to the point of view (dollars spent, time gained, miles covered, …), or of successive levels on a qualitative scale (a hindrance difficult to bear, a hindrance to which one becomes quickly accustomed, neither good nor bad, rather nice on the whole, exceptionally comfortable).

To complete the definition of a criterion g, we must make clear the exact process which allows us to associate any potential action a with a number g(a). This process may require more or less complicated calculations (forecasting a cost), the use of sophisticated models (generating and assigning traffic to evaluate time gained), surveys (reaction to a product ready to be launched on the market) or the advice of experts (risk evaluation).

Criterion g thus appears generally as a construct, a product of multiple options. The choice between various possible options is not always an easy task and should be guided by the objective of obtaining an uncontroversial preference modelling on the basis of the criterion.

As an example, let us examine a simple case in which criterion g takes into account a single, well-defined consequence, *e.g.* a waiting time. Let us suppose that, for a given action a, this waiting time is not precisely known but can be modelled using a probability distribution p_a ($p_a(t).dt$ being the probability that the wait lies in the interval [t, t + dt]). It may seem reasonable (above all if this wait is repeated) to compare actions according to the average wait they entail. This leads us to postulate (if the wait is found between the minimum m and the maximum M):

$$g(a) = \int_m^M p_a(t)t\,dt$$

This way of taking uncertainty into account may be unsatisfactory however. It may indeed be entirely legitimate to prefer an action a, characterized by an average wait of 30 units and with dispersion very close to this average (m = 28, M = 30), to an action b characterized by an average wait of 25 but with a larger range of dispersion (m = 5, M = 60). To take this aversion to a potentially long wait into account, we can still use an averaging principle to build g(a), replacing the waiting time t by an increasing transformation u(t) aiming at capturing the disutility induced by a waiting time t. The formula above is thus replaced by:

$$g(a) = \int_m^M p_a(t)u(t)\,dt$$

In order to be able to use such a criterion it must be assumed that it is possible to objectively reason the function u(t) and to assess the probability distributions p_a with a minimum of realism (on these questions, see Keeney and Raiffa (1976)). This is not always the case and other ways of dealing with imprecision, uncertainty and inaccurate determination in the construction of a criterion can be envisaged. In this respect discrimination thresholds are useful tools. Certainly, the value of such thresholds does not generally emerge in an obvious manner.

It is, however, possible to treat them as parameters likely to vary within a given interval and to study the impact of such changes on the final conclusions.

4.3. Monocriterion or multicriteria analysis ?

When the "cloud" of consequences is not too complex, it is possible to build a single criterion apprehending all the pertinent consequences. In such a monocriterion analysis one must be able to define a point of view taking all consequences into account and having a more or less concrete meaning: benefits, rate of return, utility. It should be emphasized that such an analysis implies that it is possible to measure all consequences on a common scale.

Such a monocriterion analysis should avoid:

- excluding (more or less consciously) from the definition of the single criterion those aspects of the consequences that are difficult to grasp in such a system of representation;
- using reference prices or conversion rates, both necessary for bringing together heterogeneous consequences on a common scale and yet difficult to evaluate in an uncontroversial manner;
- ending up with a complex formula based on obscure logic and not conducive to communication among the various actors in the decision process.

When the consequences are so heterogeneous that the preceding difficulties cannot be avoided, it is preferable to proceed to a multicriteria analysis. This consists of building a family $F = \{g_1, g_2, \ldots, g_n\}$ of several criteria, each one apprehending a homogeneous category of consequences. Building a family of criteria, which can be seen as an intermediate step in the decision-aid process, often allows the analyst to avoid most of the difficulties mentioned above. However this family will only be helpful if it possesses a number of consistency properties (e.g, exhaustiveness, non-redundancy), is intelligible to the different actors that are involved in the decision process and accepted by them as the basis of their work. Under such conditions, this family often constitutes a useful tool for communication providing the basis for reasoning, transforming and arguing preferences.

As we shall see in section 7, these several criteria may well be aggregated into a single one at a later stage of the study. Such an aggregation into a single criterion should not be confused with monocriterion analysis. When a family of criteria is not explicitly built, consequences recognized by everyone are often confused from the very outset of the analysis with parameters used to reduce them to a common unit by a conversion procedure, a procedure inevitably marked by arbitrariness and generally influenced by a particular system of values.

5- Search for an optimum

When the set A of potential actions possesses the three properties mentioned in 3.2 and, in particular, when the analysis of the consequences of these actions has led to the construction of a single criterion, we can, as part of problem formulation P.α, attempt to solve the optimization problem:

find $a^* \in A$ such that $g(a^*) = \max_{a \in A} g(a)$.

When A is finite and contains few actions, the solution to this problem is trivial. It is quite a different matter when A is infinite (for example when A is a subset of Re^k) or when the cardinal of A prohibits any search for an optimum through simple enumeration (combinatorial problems). We must then turn to specific techniques (linear, non-linear, dynamic programming, graph theory) which are part of the core of Operational Research and which we shall not touch here on. The following four sections present a certain number of concepts and techniques for decision-aid that can be used in the numerous cases in which optimization methods cannot be used because of the nature of A, the problem formulation or the presence of several criteria.

6- The problem of aggregation

When analyzing the consequences of the potential actions leads us to build several criteria, decision-aid can no longer be formulated as simply as it was in the preceding section. It is useful, in this case, to summarize the results of the analysis of the consequences in a table of scores (see table 2). Based on such a table which synthesizes the performances of all or some of the potential actions on various criteria (possibly endowed with discrimination thresholds), the problem consists of knowing which reasoning, calculations and deductions can justify the proposition "taking all criteria into consideration, action a is at least as good as action b", which we denote by a S b. This is called the problem of the aggregation of scores.

Let us first consider a simple case in which the n criteria are unanimous in declaring that a is as at least as good as b, *i.e.* a P_j b, a Q_j b or a I_j b for all $j \in \{1, 2, \dots, n\}$. We say that the criteria are not conflicting in the comparison of a and b and that "a dominates b".

When "a dominates b", we have grounds for maintaining that "a is at least as good as b taking all criteria into consideration" regardless of the value system of the actors involved in the process.

Action a is said to be efficient if there is no other action $a' \in A$ such that a' dominates a and a does not dominate a'. In certain cases, we are only interested in the subset $A^* \subseteq A$ of all efficient actions. In the case where A is infinite or contains a large number of actions, numerous techniques have been developed, either for testing whether an action is efficient or not, or for determining A* (see, *e.g.*, Zeleny (1982), Chankong and Haimes (1983), Goicoechea *et al.* (1982)). Let us note, however, that restricting our attention to the set A* of efficient actions is only meaningful within the framework of P.α.

Table 2 : Table of Scores

	g_1	g_2	...	g_j	...	g_n
	p_1	p_2	...	p_j	...	p_n
	q_1	q_2	...	q_j	...	q_n
a_1				.		
a_2				.		
...				.		
a_i	. . .	. . .	. . .	$g_j(a_i)$		
...						
a_m						

In all cases where there is no dominance, we say that criteria are conflicting in the comparison between a and b. To maintain that, taking all criteria into consideration, "a is better than b"or that "a is indifferent to b" amounts to taking a stand on the outcome of these conflicts. Formulating this outcome will depend on numerous factors among which are:

- the system of values of the actors involved and, in particular, the importance that they grant to each criterion;
- the technique to be used for the aggregation of scores;
- the precision and nature of the evaluations contained in the table of scores.

The techniques of aggregation, examined in the three following sections, formalize a number of simple ways of reasoning for determining the outcome of the conflicts that are briefly presented here.

Let us consider as an example the following table of scores:

	g_1	g_2	g_3	g_4	g_5
a	10	30	80	90	60
b	80	70	50	40	40

It is not restrictive to suppose that preference increases with score and that on all criteria the difference between the scores of a and b is greater than the corresponding preference threshold.

Let us examine, on the basis of this example, how an actor could argue in favor of the proposition "a is at least as good as b".

The first type of reasoning consists of considering the set $C(a\ S\ b) = \{g_j \in F : a\ P_j\ b \text{ or } a\ Q_j\ b \text{ or } a\ I_j\ b\}$ of criteria agreeing with the proposition a S b. Here we have: $C(a\ S\ b) = \{g_3 , g_4 , g_5\}$. If each criterion were represented by a voting individual, 3 voters out of 5 would vote for the proposition a S b. A straightforward generalization of this type of reasoning leads to the idea of concordance analysis in which the proposition a S b is accepted only if the coalition C(a S b) is "sufficiently" important. Note that this reasoning uses only the ordinal properties of the various scores and does not allow the idea of compensation to come into play.

Confronted with the same table of scores, another actor may well consider that the proposition a S b is unfounded. He calls attention to the fact that the preceding analysis, even if it brings out a majority of criteria in favor of a, neglects the fact that the difference $g_1(b) - g_1(a) = 70$ in favor of b seems so important that the proposition a S b is subject to caution. This type of reasoning grants certain criteria, which are discordant with the proposition a S b, the power to "veto" this proposition as soon as the difference between the scores of b and a is "large" on these criteria[3].

Quite another type of reasoning would consist of appreciating the way in which differences in scores which favor a (criteria g_3, g_4 and g_5) compensate for the differences in favor of b (criteria g_1 and g_2). To do this, we could ask ourselves if a difference of 70 according to g_1 is or is not compensated for by a difference in the opposite direction of 50 according to g_4. A simple way of resolving the problem attempts to determine the "worth" of a unit of criterion g_5 when expressed in the "money" of criterion g_1, in a way analogous to choosing the coefficients of grades given on exam papers. If we were able to define such "conversion rates" (commonly called substitution rates) in that way, we would then be able to convert all the differences onto a single common scale and thus rank the potential actions.

7- Aggregation into a single criterion

This approach is probably the most traditional one. It consists of building a single criterion by using an aggregation function V by letting:

$$g(a) = V(g_1(a), g_2(a), ..., g_n(a)).$$

Any two actions can thus be compared in terms of indifference or of strict preference (or weak preference if discrimination thresholds are introduced) on the basis of the values g(a). Once this single criterion is obtained, we can simply develop a prescription in P.α, P.β or P.γ.

The aggregation function V frequently assumes one of the two forms:

$$g(a) = \sum_{j=1}^{n} k_j\, g_j(a) \qquad \text{(weighted sum aggregation) or} \qquad (1)$$

$$g(a) = \sum_{j=1}^{n} k_j\, v_j[g_j(a)] \qquad \text{(additive utility)} \qquad (2)$$

where k_j are strictly positive coefficients and v_j strictly increasing functions on the real line. It is not restrictive to impose $\sum_{j=1}^{n} k_j = 1$ and $0 \leq v_j[g_j(.)] \leq 1$.

Other more complex aggregation functions can be envisaged (especially when the g_j are expected utility criteria; see Keeney and Raiffa (1976)).

This approach of aggregation into a single criterion does not tolerate incomparability and, thus, requires rich "inter-criteria information". The "rate of conversion" between units of various criteria mentioned in the preceding section are essential for reasoning out the construction of V.

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[3] This type of reasoning is different from that which uses "reservation levels" in order to separate A between satisfactory and unsatisfactory actions. Using such reasoning, a reservation level g_j^* is defined on each criterion j and it is declared that an action a is satisfactory if:

- $g_j(a) \geq g_j^*$ for all $j \in \{1, 2, ..., n\}$ (conjunctive viewpoint) or,
- $g_j(a) \geq g_j^*$ for some $j \in \{1, 2, ..., n\}$ (disjunctive viewpoint).

Formally, the substitution rate between criteria g_j and g_h at the point $\mathbf{g}^0 = (g_1^0, g_2^0, ..., g_n^0)$ in the space of scores is the variation on criterion g_j allowing to compensate for (this notion of compensation being central to this approach) a reference change on criterion g_h. Supposing in a non-restrictive manner that the reference change on criterion g_h be 1, this substitution rate $r_{jh}(g^0)$ is the number such that the action characterized by the scores of $\mathbf{g}^0$ is judged indifferent to the action having the following vector of scores:

$$(g_1^0, g_2^0, ..., g^0_{j-1}, g_j^0 + r_{jh}(g^0), g^0_{j+1}, ..., g^0_{h-1}, g_h^0 - 1, g^0_{h+1}, ..., g^0_n).$$

Subject to differentiability conditions, the usual definition of the substitution rate can be found when the discrimination thresholds are zero and when the change in reference on the criterion g_h tends towards 0. Thus we have:

$$r_{jh}(\mathbf{g}^0) = [\partial V(\mathbf{g}^0)/ \partial g_j]/[\partial V(\mathbf{g}^0)/ \partial g_h]$$

Using a weighted sum aggregation amounts to supposing that the substitution rates $r_{jh} = k_j/k_h$ are constant. In the more general additive form of type (2), the substitution rates $r_{jh}(\mathbf{g}^0)$ are no longer constant but are independent of the scores of criteria other than g_j and g_h.

The aggregation function described in (1) and (2) are used in a large number of methods, in particular:

- those stemming from "Goal Programming" where one looks to find an action differing as little as possible from a goal on each criterion (see, *e.g.*, Ignizio (1976));
- those stemming from "Compromise Programming" where one looks to find an action as close as possible to an ideal point in terms of a certain distance (see Zeleny (1982));
- those using the notion of ordinal regression (see Jacquet-Lagrèze and Siskos (1983));
- those stemming from MAUT (see Keeney and Raiffa (1976));
- those using paired comparisons as in the AHP (see Saaty (1980)).

8- Aggregation into an outranking relation

This approach aims at building a binary relation S on A (often called an outranking relation) that is richer than the dominance relation. Contrary to the preceding approach, the relation S is not built via a single criterion. Here S aims at capturing the part of the preferences that can be well established at this stage of the process. The model of global preferences that is developed tolerates incomparability (*i.e.* situations for which, considering the information available at this stage of the decision-aid process, it does not seem possible to establish a clear situation of preference between two actions) and/or intransitivity. Thus, contrary to the first approach, establishing a prescription in one of the three problem formulations α, β, γ may not be an easy task and may imply the use of specific techniques.

With this approach, the relation S is generally built by applying a "test" to all ordered pairs of alternatives. Such methods are usually applied when A contains a finite and restricted number of potential actions — this is not an imperative limitation however. These methods were devised in France and have been primarily developed in French-speaking countries (see Schärlig (1985)).

With most methods, the outranking test uses the notions of concordance and discordance introduced in section 6. As an example, we shall briefly present ELECTRE I when it is applied to a family of criteria without discrimination thresholds. In ELECTRE I, the proposition a S b is accepted if the concordant coalition $C(a\ S\ b) = \{g_j \in F : g_j(a) \geq g_j(b)\}$ is "sufficiently" important (condition of concordance) and if on the other criteria the differences $g_j(b) - g_j(a)$ are not "too" large (condition of non-discordance). Like most of the methods stemming from this approach, we are led, in order to give a specific content to the condition of concordance, to assign a weight k_j to each criterion. In ELECTRE I, the importance of a coalition is represented by the sum of the weights of the criteria belonging to that coalition. Thus the index c(a, b) defined by:

$$c(a, b) = \sum_{j \in C(aSb)} k_j / \sum_{i}{}^{n}{}_{=1} k_j$$

represents the relative importance of C(a S b) among the set of all criteria. Whether or not C(a S b) is "sufficiently" important is then judged comparing c(a, b) to a threshold $s \geq 1/2$ called concordance threshold.

In order to determine which differences on the discordant criteria are judged "too" large, a veto threshold v_j (that may vary with g_j) is defined on each criterion in such a way that the existence of a discordant criterion such that $g_j(b) - g_j(a) \geq v_j$ prohibits accepting of a S b whatever the value c(a, b).

Thus, in ELECTRE I:

$$a\ S\ b \Leftrightarrow [c(a, b) \geq s \text{ and } g_j(b) - g_j(a) < v_j \text{ for all } j \notin C(a\ S\ b)]$$

It is easy to see that if s = 1 or if $v_j = 0$ for all j, the relation S is nothing but the dominance relation. Simple examples, inspired by Condorcet's well-known paradox, show that, in general, S is neither complete nor transitive.

It is important to note that in the preceding formula, the weights k_j cannot be interpreted as substitution rates. As such, they are fundamentally different from the k_j used in the preceding section. This illustrates that the often vague and inaccurate notion of the importance of a criterion acquires meaning only as part of the particular procedure of aggregation.

ELECTRE I was originally designed to cope with a problem formulation P.α . Once the outranking relation S is built the method then seeks to determine the minimum set of actions not outranking each other such that all the actions outside of this set are outranked by at least one action from this set (that is, the kernel of the graph (A, S) after detection and elimination by reduction of possible circuits). However the outranking relation built in ELECTRE I can be used in other problem formulations. In P.γ, for example, we could determine a complete or partial ranking that would be the closest possible to S according to some distances, or determine a ranking based on the number of actions that each action outranks (this is the notion of distillation used in the methods ELECTRE II, III, IV and PROMETHEE, see Schärlig (1985)).

However, the outranking approach is not exclusively tied to the notion of concordance-discordance. Outranking relations can easily be built based on other principles such as, for example, the use of intervals for substitution rates in (2), the simultaneous consideration of a set of aggregation functions or the consideration of dominance relations taking into account discrimination thresholds (see Jacquet-Lagrèze and Siskos (1983)).

9. Interactive Approach

In this approach, contrary to what has been done in the other two, we are not looking for a synthetic, exhaustive and definitive rule to aggregate scores. The aggregation no longer originates in a rule, even a partial or provisional one, but in a sequence of *ad hoc* judgments that the decision-maker formulates. The judgments have only a limited import inasmuch as they involve only a single action and its environment within the space of scores, or a very small number of actions that it seems pertinent and judicious to compare because they are similar.

This approach rests on a protocol of interaction between the questioned entity D (decision-maker or actor) and the questioning entity C (analyst or computer). This protocol can be broken down into dialogue steps in which C collects and assembles the reactions of D (which can be expressed under various forms: indications of a substitution rate, choice of aspiration or reservation levels, choice of one action among several, choice of one or more criteria to improve or worsen) and into calculation steps in which C integrates the answers provided by D in such a way as to make him react again.

Such an interactive procedure can be viewed as a decision-aid procedure if this alternation between dialogue steps and calculation steps leads either to D discovering partial answers to his or her preoccupations or to C assembling the material necessary for the development of his or her prescription. This process by which an opinion or a conviction is formed cannot be exempt from trial and error. In such procedures, it should be possible for D to reconsider his or her answers at any time in the procedure. This means that the usual idea of "convergence" of an algorithm has little importance here and it is not unlikely that a change of mind from D forces the interactive procedure back to a stage which has already been encountered. But this should not be considered a "loop" since in this "loop" D may well have acquired information, refined his or her preferences and/or improved his or her perception of the set of actions. The interactive procedure stops then either because D is satisfied (for example, because C has brought to light a compromise action that is judged satisfactory), or because D is weary or even because C is unable to or does not think it necessary to continue the dialogue. In this light, an interactive procedure seeks to achieve "psychological convergence" as opposed to the traditional "algorithmic convergence" (see Vincke (1989)). It should be emphasized that this idea of psychological convergence does not eliminate the necessity for interaction protocols to satisfy a number of consistency properties, *e.g.* proposing compromises corresponding to efficient actions in a problem formulation P.α.

Many methods have been developed within the framework of this approach (see Hwang *et al.* (1979) or Steuer (1986)). They are, for the most part, designed to treat cases where the set A is infinite in the problem formulation P.α.

10. Some final notes on the validity of decision-aid

Some specialists believe that the validity of decision-aid methods and procedures is restricted only by a lack of sufficient means (time and money). Others consider decision-aid as a highly subjective and partial process which, consciously or not, is nothing but a strategic tool used by some actors in the decision process. Somewhere between these two extreme positions lies what we can legitimately expect from a scientific attitude towards decision-aid.

We would like to emphasize that decision-aid cannot be seen as entirely founded on scientific grounds. Contrary to their counterparts in the physical sciences, the models and instruments of decision-aid do not claim to describe a reality that would be independent of the observer and exist independently of other human actors personalities. In the majority of contexts involving decisions, we must admit that the various participants, by their judgments as much as by their behavior, interact with reality and contribute to forging what we would like to describe as an external object. Thus, the very way in which the questions are formulated, giving more weight to certain factors or forms of logic, is likely to disturb that part of reality that we would like to observe and isolate to use as a support for deduction. Even when this does not occur, the truths discovered through the use of models and instruments remain contingent upon multiple options (definition of the problem; modelling of the consequences; management of uncertainty, imprecision, inaccurate determination) as well as upon one or several systems of values. In what way might an advantage from one point of view compensate for a disadvantage from another point of view ? Are there disadvantages that should not be compensated for ? How can we arbitrate between a risky but potentially profitable action and a safer but probably less profitable action ? Apart from a few exceptional cases, science cannot answer such questions.

However, we think that there is room for a scientific attitude towards decision-aid. Thanks to rigorous concepts, well-formulated models, precise calculations and axiomatic considerations, we are able to clarify decisions by separating what is objective from what is less objective, by separating strong conclusions from weaker ones, by dissipating certain forms of misunderstanding in communication, by avoiding the trap of illusionary reasoning, by bringing out certain counter-intuitive results. This "decision-aid science", which is still taking shape, can only truly come to fruition if particular attention is given to the way the tools and procedures it produces are integrated into decision-making processes.

Between the two extreme positions outlined above, we think, along with the majority of specialists, that if a scientific approach to decision problems can bring effective aid to the decision makers, it cannot claim to dictate their behavior. A margin of incompressible freedom remains. No objective procedure founded only on reason can demonstrate the optimality or even the reasonableness of a system of values or a means of anticipating the future. Influences exerted by the personality of actors involved in decision processes elude scientific analysis in many other aspects, notably in their strategic behavior, their capacity to perceive possibilities and evaluate consequences, to question any of their convictions, to influence the convictions of others or to create irreversible situations. Nonetheless, decision-aid, founded on appropriate concepts and procedures, can and does play an important and beneficial role in decision processes.

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