Social choice theory

A brief introduction

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Introduction

Aims

- analyze a number of properties of electoral systems
- present a few elements of the classical theory

What is Social Choice Theory?

Social Choice Theory

- aim: study decision problems in which a group has to take a decision among several alternatives
- abstract theory
  - nature of the decision
  - size of the group
  - nature of the group
- many (deep) results
  - Economics, Political Science, Applied Mathematics, OR
  - two Nobel Prizes: Kenneth J. Arrow, Amartya Sen

Areas of applications

Applications

- political elections
- other types of elections
  - fewer voters and candidates (e.g., electing a Dean)
- decision with multiple criteria
- artificial intelligence
  - multiple agents
  - multiple rules

Problem

Vocabulary: political elections

- group
  - society
- members of the group
  - voters
- alternatives
  - candidates

Problem

- study election problems in which a society has to take a decision among several candidates

Today's problem

Problem

- choice of *one* among several candidates
  - French or US presidential elections

Electing several candidates: assembly

- apply same rules in each electoral district
- many specific problems: gerrymandering, technical problems (as sometimes seen in the USA)

Proportional representation

- PR does not solve the decision problem in the Parliament!
  - one bill will adopted on each issue
- PR raises many difficult problems (What is a just PR? How to achieve it? PR and Power indices)

A glimpse at PR

Problem 1: # of seats and power

- Parliament: 100 MPs
- voting rule in the Parliament: simple majority (> 50 %)
- # of votes exactly proportional to # of seats

Example

- party A: 45 % of votes
- party B: 15 % of votes
- party C: 40 % of votes

- all coalitions of 1 party are loosing coalitions
- all coalitions of at least 2 parties are winning coalitions
- entirely symmetric situation
- all parties have the same power

A glimpse at PR

Problem 2: obtaining a fair PR

- in general # of voters $\gg$ # of MPs
- # of MPs must be integer!
- rounding off procedures

Hamilton's rule

- 2 100 000 voters, 3 parties, 20 MPs
- results
  - party A: 928 000, quota: $r_A = 928\,000/2\,100\,000 = 8.84$
  - party B: 635 000, quota: $r_B = 635\,000/2\,100\,000 = 6.05$
  - party C: 537 000, quota: $r_C = 537\,000/2\,100\,000 = 5.11$
- party x gets at least $\lfloor r_x \rfloor$ seats
- if all seats are allocated: done
- if not: allocate the remaining seats according to the $r_x - \lfloor r_x \rfloor$

Hamilton's rule

- party A: 928 000, quota: $r_A = 8.84 = 928\,000/2\,100\,000$
- party B: 635 000, quota: $r_B = 6.05 = 635\,000/2\,100\,000$
- party C: 537 000, quota: $r_C = 5.11 = 537\,000/2\,100\,000$

Results

- party A gets 8 seats
- party B gets 6 seats
- party C gets 5 seats
- $8 + 6 + 5 = 19 < 20$
- party A gets the extra seat because $0.84 > 0.11 > 0.05$

Example

20 seats

- party A: $r_A = 8.84$, $8 + 1 = 9$ seats
- party B: $r_B = 6.05$, 6 seats
- party C: $r_C = 5.11$, 5 seats

21 seats

- party A: $r_A = 9.28$, 9 seats
- party B: $r_B = 6.35$, 6 seats
- party C: $r_C = 5.37$, $5 + 1 = 6$ seats

22 seats: Alabama paradox (1881)

- party A: $r_A = 9.72$, $9 + 1 = 10$ seats
- party B: $r_B = 6.65$, $6 + 1 = 7$ seats
- party C: $r_C = 5.63$, 5 seats

Election of one candidate

Common sense

- the choice of the candidate will affect all members of the society
- the choice of the candidate should take the opinion of all members of society into account

Intuition

Democracy $\Rightarrow$ Elections $\Rightarrow$ Majority

Elections

Philosophical problems

- "general will" and elections
- majority and protection of minorities
- formal vs real freedom

Political problems

- direct or undirect democracy?
- rôle of parties?
- who can vote? (age, sex, nationality, paying taxes, ...)
- who can be candidate?
- what type of mandate?
- how to organize the campaign?
- rôle of polls?

Technical problems

Majority

When there are only two candidates

- elect the one receiving the more votes

Majority

When there are more than candidates

- many ways to extend this simple idea
- not equivalent
- sometimes leading to unwanted results

Typology of elections

Two main criteria

1. type of ballots admitted
   - one name
   - ranking of all candidates
   - other types (acceptable candidates, grading candidates, etc.)
2. method for organizing the election and for tallying ballots

Consequences

- many possible types of elections
- many have been proposed
- many have have been used in practice

Two hypotheses

Hypotheses

1. all voters are able to rank order the set of all candidates (ties admitted)

$$a \succ b \succ [d \sim e] \succ c$$

   - each voter has a weak order on the set of all candidates
2. voters are sincere
   - if I have to vote for one candidate, I vote for a

Plurality voting: UK

Rules

- one round of voting
- ballots with one name
- “first past the post”

Remark

- ties are neglected (unlikely)
  - one voter has special power (the Queen chooses in case of a tie)
  - one candidate receives special treatment (the older candidate is elected)
  - random tie breaking rule

Plurality voting

Example

- 3 candidates $\{a, b, c\}$
- 21 voters (or 21 000 000 or 42 000 000. . .)

10 voters:	$a \succ b \succ c$
6 voters:	$b \succ c \succ a$
5 voters:	$c \succ b \succ a$

Results

$$a : 10 \quad b : 6 \quad c : 5$$

- a is elected. . .
- but an absolute majority of voters (11/21) prefer all loosing candidates to the elected one!

a: Tory, b: Labour, c: LibDem

Plurality voting

Remarks

- problems are expected as soon as there are more than 2 candidates
- a system based on an idea of "majority" may well violate the will of a majority of voters
- sincerity hypothesis is heroic!

Plurality with runoff: France

Rules

- ballots with one name
- first round
  - the candidate with most votes is elected if he receives more than 50% of votes
  - otherwise go to the second round
- second round
  - keep the two candidates having received more votes
  - apply plurality voting

Variants

- rule are slightly different for the “élections législatives”

Plurality with runoff

Previous example

- 3 candidates $\{a, b, c\}$
- 21 voters

10 voters:	$a \succ b \succ c$
6 voters:	$b \succ c \succ a$
5 voters:	$c \succ b \succ a$

Results

$$a : 10 \quad b : 6 \quad c : 5$$

- absolute majority: $\lceil 21/2 \rceil = 11$ votes
- go to the second round with a and b

$$a : 10 \quad b : 11$$

- b is elected
- no candidate is preferred to b by a majority of voters

Plurality with runoff

Example

- 4 candidates $\{a, b, c, d\}$
- 21 voters (may be also 21 000 000 or 42 000 000)

10 voters:	$b \succ a \succ c \succ d$
6 voters:	$c \succ a \succ d \succ b$
5 voters:	$a \succ d \succ b \succ c$

Results: 1st round

$a : 5 \quad b : 10 \quad c : 6 \quad d : 0$

- absolute majority: $\lceil 21/2 \rceil = 11$ votes
- go to the second round with b and c

Results: 2nd round

$b : 15 \quad c : 6$

- b is elected (15/21)
- an absolute majority of voters (11/21) prefer a and d to b

Plurality with runoff

Plurality vs plurality with runoff

- the French system does only a little better than the UK one
- preferences used in the above example are not bizarre
  - try replacing a, b, c, d by MoDem, UMP, PS, PCF, FN, etc.
- sincerity and wasted votes

Plurality with runoff: manipulation

Example

- 4 candidates $\{a, b, c, d\}$
- 21 voters

10 voters:	$b \succ a \succ c \succ d$
6 voters:	$c \succ a \succ d \succ b$
5 voters:	$a \succ d \succ b \succ c$

- b is elected

Non-sincere voting

- the 6 voters for which $c \succ a \succ d \succ b$ vote as if their preferences were $a \succ c \succ d \succ b$

Results

- a is elected at the first round (11/21)
- profitable to the six manipulating voters (for them $a \succ b$)

Manipulable voting rules

Definition

a voting rule is manipulable if it may happen that some voters may have an interest to vote in a non-sincere way

Problems

- elections are no more a means to reveal preferences
  - manipulations and counter-manipulations
  - equilibrium
- bonus to clever voters

Plurality with runoff: monotonicity

Example: before campaign

- 3 candidates $\{a, b, c\}$
- 17 voters

6 voters:	$a \succ b \succ c$
5 voters:	$c \succ a \succ b$
4 voters:	$b \succ c \succ a$
2 voters:	$b \succ a \succ c$

Results: before campaign

absolute majority: $\lceil 17/2 \rceil = 9$

$$a : 6 \quad b : 6 \quad c : 5$$

$$a : 11 \quad b : 6$$

- a is elected!
- a gets more money to campaign against b

Plurality with runoff

6 voters:	$a \succ b \succ c$
5 voters:	$c \succ a \succ b$
4 voters:	$b \succ c \succ a$
2 voters:	$b \succ a \succ c$

- 2 voters $b \succ a \succ c$ change their minds in favor of a
- new preference: $a \succ b \succ c$

absolute majority: $\lceil 17/2 \rceil = 9$

$$a : 8 \quad b : 4 \quad c : 5$$

$$a : 8 \quad c : 9$$

- c is elected!
- the good campaign of a is fatal to him/her
- non-monotonic method: increasing possibilities of manipulation

skip participation

Plurality with runoff: participation

Example

- 3 candidates $\{a, b, c\}$
- 11 voters

4 voters:	$a \succ b \succ c$
4 voters:	$c \succ b \succ a$
3 voters:	$b \succ c \succ a$

Results

absolute majority: $\lceil 11/2 \rceil = 6$

$$a:4 \quad b:3 \quad c:4$$

$$a:4 \quad c:7$$

- c is elected
- this is not a nice outcome for the first 4 voters
- 2 of them go fishing and abstain (at the two rounds)

Before

4 voters:	$a \succ b \succ c$
4 voters:	$c \succ b \succ a$
3 voters:	$b \succ c \succ a$

- c elected

After

2 voters:	$a \succ b \succ c$
4 voters:	$c \succ b \succ a$
3 voters:	$b \succ c \succ a$

Results

absolute majority: $\lceil 11/2 \rceil = 6$

$$a:2 \quad b:3 \quad c:4$$

$$b:5 \quad c:4$$

- b is elected
- the abstention of the two voters who think $b \succ c$ has been very rational

Plurality with runoff: separability

Example

- 3 candidates $\{a, b, c\}$
- 26 voters in two districts $(13 + 13)$

District 1

4 voters:	$a \succ b \succ c$
3 voters:	$b \succ a \succ c$
3 voters:	$c \succ a \succ b$
3 voters:	$c \succ b \succ a$

$$a : 4 \quad b : 3 \quad c : 6$$

$$a : 7 \quad c : 6$$

- a is elected $(7/13)$

District 2

4 voters:	$a \succ b \succ c$
3 voters:	$c \succ a \succ b$
3 voters:	$b \succ c \succ a$
3 voters:	$b \succ a \succ c$

$$a : 4 \quad b : 6 \quad c : 3$$

$$a : 7 \quad b : 6$$

- a is elected $(7/13)$

Plurality with runoff

Nationwide

4 voters:	$a \succ b \succ c$
3 voters:	$b \succ a \succ c$
3 voters:	$c \succ a \succ b$
3 voters:	$c \succ b \succ a$
4 voters:	$a \succ b \succ c$
3 voters:	$c \succ a \succ b$
3 voters:	$b \succ c \succ a$
3 voters:	$b \succ a \succ c$

$$a : 8 \quad b : 9 \quad c : 9$$

- a looses at the first round
- method is not separable
- decentralization of decisions?

Summary

French vs UK system

- the French system does only a little better better than the UK one on the "democratic side"
- it has many other problems
  - manipulable
  - not monotonic
  - no incentive to participate
  - not separable
- are there other (hopefully better!) systems?
- conventional wisdom ("*au premier tour on choisit, au deuxième tour on élimine*") must be used with great care!

Amendment procedure

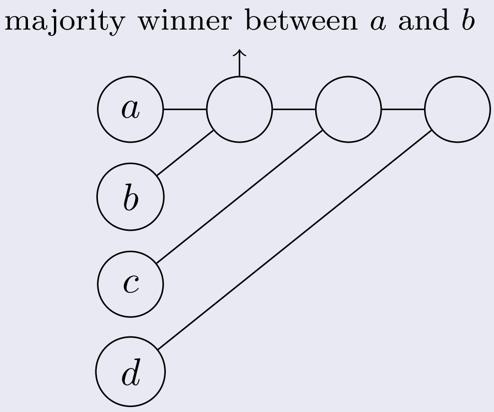
Remarks

- the majority method works well with two candidates
- when there are more than two candidates, organize a series of confrontations between two candidates according to an agenda
- method used in most parliaments
  - a bill is proposed
  - amendments to the bill are proposed
  - compare the amended bill vs the status quo

Amendment procedure

Example

- 4 candidates $\{a, b, c, d\}$
- agenda: a, b, c, d



- a is a bill
- b, c are amendments
- d is the status quo

Amendment procedure

Example

- 3 candidates $\{a, b, c\}$
- 3 voters

1 voter:	$a \succ b \succ c$
1 voter:	$c \succ a \succ b$
1 voter:	$b \succ c \succ a$

- agenda a, b, c: c is elected
- agenda b, c, a: a is elected
- agenda c, a, b: b is elected

- results depending on the (arbitrary) choice of the agenda
  - power given to the agenda-setter
- candidates not treated equally
  - late-coming candidates are favored
  - method is not neutral

Amendment procedure

Example

- 4 candidates $\{a, b, c, d\}$
- 30 voters
- agenda a, b, c, d

10 voters:	$b \succ a \succ d \succ c$
10 voters:	$c \succ b \succ a \succ d$
10 voters:	$a \succ d \succ c \succ b$

Results

- b beats a
- c beats b
- d beats c
- d is elected...
- 100% of the voters prefer a to d!
- method is not unanimous!

Ballots: ordered lists

Ballots with a single name

- poor performances...
- may be due to poor information on preferences
- ask for the full preference on each ballot

Remarks

- much richer information
  - practice?
- ballots with one name are a particular case

Condorcet

Principles

- compare all candidates by pair
- declare that a is "socially preferred" to b if (strictly) more voters prefer a to b (social indifference in case of a tie)
- Condorcet's principle: if one candidate is preferred to all other candidates, it should be elected
- Condorcet Winner (CW: must be unique)

Remarks

- UK and French systems violate Condorcet's principle
- the UK system may elect a Condorcet looser
- Condorcet's principle does not solve the "dictature of the majority" difficulty
- a Condorcet winner is not necessarily "ranked high" by voters

Condorcet and plurality

Example

- 3 candidates $\{a, b, c\}$
- 21 voters

10 voters:	$a \succ b \succ c$
6 voters:	$b \succ c \succ a$
5 voters:	$c \succ b \succ a$

- a is the plurality winner
- a is the Condorcet looser
- b is the CW
  - b beats a (11/21)
  - b beats c (16/21)

Condorcet and plurality with runoff

Example

- 4 candidates $\{a, b, c, d\}$
- 21 voters

10 voters:	$b \succ a \succ c \succ d$
6 voters:	$c \succ a \succ d \succ b$
5 voters:	$a \succ d \succ b \succ c$

- b is the plurality with runoff winner (beats c in the second round)
- a is the CW
  - a beats b (11/21)
  - a beats c (15/21)
  - a beats d (21/21)

Condorcet and ranks

Example

- 5 candidates $\{a, b, c, d, e\}$
- 50 voters

10 voters:	$a \succ b \succ c \succ d \succ e$
10 voters:	$b \succ c \succ e \succ d \succ a$
10 voters:	$e \succ a \succ b \succ c \succ d$
10 voters:	$a \succ b \succ d \succ e \succ c$
10 voters:	$b \succ d \succ c \succ a \succ e$

- a is the CW (beats 30/20 all other candidates)

Ranks

	1	2	3	4	5
a	2	1	0	1	1
b	2	2	1	0	0

Condorcet and dictature of the majority

Example

- 26 candidates $\{a, b, c, \ldots, z\}$
- 100 voters

51 voters:	$a \succ b \succ c \succ \cdots \succ y \succ z$
49 voters:	$z \succ b \succ c \succ \cdots \succ y \succ a$

- a is the CW
- b could be a reasonable choice

Condorcet's paradox

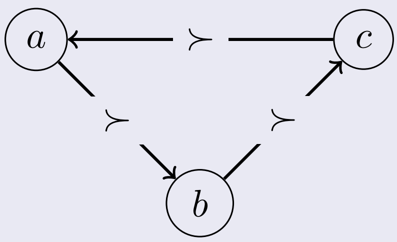
Electing the CW

- attractive ...
- but not always effective!

Condorcet's paradox

- 3 candidates $\{a, b, c\}$
- 3 voters

1 voter:	$a \succ b \succ c$
1 voter:	$c \succ a \succ b$
1 voter:	$b \succ c \succ a$



Condorcet

Condorcet's paradox

- the social strict preference relation may have circuits
  - prob. $\approx 40\%$ with 7 candidates and a large number of voters (impartial culture)
- McGarvey's theorem

Dealing with Condorcet's paradox

- weaken the principle so as to elect candidates that are not strictly beaten (Weak CW)
  - they may not exist
  - there may be more than one
- find what to do when there is no (weak) Condorcet winner

Schwartz

Principle

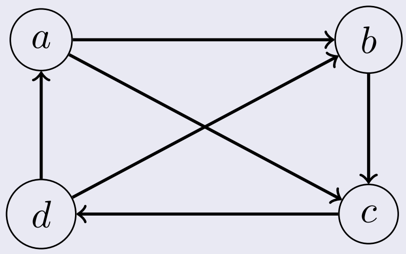
- build the social preference à la Condorcet
- the strict social preference may not be transitive
  - take its transitive closure
  - take the maximal elements of the resulting weak order

Schwartz

Example

- 4 candidates $\{a, b, c, d\}$
- 30 voters

10 voters:	$a \succ b \succ c \succ d$
10 voters:	$d \succ a \succ b \succ c$
10 voters:	$c \succ d \succ a \succ b$



- taking the transitive closure gives a clique
- all candidates are declared socially indifferent
- but 100% of voters prefer a to b!

Copeland

Principles

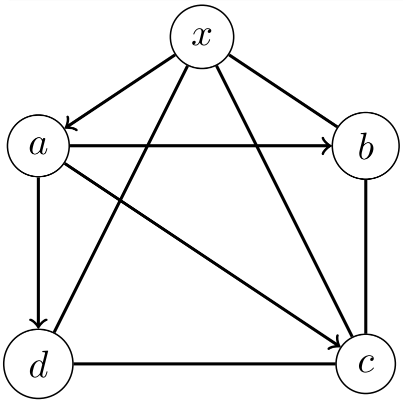
- build the social preference à la Condorcet
- count the number of candidates that are beaten by one candidate minus the number of candidates that beat him (Copeland score)
- elect the candidate with the highest score
- sports league
  - +2 for a victory, +1 for a tie, 0 for a defeat
  - equivalent to Copeland's rule

Copeland

Example

- 5 candidates $\{x, a, b, c, d\}$
- 40 voters

10 voters:	$x \succ a \succ d \succ c \succ b$
10 voters:	$x \succ a \succ b \succ c \succ d$
10 voters:	$a \succ d \succ c \succ b \succ x$
10 voters:	$b \succ c \succ d \succ x \succ a$



x	a	b	c	d
1	2	−2	−1	0

- a is elected!
- x is the unique weak CW

Borda

Principles

- each ballot is an ordered list of candidates (exclude ties for simplicity)
- on each ballot compute the rank of the candidates in the list
- rank order the candidates according to the decreasing sum of their ranks

Remarks

- simple
- efficient: always lead to a result
- separable, monotonic, participation incentive

Borda and Condorcet principle

Example

- 4 candidates $\{a, b, c, d\}$
- 3 voters

2 voters:	$b \succ a \succ c \succ d$
1 voter:	$a \succ c \succ d \succ b$

Borda scores

a	b	c	d
5	6	8	11

Results

- a is elected
- b is the obvious CW

Borda and withdrawals

Example

- 4 candidates $\{a, b, c, d\}$
- 3 voters

2 voters:	$b \succ a \succ c \succ d$
1 voter:	$a \succ c \succ d \succ b$

Borda scores

a	b	c	d
5	6	8	11

- a is elected

Example

- c and d are withdrawing
- 2 candidates $\{a, b\}$
- 3 voters

2 voters:	$b \succ a$
1 voter:	$a \succ b$

Borda scores

a	b
5	4

- b is elected!
- door wide open for manipulations
- introduce dummy candidates

Summary

Example

- 4 candidates $\{a, b, c, d\}$
- 27 voters

5 voters:	$a \succ b \succ c \succ d$
4 voters:	$a \succ c \succ b \succ d$
2 voters:	$d \succ b \succ a \succ c$
6 voters:	$d \succ b \succ c \succ a$
8 voters:	$c \succ b \succ a \succ d$
2 voters:	$d \succ c \succ b \succ a$

Results

- a is the plurality with runoff winner
- d is the plurality winner
- b is the Borda winner
- c is the CW

What are we looking for?

Democratic method

- always giving a result like Borda
- always electing the Condorcet winner
- consistent w.r.t. withdrawals
- monotonic, separable, incentive to participate, not manipulable
- etc.

Arrow

Framework

- $n \geq 3$ candidates (otherwise use plurality)
- m voters ($m \geq 2$ and finite)
- ballots: ordered list of candidates

Problem

- find all electoral methods respecting a small number of "desirable" principles

Arrow

Principles

- universality
  - the method should be able to deal with any configuration of ordered lists
- transitivity
  - the result of the method should be an ordered list of candidates
- unanimity
  - the method should respect a unanimous preference of the voters
- absence of dictator
  - the method should not allow for dictators
- independence
  - the comparison of two candidates should be based only on their respective standings in the ordered lists of the voters

Arrow's theorem (1951)

Theorem

There is no method respecting the five principles

Borda

- universal, transitive, unanimous with no dictator
- cannot be independent

Condorcet

- universal, independent, unanimous with no dictator
- cannot be transitive

Sketch of proof

Decisive coalitions

$V \subseteq N$ is decisive for (a, b) if

$$a \succ_i b \text{ for all } i \in V \Rightarrow a \succ b$$

Almost decisive coalitions

$V \subseteq N$ is almost decisive for (a, b) if

$$\left.\begin{array}{r} a \succ_i b \text{ for all } i \in V \\ b \succ_j a \text{ for all } j \notin V \end{array}\right\} \Rightarrow a \succ b$$

Lemma 1

Lemma

If V is almost decisive over some ordered pair (a, b), it is decisive over all ordered pairs.

Sketch of proof

Take $\{a, b, x, y\}$ and use universality to obtain:

$$V : x \succ a \succ b \succ y$$
$$N \setminus V : x \succ a, b \succ y, b \succ a$$

The relative position of x and y for $N \setminus V$ is not specified.
Unanimity implies $x \succ a$ and $b \succ y$.
Almost decisiveness of V for (a, b) implies $a \succ b$.
Transitivity implies $x \succ y$.
Independence implies that this does not depends on the position of a and b.
Hence V is decisive for (x, y). □

Lemma 2

Lemma

If V is decisive and $|V| > 1$, some proper subset of V is decisive

Sketch of proof

Partition V into $V1$ and $V2$.
Take $\{x, y, z\}$ and use universality to obtain:

$$V1 : x \succ y \succ z$$
$$V2 : y \succ z \succ x$$
$$N \setminus V : z \succ x \succ y$$

Decisiveness of V implies $y \succ z$.
If $x \succ z$ then $V1$ is almost decisive for (x, z) and use Lemma 1 to conclude.
Otherwise, we have $z \succsim x$, so that $y \succ x$. This implies that $V2$ is almost decisive for (y, x) and use Lemma 1 to conclude. □

Proof

Proof

- unanimity implies that N is decisive
- since N is finite, the iterated use of Lemma 2 leads to the existence of a dictator □

Analysis of principles

Principles

- Unanimity: no apparent problem
- Absence of dictator: minimal requirement of democracy!
- Universality: a group adopting functioning rules that would not function in "difficult situations" could be in big trouble!

Unimodal preferences

Consequences

- if the preferences of all voters are unimodal with the same underlying axis
- Condorcet's paradox cannot occur

Problem

- not true if more than one axis!

Independence

Interpretation

- no intensity of preference considerations
  - I "intensely" or "barely" prefer a to b
  - practice: manipulation, interpersonal comparisons?
- no consideration of a third alternative to rank order a and b

Borda and independence

Example

- 4 candidates $\{a, b, c, d\}$
- 3 voters

2 voters:	$c \succ a \succ b \succ d$
1 voter:	$a \succ b \succ d \succ c$

Borda scores

a	b	c	d
5	6	8	11

- a is elected

Example

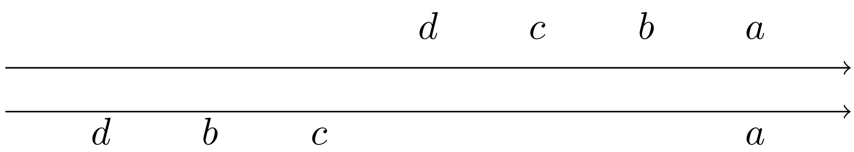
- 4 candidates $\{a, b, c, d\}$
- 3 voters

2 voters:	$c \succ a \succ b \succ d$
1 voter:	$a \succ c \succ b \succ d$

Borda scores

a	b	c	d
5	9	4	12

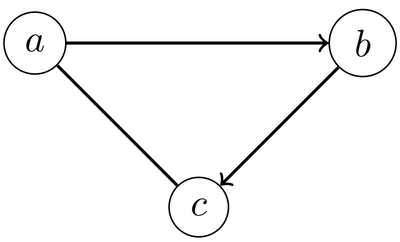
- c is elected



Transitivity

Remarks

- maybe too demanding if the only problem is to elect a candidate
  - absence of circuit is sufficient
- but... guarantees consistency



- in $\{a, c\}$, the maximal elements are a and c
- in $\{a, b, c\}$, the maximal element is a

Relaxing transitivity

From weak orders to…

- semi-orders and interval orders
  - no change (if more than 4 candidates)
- transitivity of strict preference
  - oligarchy: group O of voters st

$$a \succ_i b, \forall i \in O \Rightarrow a \succ b,$$
$$\exists i \in O : a \succ_i b \Rightarrow Not[b \succ a]$$

- absence of circuits
  - some voter has a veto power

$$a \succ_i b \Rightarrow Not[b \succ a]$$

Underlying message

Naive conclusion

- despair

But…

- the existence of an "ideal" method would be dull!
  - analyze the pros and cons of each method
  - beware of "method-sellers"
- a group is "more complex" than an individual

Extensions

Impossibility results

- logical tension between conditions
- Arrow
- Gibbard-Sattherthwaite
  - all "reasonable methods" may be manipulated (more or less easily or frequently)
- Moulin
  - no separable method can be Condorcet
  - no Condorcet method can give an incentive to participate
- Sen
  - tensions between unanimity and individual freedom

Paretian Liberal Paradox

Remarks

- obvious tensions between the majority principle and the respect of individual rights
- tensions between the respect of individual rights and the unanimity principle

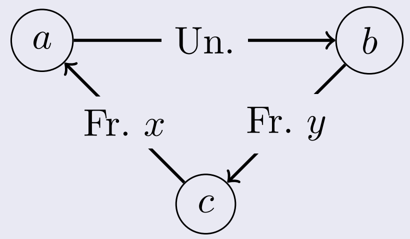
Theorem (Sen, 1970)

The combination of unanimity, universality and respect of individual rights implies problems

Sen: Paretian liberal paradox

Example

- 2 (male) individuals on a desert island
  - x the Puritan
  - y the Liberal
- a pornographic brochure
- 3 social states
  - a: x reads
  - b: y reads
  - c: nobody reads
- preferences
  - x: $c \succ a \succ b$
  - y: $a \succ b \succ c$



Extensions

Characterization results

- find a list of properties that a method is the only one to satisfy simultaneously
  - Borda
  - Copeland
  - Plurality

Example of result

- neutral, anonymous and separable method are of Borda-type (Young, 1975)

Extensions

Analysis results

- find a list of desirable properties
  - not an easy task!
- fill up the methods / properties table

Ideally

- characterization results will use intuitive axioms
- analysis results will lead to characterization and/or impossibility results

Extensions

Other aspects

- institutional setting
- welfare judgments
  - voting on taxes
- direct vs indirect democracy
- electoral platforms
- paradox of voting (why vote?)

Why vote?

Voting has a cost

- I have to go to the polling station
- I had rather go fishing

Analysis

- the probability that my vote will change the results is nil
- why should I bother?

Models

- economic explanations
- sociological explanations
    - not fully convincing on their own

Ostrogorski's Paradox

Representative democracy

- you vote for a party that has a position on several issues (economic, social, international, etc.)
- no party can be expected to represent your opinion on every issue
- why vote for parties instead of issues?

Ostrogorski's Paradox

Example

- 5 voters, 2 parties (X and Y), 3 issues

	issue 1	issue 2	issue 3
voter 1	X	Y	Y
voter 2	Y	X	Y
voter 3	Y	Y	X
voter 4	X	X	X
voter 5	X	X	X

- on issue 1, voter 1 agrees with party X

- if each voter votes for the party with which he agrees on a majority of issues, Y wins
- the loosing party X agrees with a majority of voters on each issue!

Anscombe's paradox

Example

- 5 voters, 2 parties (X and Y), 3 issues

	issue 1	issue 2	issue 3	
voter 1	X	X	Y	minority
voter 2	Y	Y	Y	minority
voter 3	Y	X	X	minority
voter 4	X	Y	X	majority
voter 5	X	Y	X	majority
result	X	Y	X	

- on issue 1, voter 1 agrees with party X

Analysis

- vote on issues
- a majority of voters can be frustrated on a majority of issues!

Direct and undirect democracy

Referendum paradox

- direct democracy: referendum
- indirect (representative) democracy: parliament

Paradox

- these two methods can lead to different results...
- even if each MP votes according to the opinion of the majority of his electors

	MP1	...	MP167	MP168	...	MP200
Yes	7 000	...	7 000	15 000	...	15 000
No	8 000	...	8 000	0	...	0

- "No" wins in assembly (167/200 = 83%)
- "Yes" wins in referendum (55%)