Monotonicity of ‘ranking by choosing’ (SCW, 2004, 23:249–273) Erratum: proof of Proposition 2, page 266

Denis Bouyssou

8 June 2006

Marc Pirlot and Brice Mayag have brought to my attention the fact that the published proof of Proposition 2 in my paper Monotonicity of ‘ranking by choosing’ (SCW, 2004, 23:249–273) is rather imprecise. Although what has been published is correct, here is a more direct and hopefully clearer version of this proof.

Proposition 2 (Covering compatibility and $Aizerman$)
Let $\mathcal{S}$ be a local, neutral and monotonic choice procedure satisfying $Aizerman$. If $\mathcal{S}$ refines UC then $\succsim_{\mathcal{S}}$ is not monotonic.

Proof
The claim will be proved if we can show that, for all neutral and monotonic choice procedures refining UC and satisfying $Aizerman$, there is a comparison function π such that $x \in \mathcal{S}(X, \pi)$ $y \notin \mathcal{S}(X, \pi)$ and $y \in \mathcal{S}(X, \pi^{x\uparrow})$. Indeed, this will imply that $x \succ_{\mathcal{S}}(\pi)\ y$ and $y \succsim_{\mathcal{S}}(\pi^{x\uparrow})\ x$, violating monotonicity. The following example suffices.

Example 9
Let $X = \{a, b, c, d, e\}$. Consider the tournament T on X defined by:

$$
\begin{gathered}
aTd, aTe, \\
bTa, \\
cTa, cTb, \\
dTb, dTc, dTe, \\
eTb, eTc.
\end{gathered}
$$

We have $UC(X, T) = \{a, c, d\}$ and aTd, dTc and cTa. Therefore, since $\mathcal{S}$ refines UC, we have $\mathcal{S}(X, T) \subseteq \{a, c, d\} \subseteq X$. Since $\mathcal{S}$ satisfies $Aizerman$, $\mathcal{S}(\{a, c, d\}, T) \subseteq \mathcal{S}(X, T)$.

Because $\mathcal{S}$ is local and neutral, we know that $\mathcal{S}(\{a, c, d\}, T) = \{a, c, d\}$. Hence we must have $\mathcal{S}(X, T) = \{a, c, d\}$.

Consider now the tournament V identical to T except that aVc. Using the same reasoning as above, it is easy to check that $\mathcal{S}(X, V) = UC(X, V) = \{a, b, d\}$. Hence b enter the choice set while a is improved and $\succsim_{\mathcal{S}}$ is not monotonic.

$\square$