Minimax Strikes Back

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ABSTRACT

Deep Reinforcement Learning reaches a superhuman level of play in many complete information games. The state of the art algorithm for learning with zero knowledge is AlphaZero. We take another approach, the Descent framework, which uses a different, Minimax-based, search algorithm, as well as different learning targets and that does not use a policy. We show that for multiple games it is much more efficient than the reimplementation of AlphaZero: Polygames. It is even competitive with Polygames when Polygames uses 100 times more GPU (at least for some games). One of the keys to the superior performance is that the cost of generating state data for training is approximately 296 times lower with the Descent framework. With the same reasonable ressources, the Descent framework without reinforcement heuristic is 7 times faster than Polygames and much more than 30 times faster with reinforcement heuristic.

KEYWORDS

Minimax, Reinforcement Learning, Zero Learning, Games, Tree Search

ACM Reference Format:
Quentin Cohen-Solal and Tristan Cazenave. 2023. Minimax Strikes Back. In *Proc. of the 22nd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2023), London, United Kingdom, May 29 – June 2, 2023*, IFAAMAS, 9 pages.

1 INTRODUCTION

Monte Carlo Tree Search (MCTS) [3, 12, 18] and its refinements [6, 7, 23] are the current state of the art in complete information games search algorithms. Historically, at the root of MCTS were random and noisy playouts. Many such playouts were necessary to accurately evaluate a state. Since *AlphaGo* [23] and *AlphaZero* [24] it is not the case anymore. More precisely, strong policies and evaluations are now provided, by neural networks that are trained with Reinforcement Learning. These evaluations are stronger and faster to calculate.

In AlphaGo and its descendants the policy is used as a prior in the PUCT bandit to explore first the most promising moves advised by the neural network policy. In addition, the evaluations replace the playouts. Moreover, in AlphaGo, before the reinforcement learning process, data from matches played between humans are used during a supervised learning process. It is not the case with the latest version, i.e. AlphaZero, where a very high level of play can be achieved without the use of knowledge. For example, AlphaZero has surpassed the level of the program Stockfish in Chess (Grandmaster level) [24]. In this paper, we advocate that with reinforcement learning, MCTS might not be the best algorithm anymore. Minimax algorithms are serious challengers when equipped with a strong evaluation function from reinforcement learning.

In this paper, we show that minimax-based algorithms are competitve with MCTS-based algorithms, and even superior for at least many games. More precisely, we make a comparison between a recent Minimax-based reinforcement learning framework, called *Descent*, with AlphaZero, the state of the art of reinforcement learning, which had not been done before. Unlike the AlphaZero approach based on MCTS, the search algorithm of the Descent framework for confrontations is a variant of Minimax, called *Unbounded Minimax*.

The remainder of the paper is organized as follows. The second section deals with related work. In particular, the section 2.2 presents the two zero learning algorithms we use in this paper (the Descent framework and the AlphaZero framework, reimplemented in the *Polygames* program). This section also compares their characteristics. Sections 3 experimentally compares the two learning algorithms. Section 4 is a discussion about the results and Section 5 concludes the article.

Important note: Details of experiments, removed for a reason of space, are described in Technical Appendix, which includes the description of the used games, the used neural network architectures, the learning parameters, algorithms, other experiments, and additional performance curves and tables.

2 BACKGROUND

2.1 Related Work

There are many search algorithms for perfect information games. The two standard algorithms are Monte Carlo Tree Search and Minimax with alpha-beta pruning.

MCTS has its roots in computer Go [12]. It was theoretically defined with the UCT algorithm [18] that converges to the Nash equilibrium and uses a well defined bandit, Upper Confidence Bounds (UCB), which minimizes the cumulative regret at each node [2]. Theoretical bandits were soon replaced with empirical bandits, giving better results. First the RAVE algorithm [14] improved greatly on UCT for the games of Go and Hex [9]. A later refinement is GRAVE that improves on RAVE for many different games [6] and is used for General Game Playing, for example in Ludii [4].

MCTS was combined with neural networks in AlphaGo, surpassing professional level in the game of Go [23]. The search algorithm used in AlphaGo is MCTS with PUCT, a bandit that uses the policy given by the neural network to bias the moves to explore. Later AlphaGo was redesigned to learn from zero knowledge, leading to AlphaGo Zero [25]. It was then applied to other games, namely Shogi and Chess, with the more general AlphaZero program [24]. Many teams have replicated the AlphaZero approach for Go and for other games: Elf/OpenGo [27], Leela Zero [22], Crazy Zero by

Proc. of the 22nd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2023), A. Ricci, W. Yeoh, N. Agmon, B. An (eds.), May 29 – June 2, 2023, London, United Kingdom.

Rémi Coulom, KataGo [28], Galvanise Zero [13] and Polygames [8].

As we use Polygames as a sparring partner, we will give more details about it. Polygames (MIT License) replicated the AlphaZero approach and applied it to many games with success. There are multiple innovations in Polygames. It can train neural networks with an architecture independent of the size of the board. To do so it uses a fully convolutional policy, meaning that there is no dense layer between the last convolutional planes and the policy. The value head is also independent of the size of the board since it uses global average pooling before the dense layers connected to the evaluation output neuron. It is much more difficult to train a network for Hex 19 (board size 19×19) or Havannah 10 than training it on a smaller size board. Polygames did succeed in these games by scaling its neural networks trained on smaller sizes to the difficult board sizes. It played on the Little Golem game server and beat the best players at these games that were considered too difficult for Zero Learning. Other innovations of Polygames include a pool of neural networks during self-play matches in order to avoid catastrophic forgetting.

The other kinds of algorithms used in computer games are the $\alpha\beta$ family of algorithms, whose consecration took place with Deep Blue [5], the first program beating a world chess champion. $\alpha\beta$ dominated the field of perfect information games until the advent of MCTS in 2006. Still, many current strong Chess programs use $\alpha\beta$ [15]. The latest versions are combined with NNUE neural networks [21]. There has been a lot of research on the optimizations of the $\alpha\beta$ algorithm [20]. Many of them deal with move ordering since move ordering can drastically improve the search time of $\alpha\beta$ based programs [17]. In [26], $\alpha\beta$ was also combined with a policy within a reinforcement learning architecture and and it reaches a good level at Hex (the policy is used to prune actions in order to reduce the branching factor).

The search algorithms we use to learn and play games are close to Unbounded Best-first Minimax Search [19]. There is very little study on this algorithm and it seems little or not applied in practice, except in the work of [10]. In that work, variants and improvements of Unbounded Minimax are proposed with several complementary techniques of zero learning that do not require the use of policies. These zero techniques still manage to achieve a state-of-the-art level of play at the game of Hex (size 11, 13, and 19). In the context of the experiments of [10], the proposed approach is the best zero learning approach not using a policy: in particular, replacing the used variant of Unbounded Minimax, called *Descent*, by Unbounded Minimax, by $\alpha\beta$ or by MCTS (with UCT) gives less good results. Moreover, in the experiments of that work, another variant of Unbounded Minimax, called *Unbounded Minimax with Safe decision*, is shown best than the Unbounded Minimax or $\alpha\beta$ for the confrontations ("What is the best search algorithm for winning a game?" is a different question than "What is the best search algorithm to learn faster?"). Note that, it has been proved that, with enough time, Descent and Unbounded Minimax find the best game strategy [11].

2.2 Deep Reinforcement Learning Algorithms Compared in this Paper

We detail in this section the two zero learning frameworks used in the experiments of this article.

2.2.1 Polygames Learning Algorithm. Polygames uses its search algorithm, MCTS with PUCT, to generate matches, by playing against itself. It uses the information from these matches to update its neural network. This neural network is used by the search algorithm to evaluate states by a value and by a policy (i.e. a probability distribution on the actions playable in that state). For each finished match, the network is trained to associate with each state of the state sequence of this match the result of the end of that match (which is -1 for a loss, 0 for a draw, and $+1$ for a win). It is also trained, at the same time, to associate with each state a particular policy. In that "target" policy, the probability of an action is proportional to $N^{1/\tau}$ where N is the number of times this action has been selected in the search from that state and τ is a parameter. Note that, during each Polygames learning process, several games are performed in parallel and their evaluations are batched in order to be evaluated in parallel on the GPU.

2.3 More Details about AlphaZero/Polygames

During a learning process using AlphaZero/Polygames, as long as there is time left, a new match phase is performed. A phase consists of a match against oneself, where in each turn the move to be played is decided after carrying out a search with MCTS + PUCT. The algorithm of MCTS is similar to Unbounded Minimax. The first main difference is that the value of a state is not the minimax value in the partial game tree but the average of the leaves in the subtree starting from that state. The second main difference is that the tree

Function `descent_iter`(s, S, T, f_θ, f_t)
- **if** `terminal`(s) **then**
  - $S \leftarrow S \cup \{s\}$
  - $v(s) \leftarrow f_t(s)$
- **else**
  - **if** $s \notin S$ **then**
    - $S \leftarrow S \cup \{s\}$
    - **foreach** $a \in$ `actions`(s) **do**
      - **if** `terminal`$(a(s))$ **then**
        - $S \leftarrow S \cup \{a(s)\}$
        - $v(s,a) \leftarrow f_t(a(s))$
        - $v(a(s)) \leftarrow v(s,a)$
      - **else**
        - $v(s,a) \leftarrow f_\theta(a(s))$
  - $a_b \leftarrow$ `best_action`(s)
  - $v(s,a_b) \leftarrow$ `descent_iter`$(a_b(s), S, T, f_\theta, f_t)$
  - $a_b \leftarrow$ `best_action`(s)
  - $v(s) \leftarrow v(s,a_b)$
- return $v(s)$

Function `descent`$(s, S, T, \tau, f_\theta, f_t)$
- $t =$ `time`$()$
- **while** `time`$() - t < \tau$ **do** `descent_iter`(s, S, T, f_θ, f_t)
- return S

Algorithm 1: *Descent* algorithm (symbol definitions in Table 1).

Symbols	Definition
actions (s)	action set of the state s for the current player
terminal (s)	true if s is an end-game state
$a(s)$	state obtained after playing the action a in the state s
time ()	current time in seconds
S	states of the partial game tree (and keys of the transposition table T)
T	transposition table (contains state labels as v or P)
$P(s)$	target policy of state s computed from the search data
D	learning data set
τ	search time per action
$t_{\max}$	chosen total duration of the learning process
$v(s)$	value of state s from the game search
$v(s,a)$	value obtained after playing action a in state s
$f_\theta(s)$	adaptive evaluation function (of non-terminal game tree leaves ; first player point of view)
$f_t(s)$	evaluation of terminal states, e.g. gain game (first player point of view)
action_selection(s, S, T)	decides the action to play in the state s depending on the partial game tree, i.e. on S and T
update(f_θ, D)	updates the parameter θ of f_θ in order for $f_\theta(s)$ is closer to v for each $(s,v) \in D$

Table 1: Index of symbols

is constructed not in choosing states of higher value, but states optimizing the value plus an exploration term depending on the policy of the neural network and the number of selection of actions during the search. After the match, the match state sequence data is added to the previous data (only the most recent data points are kept). Every K matches (K is a parameter), training is performed from a sample of this data set. The main part of this algorithm is described in Algorithm 2.

2.3.1 Descent Standard Learning Algorithm. The learning framework of [10] is based on a variant of Unbounded Minimax called *Descent*, which consists in exploring the sequences of actions until terminal states. In comparison, Unbounded Minimax and MCTS explore the sequences of actions only until reaching a leaf state. An iteration of Descent thus consists in a deterministic complete simulation of the rest of the game. The exploration is thus deeper while remaining a best-first approach. This allows the values of terminal states to be propagated more quickly to (shallower) non-terminal states. Descent is formally described in Algorithm 1.

Unlike Polygames, the learned target value of a state is not the end-game value but its minimax value in the partial game tree built during the match. This information is more informative, since it contains part of the knowledge acquired during the previous matches. In addition, contrary to Polygames, learning is carried out for each state of the partial game tree constructed during the searches of the match (not just for each state of the states sequence of the played match). In other words, with Polygames, there is one learning target per search whereas with the Descent framework, there are several learning targets per search. Therefore, there is no loss of information with Descent: all of the information acquired during the search is used during the learning process. As a result, the Descent framework generates a much larger amount of data for training from the same number of played matches than AlphaZero / Polygames. Thus, unlike the state of the art which requires to generate matches in parallel to build its learning dataset, this approach does not require the parallelization of matches (and the parallelization of Descent is not done in the experiments of this article).

During confrontations, the used search algorithm is Unbounded (Best-First) Minimax with Safe decision, denoted UBFM$_s$. It is a variant of Unbounded Best-First Minimax search which performs the same search. More precisely, it iteratively extends the best sequence of actions in the partial game tree (i.e. it adds at each iteration the leafs of the principal variation of the partial game tree). Note that, on the one hand, the best action sequence generally changes after each extension. On the other hand, in general, the worse the evaluation function is, the wider the exploration is. The difference between them is as follows: with Unbounded Minimax, the action to play, chosen after the last search, is the one with the best value, while with this variant, the chosen action is the one that is the most explored.

Finally, this approach is optionally based on a *reinforcement heuristic*, that is to say an evaluation function of terminal states more expressive than the classical gain of a game (i.e. +1 / 0 / −1). The best proposed general reinforcement heuristics in [10] are *scoring* and the *depth heuristic* (the latter favoring quick wins and slow defeats).

Note that this approach does not use a policy, so there is no need to encode actions. Consequently, this avoids the learning performance problem of neural networks for games with large number of actions (i.e. very large output size). In addition, although the Descent framework does not performed matches in parallel, it batches all the child states of an extended state together to be evaluated at one

Function `AlphaZero_main_algorithm(`$t_{\max}$`)`
- $t_0 \leftarrow$ `time()`
- **while** `time()` $- t_0 < t_{\max}$ **do**
  - **for** $k \in \{1, \ldots, K\}$ **do**
    - $s \leftarrow$ `initial_game_state()`
    - $S \leftarrow \emptyset$
    - $T \leftarrow \{\}$
    - $G \leftarrow \{s\}$
    - **while** ¬`terminal(`s`)` **do**
      - $S, T \leftarrow$ `mcts(`s, S, T, f_θ, f_t`)`
      - $a \leftarrow$ `action_selection(`s, S, T`)`
      - $s \leftarrow a(s)$
      - $G \leftarrow G \cup \{s\}$
    - $D \leftarrow \{(s', f_t(s), P(s)) \mid s' \in G\}$
  - `update(`f_θ, D`)`

Algorithm 2: Main algorithm of AlphaZero (see Table 1 for the definitions of symbols ; K is the number of matches performed between two updates, some of these matches are executed in parallel ; G is the sequence of states of the current match).

time on the GPU [10] (this is done with Descent and Unbounded Minimax).

2.4 More Details about the Descent Framework

During a learning process using the Descent Framework, as long as there is time left, a new match phase is performed. A match phase consists of a match against oneself, where in each turn the move to be played is decided after carrying out a search with Descent. The move to be played after the search is chosen according to an action selection method, depending on the result of the search. In these experiments, the action selection method used is the *ordinal law* [10] with the exploitation parameter ϵ' choosing at random uniformly between 0 and 1, each time a new action must be decided. After each match phase, the data from the associated partial game tree is added to the previous data (here, only the data of the last 100 matches are kept). Then, a training phase is carried out from a sample of this data set. Specifically, *smooth experience replay* is used [10]. The main part of this algorithm is described in Algorithm 3. The full formalization is described in [10].

Function `Descent_main_algorithm(`$t_{\max}$`)`
- $t_0 \leftarrow$ `time()`
- **while** `time()` $- t_0 < t_{\max}$ **do**
  - $s \leftarrow$`initial_game_state()`
  - $S \leftarrow \emptyset$
  - $T \leftarrow \{\}$
  - **while** $\neg$`terminal(`s`)` **do**
    - $S, T \leftarrow$`descent(`s, S, T, f_θ, f_t`)`
    - $a \leftarrow$`action_selection(`s, S, T`)`
    - $s \leftarrow a(s)$
  - $D \leftarrow \{(s, v(s)) \mid s \in S\}$
  - `update(`f_θ, D`)`

Algorithm 3: Main algorithm of the Descent framework (see Table 1 for the definitions of symbols).

3 COMPARISON OF ZERO REINFORCEMENT LEARNING ALGORITHMS

In this section, we experimentally compare the two learning algorithms Polygames / AlphaZero (see Section 2.2.1) and Descent (see Section 2.3.1). First, in the context of 8 games, we compare the data efficiency of the two algorithms, i.e. the amount of data generated during the self-play matches which are learned in order to self-improve. Second, we compare the win performances of the two algorithms in the same context (the same games and the algorithms also use the same resources). They are rated against MCTS. Then, a longer training is performed on Hex 13 and the algorithms are evaluated against Mohex 2.0 [16], the best publicly available Hex program. Finally, the Polygames networks, that have won numerous medals during the TCGA 2020 tournament, confront Descent networks that have used drastically less computational power for their learning processes. In each of these experiments, Descent is strongly better than Polygames.

layer #	C-network	R_1-network	R_2-network
1	conv. + ReLU	convolution	convolution
$\cdots$	conv. + ReLU	2 res. blocks	8 res. blocks
$N-2$	conv. + ReLU	1×1 conv.	dense + ReLU
$N-1$	dense + ReLU	dense + ReLU	dense + ReLU
N	dense layer	dense layer	dense layer

Table 2: Description of 3 neural architectures of value networks, called C-network, R_1-network, and R_2-network. Each residual block is composed of a ReLU followed by a convolution followed by a ReLU followed by a convolution followed by a ReLU. Output contains one neuron. Other parameters are: kernel is 3×3, filter number is F, neuron number in dense layers is D, with padding for R_i-network and without padding for C-network.

Game	F	D
Surakarta	132	845
Othello	132	477
Breakthrough	132c	477
Hex 13×13	132	155
Connect6	132	65
Outer-Open-Gomoku	132	111
Havannah 8	132	111
Havannah 10	132	65

Table 3: The filter numbers in convolutional layers and the numbers of neurons in dense layers for the R_2-networks for each game used with Descent for the 8 games.

3.1 Technical Details

We expose in this section the technical details common to the experiments of Sections 3.2, 3.3, and 3.4. Recall that full details of experiments of this paper are in Technical Appendix (the supplementaries document).

3.1.1 Parameters. For each learning process with Descent, the batch size of the stochastic gradient descent B is 3000, *smooth experiment replay* is used with the following parameters: $\mu = 100$ and $\delta = 3$. The neural network architecture is the same for each game: a R_2-network (see Table 2). The number of weights in each architecture is of the order of $5 \cdot 10^6$. However, this implies that the number of filters F and number of dense neurons D are different for each game. The corresponding numbers are described in Table 3.

The action distribution used during the training process is the ordinal law [10]. It is used with a uniform random variable between 0 and 1 as exploration parameter (the variable value changes after each search performed for determining the next action to play; therefore no simulated annealing is used).

Network architectures used for Polygames are adaptations of the architecture being used with Descent, in order to add a policy while keeping an analogous number of weights in the neural network (see the Supplementaries document for the details).

Evaluations of Section 3.3 are performed against the basic MCTS based on UCT (it uses 160 rollouts). For each learning process (i.e.

each learned neural network), each evaluation consists of 400 games (200 in first player and 200 in second player).

3.1.2 Computational Resources. In this section, we present the used computational resources for the experiments of this paper.

For the performed training runs and the performed confrontations, we use the following hardware: GPU Nvidia Tesla V100 SXM2 32 Go, 2 to 10 CPU (processors Intel Cascade Lake 6248 2.5GHz) on RedHat. There is an exception, for the performed confrontations (in Section 3.5) against Polygames tournament networks, we use the following hardware: GeForce GTX 1080 Ti, 2 to 8 CPU (Intel(R) Xeon(R) CPU E5-2603 v3 1.60GHz) on Ubuntu 18.04.5 LTS.

Descent programs (Descent learning, Unbounded Minimax, ...) are coded in Python (using tensorflow 1.15). Games and Search in Polygames are coded in C/C++. For confrontations, Polygames num_actor parameter is 8 (threads doing MCTS).

3.2 Comparison of Generated Learning Data

In this section, we experimentally compare the number of state data, the number of state evaluations, and the number of neural network evaluations performed during a Descent training and a Polygames training having the same duration (15 days). In total, 8 trainings were carried out with Descent and 5 with Polygames for each of the following games: Connect6, Outer-Open-Gomoku, Hex 13, Havannah 8, Havannah 10, Othello, Breakthrough, and Surakarta.

We start by comparing the number of evaluations. In average, the neural network evaluations of Descent is 12.7 times smaller than that of Polygames. In addition, the average number of state evaluations of Descent is 2.5 times smaller than that of Polygames. In other words, Polygames is more efficient to perform evaluations. However, this is not an intrinsic characteristic, because this difference is mainly explained by two facts. Firstly, Descent is coded in Python and searches and game mechanisms for Polygames are coded in C/C++ , which allows it to be 2 to 5 times faster (depending on the game). Second, Polygames performs many matches in parallel, whereas Descent, in its implementation, is purely sequential. This thus makes it possible for Polygames to carry out matches in parallel, and therefore to increase the number of evaluations for the same period of time. The detailed numbers for each game are described in Table 4. In summary, in this experiment, Polygames performs more evaluations but it could be counterbalanced by implementing Descent in C/C++ and/or parallelizing it (i.e. make a fair comparison between Descent and Polygames instead of this current comparison which is to Polygames' advantage).

Now we compare *learned states*: the number of state data used during the learning processes. Descent generates 296 times more numerous learned states than Polygames (despite performing fewer evaluations as we saw in the previous paragraph). This is due to the fact that Descent uses tree learning: it learns all the data generated during the search, i.e. it learns all the partial game tree build during the search. By contrast, Polygames / AlphaZero only learns a synthesis of this search, namely a policy and a state value for the state analyzed during the search. Note that since determining a policy for a state requires that its children be sufficiently explored, it does not seem possible to learn a policy for each state of the search (i.e. for each state of the partial game tree), that is to say to perform tree learning for the policy with Polygames. The same remark applies for the state value. Indeed, since the learning target for the state value is the endgame value, it would be the same for all the states of the tree, which is more likely to negatively impact the learning in creating over-fitting than improving learning. In other words, naively modifying AlphaZero / Polygames to use a terminal tree learning and tree learning for the policy in order to decrease the cost of data generation should not improve performance. However, it is possible to change the learning target, i.e. replace the endgame value by the search state value and thus to perform classic tree learning. This has been studied in the context of MCTS without policy, and the results are much worse than with Descent and tree learning [10].

In conclusion, the cost of state data generation is 50 to 700 times better with Descent than with Polygames depending on the game (296 times better on average over the tested games), despite the fact that it performs 2.5 times fewer states evaluations. Moreover, recall that using a language other than Python with Descent would further improve its performance, most likely by a factor of at least 2. Note also that the matchs with Descent are not parallelized (unlike AlphaZero / Polygames), and parallelizing them would also increase the number of data.

3.3 Win comparison with Same Resources

In this section, we compare the learning performances of the Descent framework with the learning performances of Polygames, with respect to the win percentages against MCTS.

In particular, we compare the two algorithms using the *gain* of using Descent rather than Polygames, which is the difference in their win performances:

Definition 1. The *gain of using the algorithm A rather than the algorithm A′* is $\frac{1}{2}\left((w_A - l_A) - (w_{A'} - l_{A'})\right)$ where w_A (resp. l_A) is the win (resp. loss) percentage of A.

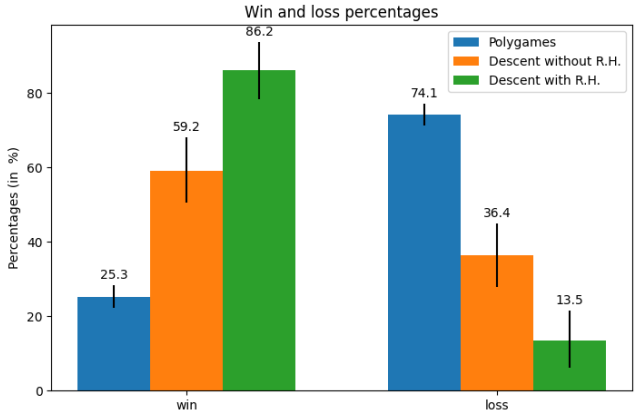
We denote by P the Polygames algorithm, D the Descent algorithm (without reinforcement heuristic), and $\mathrm{D_{RH}}$ the Descent algorithm with reinforcement heuristic.

Several trainings, each during 15 days, have been performed for each of the following games: Surakarta, Hex 13, Connect6, Outer-Open-Gomoku, Breakthrough, Othello 8, Havannah 8, Havannah 10. In total, 5 repetitions were performed with Polygames and 8 repetitions were performed with Descent for each game (4 repetitions without reinforcement heuristic and 4 repetitions with reinforcement heuristic). As the number of repetitions is small, we use the following advanced statistical evaluation procedure: *stratified bootstrap confidence interval* [1] which allows one to evaluate learning processes over several tasks even with a low number of repetitions.

The global performances against a 160-rollouts MCTS with UCT (without any knowledge nor learned policy) after the training of the Descent framework and respectively of Polygames are described in Figure 1. The details for each of the 8 games are described in Figure 2. The curves describing the evolution of the performances of the two algorithms throughout the training are described in Figure 3.

In conclusion, the performances of Descent are much better than those of Polygames. The performance superiority of Descent is even more marked when a reinforcement heuristic is used (we already knew that reinforcement heuristic is an improvement of Descent [10]). In particular, the gain of using Descent without reinforcement

Figure 1: Results in percentage of UBFM_s (resp. Polygames) using the learned neural nets from the Descent framework (resp. Polygames) with 15 days of training against MCTS with UCT. Their stratified bootstrap confidence intervals are indicated by the black lines. Descent results are detailed in function of the use of a reinforcement heuristic (abbreviated R.H.).



heuristics rather than using Polygames is 35.8% (see Def. 1), and the gain of using Descent with reinforcement heuristic rather than Polygames is 60.75%. Regarding learning speed, Descent without reinforcement heuristic achieves the performance of 15 days of learning with Polygames in only 2 days, i.e. a factor of 7 and Descent with reinforcement heuristic achieves this performance in much less than half a day, i.e. a factor of 30 (see the curve in Figure 3).

3.4 Win Comparison with Same Resources during a Long-Term Learning Process

In this section, we compare again the learning performances of the Descent framework with the learning performances of Polygames with respect to win percentages, but for a longer training (120 days), and only at Hex 13, evaluating them this time against Mohex 2.0 [16], champion program at Hex from 2013 to 2017 at the Computer Olympiads, the strongest hex program that is freely available.

For this, we have continued the learning processes of the previous section carried out on Hex 13 with the Descent framework and with Polygames. Then, we have thus evaluated them against Mohex 2.0, at different stages of their learning processes (an evaluation has been performed approximately every 4 days).

The evolution during the training of the average win percentages against Mohex 2.0 for the Descent framework (with and without reinforcement heuristic) is shown in Figure 4. Descent with reinforcement heuristic goes rather far beyond the level of Mohex 2.0. Descent without reinforcement heuristic does not reach the level of Mohex 2.0 but it still manages to beat it in certain positions. On the contrary, none of the learned Polygames networks (combined with the Polygames search algorithm) has succeeded in winning any match against Mohex 2.0, whatever the evaluation moment during their training process. In other words, the Polygames winning curve is constant and is 0%, with a confidence interval of 0%.

Therefore, in this experiment, learning with *Descent* is also widely better than with Polygames.

3.5 Comparison versus Tournaments Polygames Networks

In this section, we evaluate Descent networks against high level Polygames networks, at Breakthrough, at Othello 8 and 10.

The Polygames networks are those having won at Breakthrough and Othello 10 and finished second at Othello 8 in the TCGA 2020 tournament. They have been trained during 7 days with 100 GPU each. The used Descent networks are trained with only one GPU during 5 days and during 30 days.

The results of the confrontation of the 5-day Descent networks against Polygames networks are described in Figure 5. Although learning with Descent required 100 times less GPU (1 GPU vs. 100 GPU) and lasted slightly less time (5 days vs. a week), the Descent framework has a much better result for all three games.

The results of the confrontation of the 30-day Descent networks against the same Polygames networks are described in Figure 6. This new experience shows that the Descent networks continue to improve.

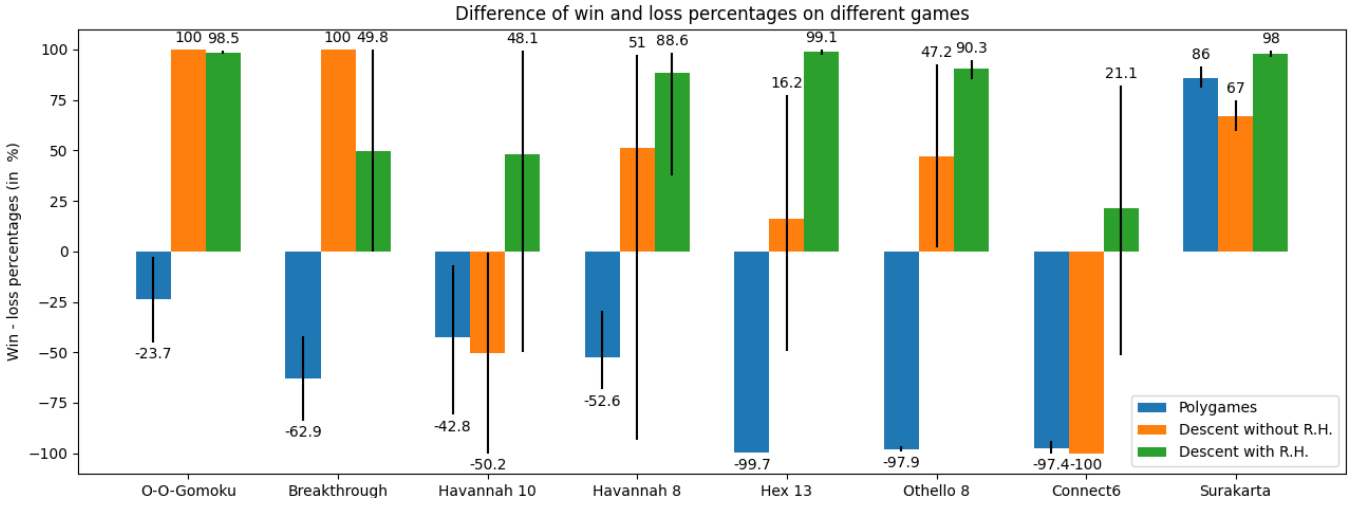
4 FINAL DISCUSSION

In [10], Descent has been compared to other reinforcement learning algorithms without knowledge that do not use learned policy. In particular, using tree learning gives better results than root learning or terminal learning (the AlphaZero / Polygames learning technique for state values). As a reminder, tree learning learns the entire partial search tree of the analyzed state while root learning and terminal learning only learns the target value for the analyzed state. More precisely, in the experiments of [10], the use of tree learning is always better than the use of terminal learning and for almost all 9 tested games, tree learning improves the final win rate by at least 40%. In this new article, we have seen that using tree learning made it possible to generate about 296 times more learning data compared to terminal learning (i.e. AlphaZero / Polygames). This is one of the reasons that allows the Descent framework to obtain better results and in particular to learn much faster, especially at the start of the training process. Note that tree learning could not be applied naturally to AlphaZero / Polygames (see Section 3.2).

The superior performance is not only due to the use of tree learning. In [10], Descent, the central algorithm, gives better results than Unbounded Minimax, itself better than Alpha-Beta and Monte Carlo Tree Search (the search algorithm of AlphaZero / Polygames). More precisely, using Descent with tree learning rather than MCTS with tree learning increases the win rate by at least 40% for all 9 tested games.

This previous study lacked a comparison with the state of the art, which, unlike the previously studied techniques, uses a policy. The question then was: does the results against the weakened state of the art (modified by non-use of a policy) generalize for the full state of the art (i.e. with policy)? This article thus fills this gap and allows one to conclude that with a reasonable hardware and an accessible time, Descent gives results largely better than AlphaZero / Polygames (see Sect 3). In addition, at least in a certain context, the performances of the Descent framework with one GPU are even better than Polygames with 100 GPUs (see Sect 3.5).

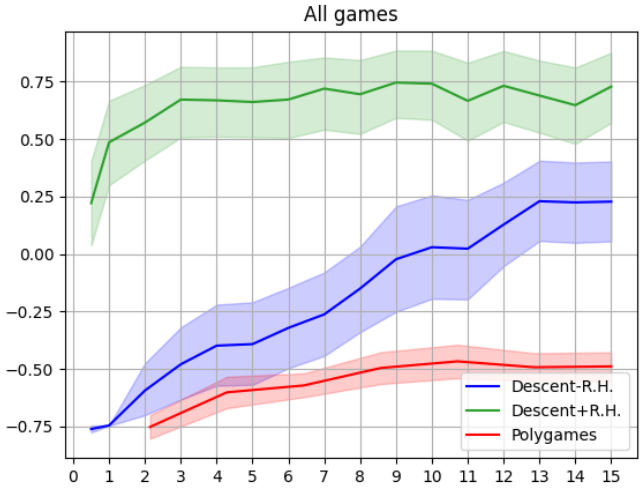
Figure 2: Average win percentages minus loss percentages for the 8 games of UBFM_s (resp. Polygames) using the learned neural nets from the Descent framework (resp. Polygames) with 15 learning days against MCTS with UCT. Descent results are detailed in function of the use of a reinforcement heuristic (abbreviated R.H.). Their bootstrap confidence intervals are indicated by the black lines.



	Connect6	Havannah 10	Havannah 8	Gomoku	Hex 13	Surakarta	Othello	Breakthrough
Learned states	55	64	111	115	359	442	529	693
Neural evaluation	0,02	0,03	0,05	0,04	0,10	0,11	0,12	0,16
States evaluation	0,37	0,30	0,37	0,49	0,65	0,40	0,10	0,49

Table 4: Ratio of Descent data over Polygames data for the same learning time and for different games (average over 5 runs for Polygames and 8 runs for Descent ; data of a run varies by a maximum of ±60% for Polygames and ±20% for Descent). For example, in Connect6, Descent learns 55 times more states, makes 50 times less neural evaluations, and makes 3 times less state evaluations.

Figure 3: Evolution of average win rates minus average loss rates of Descent with reinforcement heuristic (+R.H), of Descent without reinforcement heuristic (-R.H), and of Polygames against MCTS with UCT along the 15 days of training and their stratified bootstrap confidence intervals over the 8 games.



5 CONCLUSION

In [10], a new framework for reinforcement learning without knowledge, called Descent has been proposed. In particular, in [10], the Descent framework has been compared to different standard search algorithms and learning techniques from the literature (which does not use a policy), and it has been shown that the Descent framework obtains much better performance. However, the Descent framework has not been compared against the state of the art of reinforcement learning without knowledge, i.e. MCTS combined with a policy and a dedicated learning technique, the entire architecture being called AlphaZero. This lack of comparison is all the more critical as the use of policy in the AlphaZero framework increases the level of play considerably. A comparison with the state of the art AlphaZero is thus essential to know if the framework of Descent is better or if it is only a useful algorithm when a policy cannot be used.

Therefore, in this paper, we have made the first comparison between the Descent framework and Polygames, a re-implementation of AlphaZero. In particular, we have shown that the Descent framework had much better performances than Polygames.

Recall that the Descent framework is a Minimax approach different in many points from AlphaZero. Their basic differences are as follows. The Descent framework does not use a policy. It is based on Unbounded Minimax variants instead of MCTS. It learns as learning target the (partial) minimax value of states instead of the endgame

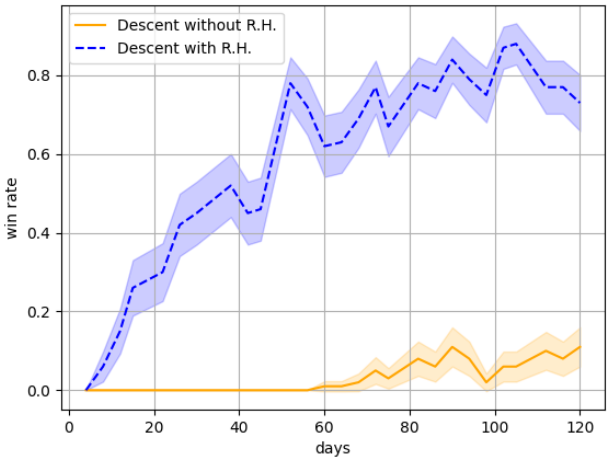


Figure 4: Evolution of average win rates of Descent networks learned with and without reinforcement heuristic, using the search UBFM$_s$**, against Mohex 2.0, during a 120 days learning process (approximately one evaluation every 4 days ; each evaluation consists of 100 matches in first player and 100 other matches in second player). The x axis is in days. Shading is the** 95% **confidence interval.**

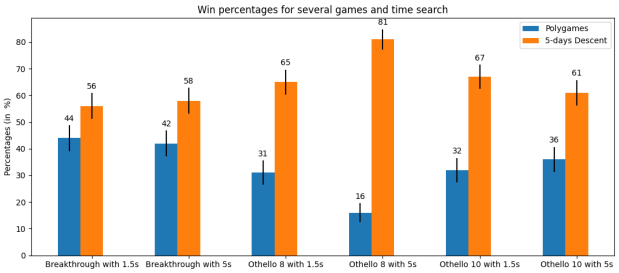


Figure 5: Results of 400 **matches of Descent networks (**5 **days of learning) with** UBFM$_s$ **at Breakthrough and Othello** 8 **and** 10 **against tournaments Polygames networks (see Technical Appendix).**

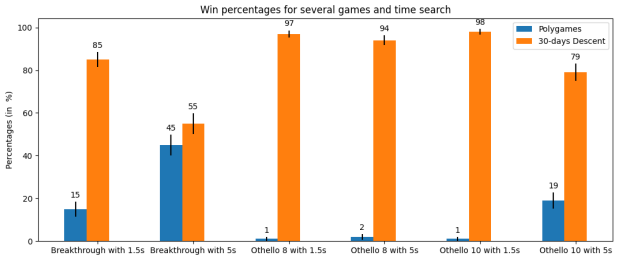


Figure 6: Results of 400 **matches of Descent networks (**30 **days of learning) with** UBFM$_s$ **at Breakthrough and Othello** 8 **and** 10 **against tournaments Polygames networks (see Technical Appendix).**

value. Finally, the Descent framework learns the values of all the states of the search tree built during the match, while AlphaZero only learns the values of the states of the match (i.e. AlphaZero only learns the data of the states sequence of the match, which is a small subset of states of the game search tree).

In our experiments, we have compared and revealed the cost of generating the learning data. Despite the fact that the Descent framework performs less than half as many state evaluations than Polygames (because contrary to Polygames, Descent is programmed in Python and Descent does not performed matches in parallel), the Descent framework generates for the same duration 296 times more learned states than Polygames.

In addition, we have thus performed a comparison of win rates against MCTS for the two zero learning algorithms on a large number of games. Descent obtains much better results than Polygames. In particular, Descent is about 7 times faster than Polygames without reinforcement heuristics and much more than 30 times faster with reinforcement heuristic. Moreover, we have performed another win rates comparison at Hex 13, with 120 days of training, against Mohex 2.0, the best freely available Hex program. Descent has obtained again much better results than Polygames.

Finally, we have made a last comparison at Othello 8, Othello 10 and Breakthrough, against top Polygames networks, having won two gold medals and one silver medal at the 2020 TCGA tournaments. These Polygames networks have been trained for a week with over 100 GPUs. It is again the Descent networks that get the best results on each game, although they have only been trained during 5 days with half a GPU.

In conclusion, all these experiments show that for many games, reinforcement learning with the *Descent* framework is widely more efficient than with Polygames / AlphaZero, at least for accessible learning times and reasonable resources use.

Note to conclude that the Descent framework faced Polygames during the 2020 Computer Olympiad and beat it at Othello 8, Othello 10 and Breakthrough. The Descent framework also beat other re-implementations of AlphaZero at Surakarta and Clobber during this competition. In fact, 5 gold medals were won by the Descent framework for the following games: Othello 10, Breakthrough, Surakarta, Amazons, and Clobber. This was the first time that the same algorithm has won so many gold medals in the same year.

The Descent framework has again competed in the 2021 Computer Olympiad. This time, it won 11 gold medals (Hex 11, Hex 13, Hex 19, Havannah 8, Havannah 10, Othello 8, Surakarta, Amazons, Breakthrough, Brazilian Draughts, Canadian Draughts; there was no competition at Othello 10 and Clobber) and notably beat Polygames at games where they met (Hex 13, Hex 19, Havannah 8, Havannah 10).

The Descent framework has again competed in the 2022 Computer Olympiad. This time, it won 5 gold medals (Surakarta, Breakthrough, Canadian Draughts, Santorini, and Ataxx) and it is still the defending champion for 13 games.

ACKNOWLEDGMENTS

Thanks to Nicholas Sowels for proofreading. This work was granted access to the HPC resources of IDRIS under the allocation 2020-AD011011461, AD011011461R1, AD011011461R2, and 2020-AD-011011714 made by GENCI. This work was supported in part by

the French government under management of Agence Nationale de la Recherche as part of the "Investissements d'avenir" program, reference ANR19-P3IA-0001 (PRAIRIE 3IA Institute).

REFERENCES

[1] Rishabh Agarwal, Max Schwarzer, Pablo Samuel Castro, Aaron C Courville, and Marc Bellemare. 2021. Deep reinforcement learning at the edge of the statistical precipice. *Advances in Neural Information Processing Systems* 34 (2021).

[2] Peter Auer, Nicolò Cesa-Bianchi, and Paul Fischer. 2002. Finite-time Analysis of the Multiarmed Bandit Problem. *Machine Learning* 47, 2-3 (2002), 235–256.

[3] Cameron Browne, Edward Powley, Daniel Whitehouse, Simon Lucas, Peter Cowling, Philipp Rohlfshagen, Stephen Tavener, Diego Perez, Spyridon Samothrakis, and Simon Colton. 2012. A Survey of Monte Carlo Tree Search Methods. *IEEE Transactions on Computational Intelligence and AI in Games* 4, 1 (March 2012), 1–43.

[4] Cameron Browne, Matthew Stephenson, Éric Piette, and Dennis J.N.J. Soemers. 2020. A Practical Introduction to the Ludii General Game System. *Advances in Computer Games. Springer* (2020).

[5] Murray Campbell, A Joseph Hoane Jr, and Feng-hsiung Hsu. 2002. Deep blue. *Artificial intelligence* 134, 1-2 (2002), 57–83.

[6] Tristan Cazenave. 2015. Generalized Rapid Action Value Estimation. In *Proceedings of the Twenty-Fourth International Joint Conference on Artificial Intelligence, IJCAI 2015, Buenos Aires, Argentina, July 25-31, 2015*. 754–760.

[7] Tristan Cazenave. 2016. Playout policy adaptation with move features. *Theor. Comput. Sci.* 644 (2016), 43–52.

[8] Tristan Cazenave, Yen-Chi Chen, Guan-Wei Chen, Shi-Yu Chen, Xian-Dong Chiu, Julien Dehos, Maria Elsa, Qucheng Gong, Hengyuan Hu, Vasil Khalidov, Li Cheng-Ling, Hsin-I Lin, Yu-Jin Lin, Xavier Martinet, Vegard Mella, Jeremy Rapin, Baptiste Roziere, Gabriel Synnaeve, Fabien Teytaud, Olivier Teytaud, Shi-Cheng Ye, Yi-Jun Ye, Shi-Jim Yen, and Sergey Zagoruyko. 2020. Polygames: Improved Zero Learning. *ICGA Journal* 42, 4 (December 2020), 244–256.

[9] Tristan Cazenave and Abdallah Saffidine. 2009. Utilisation de la recherche arborescente Monte-Carlo au Hex. *Revue d'Intelligence Artificielle* 23, 2-3 (2009), 183–202.

[10] Quentin Cohen-Solal. 2020. Learning to Play Two-Player Perfect-Information Games without Knowledge. *arXiv preprint arXiv:2008.01188* (2020).

[11] Quentin Cohen-Solal. 2021. Completeness of Unbounded Best-First Game Algorithms. *arXiv preprint arXiv:2109.09468* (2021).

[12] Rémi Coulom. 2007. Efficient Selectivity and Backup Operators in Monte-Carlo Tree Search. In *Computers and Games (Lecture Notes in Computer Science, Vol. 4630)*. Springer, 72–83.

[13] Richard Emslie. 2019. Galvanise Zero. https://github.com/richemslie/galvanise_zero.

[14] Sylvain Gelly and David Silver. 2011. Monte-Carlo Tree Search and Rapid Action Value Estimation in computer Go. *Artif. Intell.* 175, 11 (2011), 1856–1875.

[15] Guy Haworth and Nelson Hernandez. 2021. The 20th Top Chess Engine Championship, TCEC20. *ICGA Journal* 43, 1 (2021), 62–73.

[16] Shih-Chieh Huang, Broderick Arneson, Ryan B Hayward, Martin Müller, and Jakub Pawlewicz. 2013. MoHex 2.0: a pattern-based MCTS Hex player. In *International Conference on Computers and Games*. Springer, 60–71.

[17] Donald E. Knuth and Ronald W. Moore. 1975. An Analysis of Alpha-Beta Pruning. *Artif. Intell.* 6, 4 (1975), 293–326. https://doi.org/10.1016/0004-3702(75)90019-3

[18] Levente Kocsis and Csaba Szepesvári. 2006. Bandit based Monte-Carlo planning. In *17th European Conference on Machine Learning (ECML'06) (LNCS, Vol. 4212)*. Springer, 282–293.

[19] Richard E Korf and David Maxwell Chickering. 1996. Best-first minimax search. *Artificial intelligence* 84, 1-2 (1996), 299–337.

[20] Tony A Marsland. 1987. Computer chess methods. *Encyclopedia of Artificial Intelligence* 1 (1987), 159–171.

[21] Yu Nasu. 2018. Efficiently updatable neural-network-based evaluation functions for computer shogi. *The 28th World Computer Shogi Championship Appeal Document* (2018).

[22] Gian-Carlo Pascutto. 2017. Leela Zero. https://github.com/leela-zero/leela-zero.

[23] David Silver, Aja Huang, Chris J Maddison, Arthur Guez, Laurent Sifre, George van den Driessche, Julian Schrittwieser, Ioannis Antonoglou, Veda Panneershelvam, Marc Lanctot, Sander Dieleman, Dominik Grewe, John Nham, Nal Kalchbrenner, Ilya Sutskever, Timothy Lillicrap, Madeleine Leach, Koray Kavukcuoglu, Thore Graepel, and Demis Hassabis. 2016. Mastering the game of Go with deep neural networks and tree search. *Nature* 529, 7587 (2016), 484–489.

[24] David Silver, Thomas Hubert, Julian Schrittwieser, Ioannis Antonoglou, Matthew Lai, Arthur Guez, Marc Lanctot, Laurent Sifre, Dharshan Kumaran, Thore Graepel, et al. 2018. A general reinforcement learning algorithm that masters chess, shogi, and Go through self-play. *Science* 362, 6419 (2018), 1140–1144.

[25] David Silver, Julian Schrittwieser, Karen Simonyan, Ioannis Antonoglou, Aja Huang, Arthur Guez, Thomas Hubert, Lucas Baker, Matthew Lai, Adrian Bolton, et al. 2017. Mastering the game of go without human knowledge. *Nature* 550, 7676 (2017), 354–359.

[26] Kei Takada, Hiroyuki Iizuka, and Masahito Yamamoto. 2019. Reinforcement learning to create value and policy functions using minimax tree search in hex. *IEEE Transactions on Games* 12, 1 (2019), 63–73.

[27] Yuandong Tian, Jerry Ma, Qucheng Gong, Shubho Sengupta, Zhuoyuan Chen, James Pinkerton, and C Lawrence Zitnick. 2019. Elf opengo: An analysis and open reimplementation of alphazero. *arXiv preprint arXiv:1902.04522* (2019).

[28] David J Wu. 2019. Accelerating self-play learning in go. *arXiv preprint arXiv:1902.10565* (2019).