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## Report on Denis Cornaz' H.D.R. dissertation

In this HDR dissertation Denis Cornaz summarizes his contributions to graph theory as well as his recent work on voting theory.

Chapter 2 contains several elegant results. For example Theorem 1 states that a subset B of edges in a graph G belongs to a complete bipartite partial subgraph of G if and only if this is the case for every subset of B with cardinality at most the clique number of G. This is a beautiful result. Theorem 2 exhibits several optimization problems in graphs that can be solved in polynomial time (these problems can be viewed as variants of odd-circuit minimization).

In Chapter 3, the chromatic Gallai identities proved in Theorem 4 are great results. The implications for approximating the clique-partition number of a graph are striking, improving the classical approximation based on the Lovász  $\theta$  number. This is an important contribution.

Chapter 4 contains a beautiful min-max theorem relating the maximum vertex degree in a graph to the minimum number of ocm sets in an ocm covering of the graph (an *ocm set* is a vertex-disjoint union C of odd circuits and one matching M; an *ocm covering* is a collection of ocm sets  $(C_i, M_i)$  such that every edge of the graph is either in a matching  $M_i$  or in  $C_i \cap C_j$  for some  $i \neq j$ ). A classical result of König on bipartite graphs occurs as a special case.

Chapter 5 solves interesting algorithmic questions related to voting theory.

To summarize, Denis Cornaz obtained very impressive results in graph theory. This bodes well for the future.

Unfortunately, the write up is sloppy. Definitions are often vague, which is not satisfactory for a mathematical document. For example, the definition of clutter in the first two lines of Chapter 1 is imprecise, the term "edge" being used with two different meanings in the same sentence, without warning to the reader. The confusion between algorithm and formulation in the next paragraph is sloppy. The grammar is bad. As an example, "as a natural covering" (comme un recouvrement naturel) should be "has a natural covering" (a un recouvrement naturel) in the first page of Chapter 1. The sloppiness is not confined to Chapter 1. Denis Cornaz likes to use the word "eventually" but uses it incorrectly in several mathematical statements (in English it means "in the end"). One needs to know French to realize that Denis means "possibly". The title of Chapter 2 "Minimal forbidden structures in linear programs" suggests a connection with linear programming. This connection is never made. Instead, Chapter 2 presents results

in graph theory about bipartite subgraphs of a graph. What is  $\bar{\chi}(G)$  in Chapter 3? The definition is missing. Does it stand for  $\chi(\bar{G})$ ?

Denis Cornaz has published a number of beautiful results. The research is solid and Denis has the potential for continuing to pursue a brilliant career. The HDR is well deserved.

Sincerely,

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