#### IASD M2 at Paris Dauphine

#### Deep Reinforcement Learning

13: Model-Based Policy Learning

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#### Homework 3: Q-Learning and Actor-Critic Algorithms

Due on Wed **27 March**.

#### 3 outputs to submit:

- 1. Report (pdf)
- 2. (code) Submit.zip
- 3. CO O notebook



Any homework submitted late will not be graded

Ask your questions on Moodle and answer to others

### Acknowledgement

These materials are based on the seminal course of Sergey Levine CS285



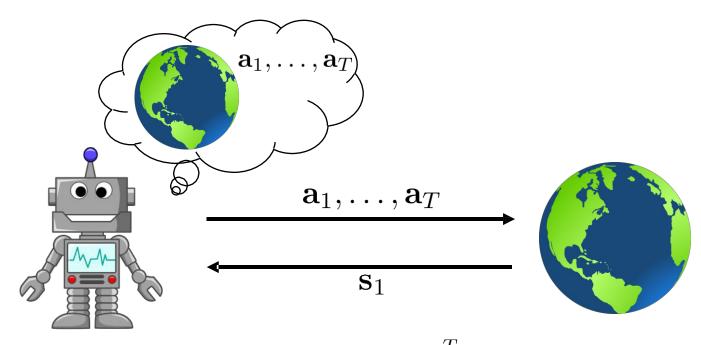
#### Last time: model-based RL with MPC

model-based reinforcement learning version 1.5:

- 1. run base policy  $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$  (e.g., random policy) to collect  $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
- 2. learn dynamics model  $f(\mathbf{s}, \mathbf{a})$  to minimize  $\sum_i ||f(\mathbf{s}_i, \mathbf{a}_i) \mathbf{s}_i'||^2$
- 3. plan through  $f(\mathbf{s}, \mathbf{a})$  to choose actions
- 4. execute the first planned action, observe resulting state s' (MPC)
- 5. append  $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$  to dataset  $\mathcal{D}$



#### The stochastic open-loop case

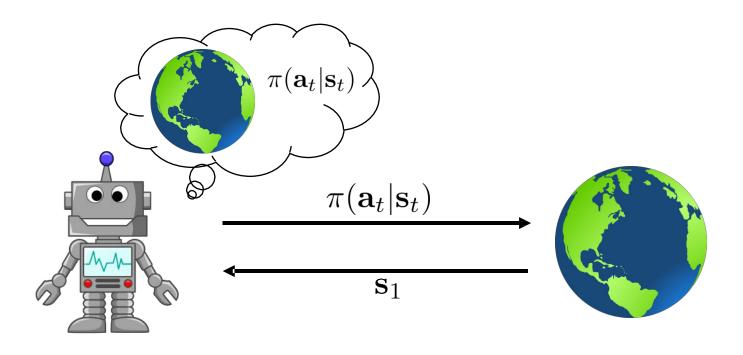


$$p_{\theta}(\mathbf{s}_1,\ldots,\mathbf{s}_T|\mathbf{a}_1,\ldots,\mathbf{a}_T) = p(\mathbf{s}_1)\prod_{t=1}^{T}p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)$$

$$\mathbf{a}_1, \dots, \mathbf{a}_T = \arg\max_{\mathbf{a}_1, \dots, \mathbf{a}_T} E\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) | \mathbf{a}_1, \dots, \mathbf{a}_T\right]$$

why is this suboptimal?

#### The stochastic closed-loop case



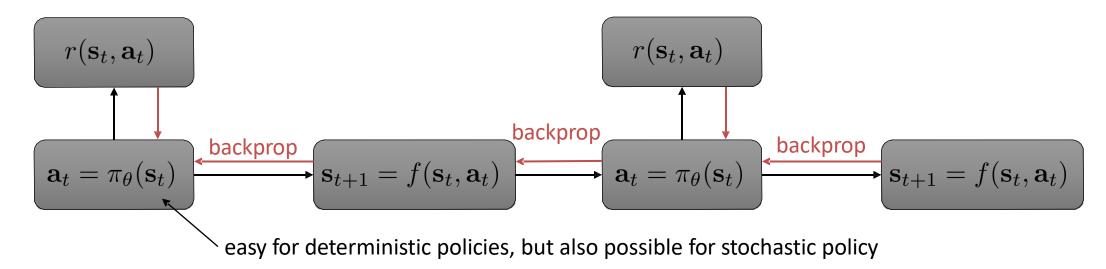
$$p(\mathbf{s}_1, \mathbf{a}_1, \dots, \mathbf{s}_T, \mathbf{a}_T) = p(\mathbf{s}_1) \prod_{t=1}^T \pi(\mathbf{a}_t | \mathbf{s}_t) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$

$$\pi = \arg\max_{\pi} E_{\tau \sim p(\tau)} \left[ \sum_{t} r(\mathbf{s}_{t}, \mathbf{a}_{t}) \right]$$

form of  $\pi$ ?

neural net  $\mathbf{s}$ time-varying linear  $\mathbf{K}_t\mathbf{s}_t+\mathbf{k}_t$ 

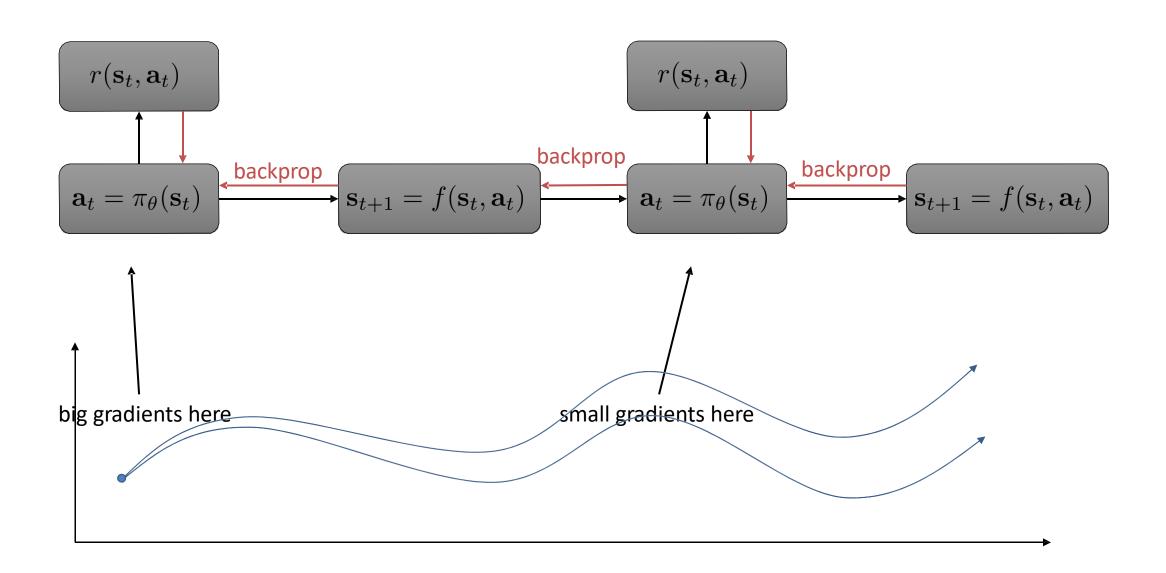
#### Backpropagate directly into the policy?



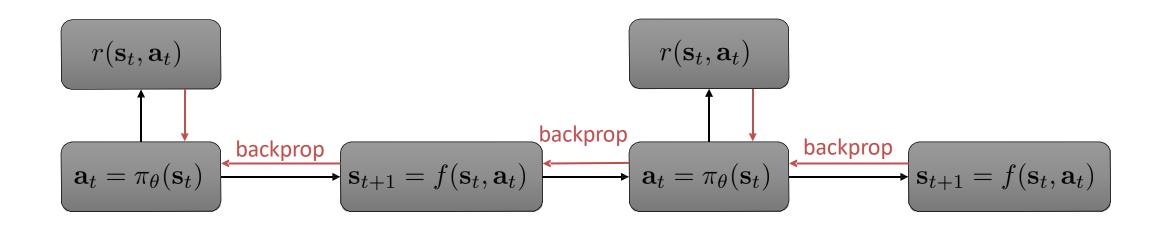
model-based reinforcement learning version 1.5:

- 1. run base policy  $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$  (e.g., random policy) to collect  $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
- 2. learn dynamics model  $f(\mathbf{s}, \mathbf{a})$  to minimize  $\sum_i ||f(\mathbf{s}_i, \mathbf{a}_i) \mathbf{s}_i'||^2$
- 3. backpropagate through  $f(\mathbf{s}, \mathbf{a})$  into the policy to optimize  $\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)$
- 4. run  $\pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)$ , appending the visited tuples  $(\mathbf{s},\mathbf{a},\mathbf{s}')$  to  $\mathcal{D}$

#### What's the problem with backprop into policy?



#### What's the problem with backprop into policy?



- Similar parameter sensitivity problems as shooting methods
  - But no longer have convenient second order LQR-like method, because policy parameters couple all the time steps, so no dynamic programming
- Similar problems to training long RNNs with BPTT
  - Vanishing and exploding gradients
  - Unlike LSTM, we can't just "choose" a simple dynamics, dynamics are chosen by nature

#### What's the solution?

- Use derivative-free ("model-free") RL algorithms, with the model used to generate synthetic samples
  - Seems weirdly backwards
  - Actually works very well
  - Essentially "model-based acceleration" for model-free RL

### Model-Free Learning With a Model

#### Model-free optimization with a model

Policy gradient: 
$$\nabla_{\theta} J(\theta) pprox rac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \hat{Q}_{i,t}^{\pi}$$

$$\text{Backprop (pathwise) gradient:} \quad \nabla_{\theta} J(\theta) = \sum_{t=1}^{T} \frac{d\mathbf{a}_{t}}{d\theta} \frac{d\mathbf{s}_{t+1}}{d\mathbf{a}_{t}} \left( \sum_{t'=t+1}^{T} \frac{dr_{t'}}{d\mathbf{s}_{t'}} \left( \prod_{t''=t+2}^{t'} \frac{d\mathbf{s}_{t''}}{d\mathbf{a}_{t''-1}} \frac{d\mathbf{a}_{t''-1}}{d\mathbf{s}_{t''-1}} + \frac{d\mathbf{s}_{t''}}{d\mathbf{s}_{t''-1}} \right) \right)$$

- Policy gradient might be more stable (if enough samples are used)
   because it does not require multiplying many Jacobians
- See a recent analysis here:
  - Parmas et al. '18: PIPP: Flexible Model-Based Policy Search Robust to the Curse of Chaos

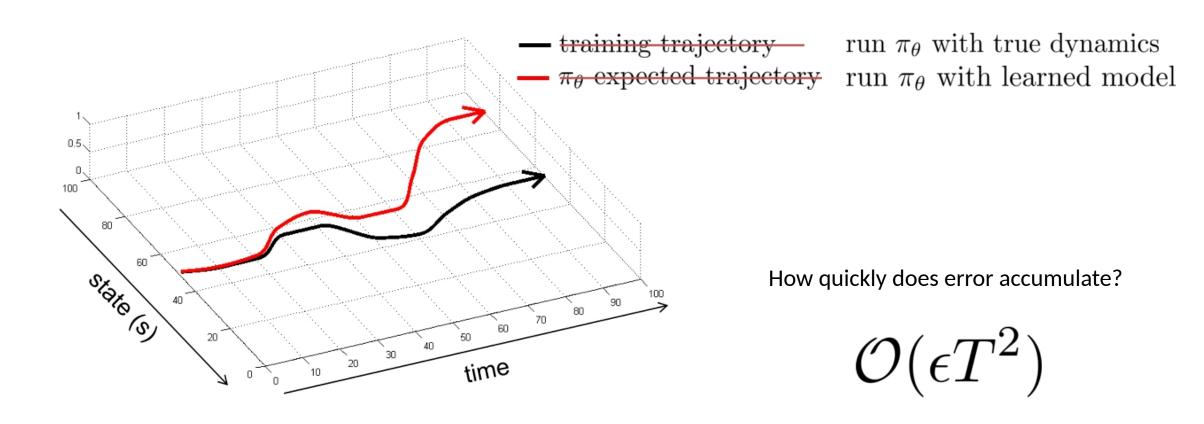
#### Model-based RL via policy gradient

model-based reinforcement learning version 2.5:

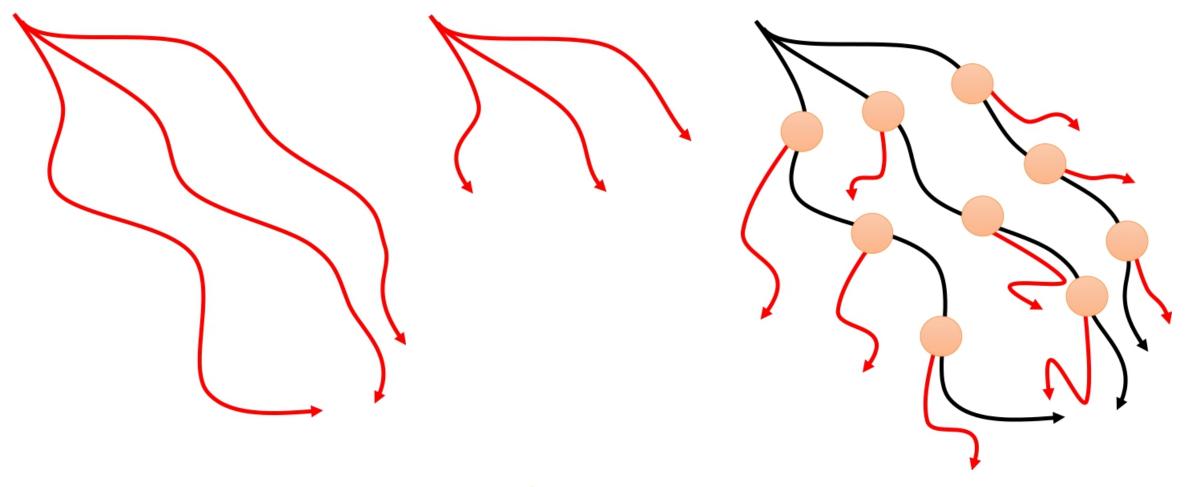
- 1. run base policy  $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$  (e.g., random policy) to collect  $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
- 2. learn dynamics model  $f(\mathbf{s}, \mathbf{a})$  to minimize  $\sum_i ||f(\mathbf{s}_i, \mathbf{a}_i) \mathbf{s}_i'||^2$
- 3. use  $f(\mathbf{s}, \mathbf{a})$  to generate trajectories  $\{\tau_i\}$  with policy  $\pi_{\theta}(\mathbf{a}|\mathbf{s})$
- 4. use  $\{\tau_i\}$  to improve  $\pi_{\theta}(\mathbf{a}|\mathbf{s})$  via policy gradient
- 5. run  $\pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)$ , appending the visited tuples  $(\mathbf{s},\mathbf{a},\mathbf{s}')$  to  $\mathcal{D}$

What's a potential **problem** with this approach?

## The curse of long model-based rollouts



## How to get away with **short** rollouts?



- huge accumulating error

- + much lower error
- never see later time steps
- + much lower error
- + see all time steps
- wrong state distribution

#### Model-based RL with short rollouts

model-based reinforcement learning version 3.0:

- 1. run base policy  $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$  (e.g., random policy) to collect  $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
- 2. learn dynamics model  $f(\mathbf{s}, \mathbf{a})$  to minimize  $\sum_i ||f(\mathbf{s}_i, \mathbf{a}_i) \mathbf{s}_i'||^2$
- 3. pick states  $\mathbf{s}_i$  from  $\mathcal{D}$ , use  $f(\mathbf{s}, \mathbf{a})$  to make short rollouts from them
- 4. use both real and model data to improve  $\pi_{\theta}(\mathbf{a}|\mathbf{s})$  with off-policy RI
- 5. run  $\pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)$ , appending the visited tuples  $(\mathbf{s},\mathbf{a},\mathbf{s}')$  to  $\mathcal{D}$

## Dyna -Style Algorithms

#### Model-based RL with short rollouts

model-based reinforcement learning version 3.0:

- 1. run base policy  $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$  (e.g., random policy) to collect  $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
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- 4. use both real and model data to improve  $\pi_{\theta}(\mathbf{a}|\mathbf{s})$  with off-policy RI
- 5. run  $\pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)$ , appending the visited tuples  $(\mathbf{s},\mathbf{a},\mathbf{s}')$  to  $\mathcal{D}$

#### Model-free optimization with a model

#### **Dyna**

online Q-learning algorithm that performs model-free RL with a model

- 1. given state s, pick action a using exploration policy
- 2. observe s' and r, to get transition (s, a, s', r)
- 3. update model  $\hat{p}(s'|s,a)$  and  $\hat{r}(s,a)$  using (s,a,s')
- 4. Q-update:  $Q(s,a) \leftarrow Q(s,a) + \alpha E_{s',r}[r + \max_{a'} Q(s',a') Q(s,a)]$
- 5. repeat K times:
  - 6. sample  $(s, a) \sim \mathcal{B}$  from buffer of past states and actions
  - 7. Q-update:  $Q(s,a) \leftarrow Q(s,a) + \alpha E_{s',r}[r + \max_{a'} Q(s',a') Q(s,a)]$

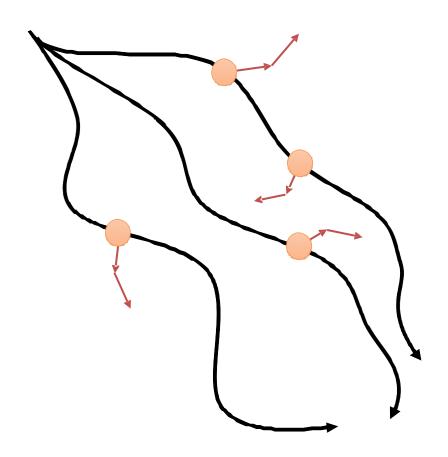
Richard S. Sutton. Integrated architectures for learning, planning, and reacting based on approximating dynamic programming.

### General "Dyna-style" model-based RL recipe

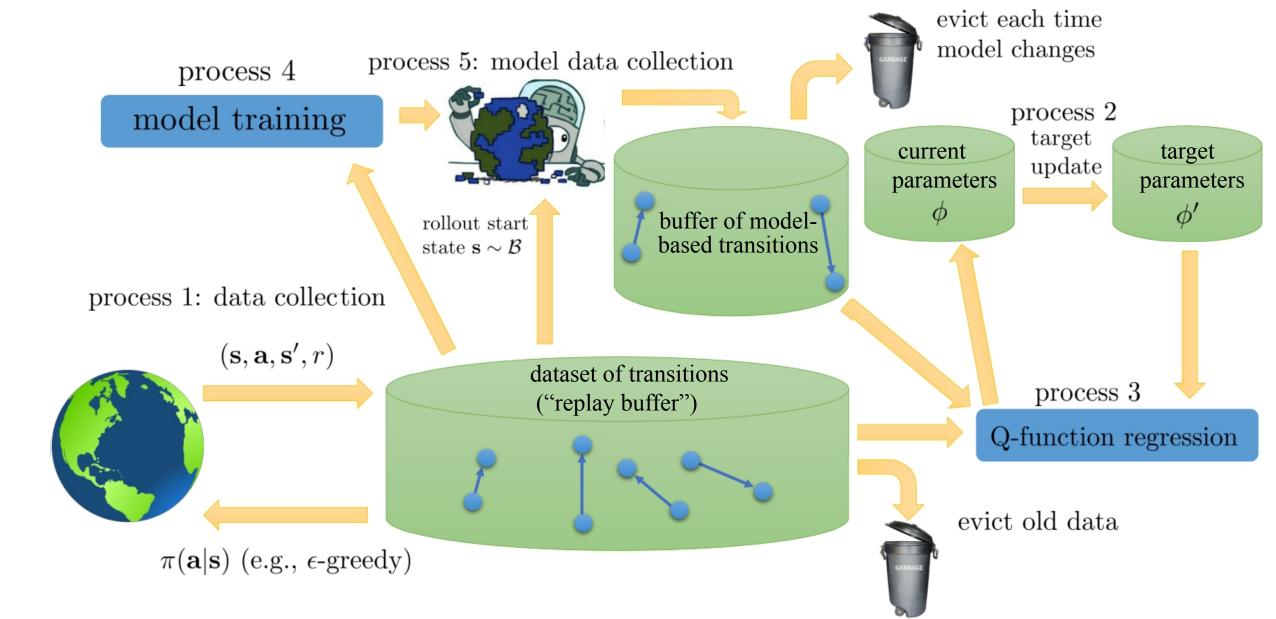
- 1. collect some data, consisting of transitions (s, a, s', r)
- 2. learn model  $\hat{p}(s'|s,a)$  (and optionally,  $\hat{r}(s,a)$ )
- 3. repeat K times:
  - 4. sample  $s \sim \mathcal{B}$  from buffer
  - 5. choose action a (from  $\mathcal{B}$ , from  $\pi$ , or random)
  - 6. simulate  $s' \sim \hat{p}(s'|s, a)$  (and  $r = \hat{r}(s, a)$ )
  - 7. train on (s, a, s', r) with model-free RL
  - 8. (optional) take N more model-based steps



+ still sees diverse states

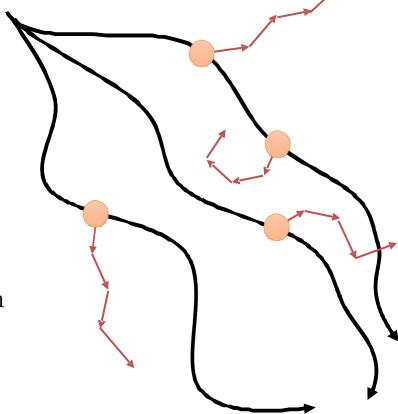


## Model-accelerated off -policy RL



# Model-Based Acceleration (MBA) Model-Based Value Expansion (MVE) Model-Based Policy Optimization (MBPO)

- 1. take some action  $\mathbf{a}_i$  and observe  $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}_i', r_i)$ , add it to  $\mathcal{B}$
- 2. sample mini-batch  $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$  from  $\mathcal{B}$  uniformly
- 3. use  $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}_j'\}$  to update model  $\hat{p}(\mathbf{s}'|\mathbf{s}, \mathbf{a})$
- 4. sample  $\{\mathbf{s}_i\}$  from  $\mathcal{B}$
- 5. for each  $\mathbf{s}_j$ , perform model-based rollout with  $\mathbf{a} = \pi(\mathbf{s})$
- 6. use all transitions  $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$  along rollout to update Q-function
- + why is this a good idea?
- why is this a bad idea?



Gu et al. Continuous deep Q-learning with model-based acceleration. '16 Feinberg et al. Model-based value expansion. '18 Janner et al. When to trust your model: model-based policy optimization. '19

Multi-Step Models & Successor Representations

## What kind of model do we need to evaluate a policy?

The job of the model is to evaluate the policy

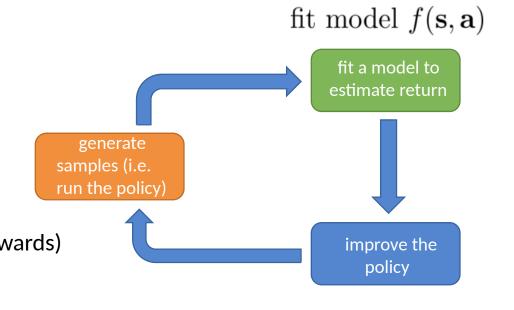
(if you can evaluate it, you can make it better)

$$J(\pi) = E_{s \sim p(s_1)}[V^{\pi}(s_1)]$$

$$V^{\pi}(s_t) = \sum_{t=t'}^{\infty} \gamma^{t'-t} E_{p(s_{t'}|s_t)} E_{\mathbf{a}_{t'} \sim \pi(\mathbf{a}_{t'}|\mathbf{s}_{t'}}[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})]$$
let's keep it simple
$$= \sum_{t=t'}^{\infty} \gamma^{t'-t} E_{p(s_{t'}|s_t)}[r(\mathbf{s}_{t'})] \quad \text{(easy to re-derive for action-dependent rewards)}$$

$$= \sum_{t=t'}^{\infty} \gamma^{t'-t} \sum_{\mathbf{s}} p(\mathbf{s}_{t'} = \mathbf{s}|\mathbf{s}_t) r(\mathbf{s})$$

$$= \sum_{t=t'}^{\infty} \left(\sum_{t=t'}^{\infty} \gamma^{t'-t} p(\mathbf{s}_{t'} = \mathbf{s}|\mathbf{s}_t)\right) r(\mathbf{s})$$



## What kind of model do we need to **evaluate** a policy?

$$V^{\pi}(\mathbf{s}_{t}) = \sum_{t=t'}^{\infty} \gamma^{t'-t} E_{p(s_{t'}|s_{t})}[r(\mathbf{s}_{t'})]$$

$$= \sum_{\mathbf{s}} \left( \sum_{t=t'}^{\infty} \gamma^{t'-t} p(\mathbf{s}_{t'} = \mathbf{s}|s_{t}) \right) r(\mathbf{s})$$

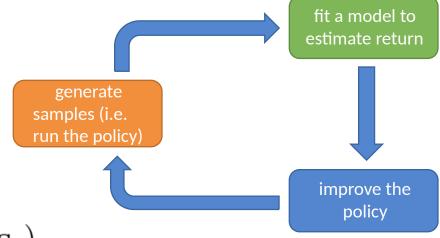
$$p_{\pi}(\mathbf{s}_{\text{future}} = \mathbf{s}|\mathbf{s}_{t})$$

$$p_{\pi}(\mathbf{s}_{\text{future}} = \mathbf{s}|\mathbf{s}_{t}) = (1-\gamma)\sum_{t'=t}^{\infty} \gamma^{t'-t} p(\mathbf{s}_{t'} = \mathbf{s}|\mathbf{s}_{t})$$

just to ensure it sums to 1

(if you can evaluate it, you can make it better)

fit model  $f(\mathbf{s}, \mathbf{a})$ 



## What kind of model do we need to evaluate a policy?

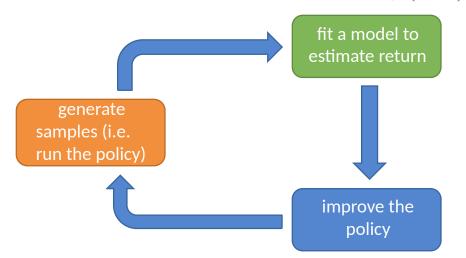
$$p_{\pi}(\mathbf{s}_{\text{future}} = \mathbf{s}|\mathbf{s}_t) = (1 - \gamma) \sum_{t'=t}^{\infty} \gamma^{t'-t} p(\mathbf{s}_{t'} = \mathbf{s}|\mathbf{s}_t)$$

$$V^{\pi}(\mathbf{s}_t) = \frac{1}{1 - \gamma} \sum_{\mathbf{s}} p_{\pi}(\mathbf{s}_{\text{future}} = \mathbf{s} | \mathbf{s}_t) r(\mathbf{s})$$
$$\mu^{\pi}(\mathbf{s}_t)^T \vec{r}$$

$$\mu_i^{\pi}(\mathbf{s}_t) = p_{\pi}(s_{\text{future}} = i|\mathbf{s}_t)$$

(if you can evaluate it, you can make it better)





This is called a **successor representation** 

#### Successor representations

$$\mu_i^{\pi}(\mathbf{s}_t) = (1 - \gamma) \sum_{t'=t}^{\infty} \gamma^{t'-t} p(\mathbf{s}_{t'} = i | \mathbf{s}_t)$$

$$= (1 - \gamma) \delta(\mathbf{s}_t = i) + \gamma E_{\mathbf{a}_t \sim \pi(\mathbf{a}_t | \mathbf{s}_t), \mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)} [\mu_i^{\pi}(\mathbf{s}_{t+1})]$$

like a Bellman backup with "reward"  $r(\mathbf{s}_t) = (1 - \gamma)\delta(\mathbf{s}_t = i)$  in practice, we can use vectorized backups for all i at once

#### A few issues...

- ➤ Not clear if learning successor representation is easier than model free RL
- > How to scale to large state spaces?
- ➤ How to extend to continuous state spaces?

#### Successor features

$$\mu_i^{\pi}(\mathbf{s}_t) = (1 - \gamma) \sum_{t'=t}^{\infty} \gamma^{t'-t} p(\mathbf{s}_{t'} = i | \mathbf{s}_t) \quad \psi_j^{\pi}(\mathbf{s}_t) = \sum_{\mathbf{s}} \mu_{\mathbf{s}}^{\pi}(\mathbf{s}_t) \phi_j(\mathbf{s}) \quad \psi_j^{\pi}(\mathbf{s}_t) = \mu^{\pi}(\mathbf{s}_t)^T \vec{\phi}_j$$

$$V^{\pi}(\mathbf{s}_t) = \mu^{\pi}(\mathbf{s}_t)^T \vec{r}$$

so what?

If the number of features is much less than the number of states, learning them is much easier!

if 
$$r(\mathbf{s}) = \sum_{j} \phi_{j}(\mathbf{s}) w_{j} = \phi(\mathbf{s})^{T} \mathbf{w}$$
  
then  $V^{\pi}(\mathbf{s}_{t}) = \psi^{\pi}(\mathbf{s}_{t})^{T} \mathbf{w}$   

$$= \sum_{j} \psi_{j}^{\pi}(\mathbf{s}_{t}) w_{j}$$

$$= \sum_{j} \mu^{\pi}(\mathbf{s}_{T})^{T} \vec{\phi}_{j} \mathbf{w}$$

$$= \mu^{\pi}(\mathbf{s}_{T})^{T} \sum_{j} \vec{\phi}_{j} \mathbf{w} = \mu^{\pi}(\mathbf{s}_{t})^{T} \vec{r}$$

#### Successor features

$$\mu_i^{\pi}(\mathbf{s}_t) = (1 - \gamma)\delta(\mathbf{s}_t = i) + \gamma E_{\mathbf{a}_t \sim \pi(\mathbf{a}_t | \mathbf{s}_t), \mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)} [\mu_i^{\pi}(\mathbf{s}_{t+1})]$$

$$\psi_j^{\pi}(\mathbf{s}_t) = \phi_j(\mathbf{s}_t) + \gamma E_{\mathbf{a}_t \sim \pi(\mathbf{a}_t | \mathbf{s}_t), \mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)} [\psi_j^{\pi}(\mathbf{s}_{t+1})]$$
special case with
$$\phi_i(\mathbf{s}_t) = (1 - \gamma)\delta(\mathbf{s}_t = i)$$

can also construct a "Q-function-like" version:

$$\psi_j^{\pi}(\mathbf{s}_t, \mathbf{a}_t) = \phi_j(\mathbf{s}_t) + \gamma E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t), \mathbf{a}_{t+1} \sim \pi(\mathbf{a}_{t+1} | \mathbf{s}_{t+1})} [\psi_j^{\pi}(\mathbf{s}_{t+1}, \mathbf{a}_{t+1})]$$

$$Q^{\pi}(\mathbf{s}_t, \mathbf{a}_t) \approx \psi^{\pi}(\mathbf{s}_t, \mathbf{a}_t)^T \mathbf{w} \qquad \text{when } r(\mathbf{s}_t) \approx \phi(\mathbf{s}_t)^T \mathbf{w}$$

#### Using successor features

#### **Idea 1:** recover a Q -function very quickly

- 1. Train  $\psi^{\pi}(\mathbf{s}_t, \mathbf{a}_t)$  (via Bellman backups)
- 2. Get some reward samples  $\{\mathbf{s}_i, r_i\}$
- 3. Get  $\mathbf{w} \leftarrow \arg\min_{\mathbf{w}} \sum_{i} ||\phi(\mathbf{s}_i)^T \mathbf{w} r_i||^2$
- 4. Recover  $Q^{\pi}(\mathbf{s}_t, \mathbf{a}_t) \approx \psi^{\pi}(\mathbf{s}_t, \mathbf{a}_t)^T \mathbf{w}$

Is this the **optimal** Q -function?

$$\pi'(\mathbf{s}) = \arg\max_{\mathbf{a}} \psi^{\pi}(\mathbf{s}, \mathbf{a})^T \mathbf{w}$$

Equivalent to **one step** of policy iteration

Better than nothing, but not optimal

#### Using successor features

#### Idea 2: recover many Q -functions

- 1. Train  $\psi^{\pi_k}(\mathbf{s}_t, \mathbf{a}_t)$  for many policies  $\pi_k$  (via Bellman backups)
- 2. Get some reward samples  $\{\mathbf{s}_i, r_i\}$
- 3. Get  $\mathbf{w} \leftarrow \arg\min_{\mathbf{w}} \sum_{i} ||\phi(\mathbf{s}_i)^T \mathbf{w} r_i||^2$
- 4. Recover  $Q^{\pi_k}(\mathbf{s}_t, \mathbf{a}_t) \approx \psi^{\pi_k}(\mathbf{s}_t, \mathbf{a}_t)^T \mathbf{w}$  for every  $\pi_k$

$$\pi'(\mathbf{s}) = \arg\max_{\mathbf{a}} \max_{k} \psi^{\pi_k}(\mathbf{s}, \mathbf{a})^T \mathbf{w}$$

Finds the highest reward policy in each state

#### Continuous successor representations

$$\mu_i^{\pi}(\mathbf{s}_t) = (1 - \gamma)\delta(\mathbf{s}_t = i) + \gamma E_{\mathbf{a}_t \sim \pi(\mathbf{a}_t | \mathbf{s}_t), \mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)} [\mu_i^{\pi}(\mathbf{s}_{t+1})]$$

always zero for any sampled state if states are continuous

Framing successor representation as *classification*:

$$p^{\pi}(F = 1|\mathbf{s}_t, \mathbf{a}_t, \mathbf{s}_{\text{future}}) = \frac{p^{\pi}(\mathbf{s}_{\text{future}}|\mathbf{s}_t, \mathbf{a}_t)}{p^{\pi}(\mathbf{s}_{\text{future}}|\mathbf{s}_t, \mathbf{a}_t) + p^{\pi}(\mathbf{s}_{\text{future}})}$$

binary classifier

F=1 means  $\mathbf{s}_{\text{future}}$  is a future state from  $\mathbf{s}_t, \mathbf{a}_t$  under  $\pi$ 

$$\mathcal{D}_{+} \sim p^{\pi}(\mathbf{s}_{\text{future}}|\mathbf{s}_{t}, \mathbf{a}_{t}) \quad \mathcal{D}_{-} \sim p^{\pi}(\mathbf{s})$$

#### Continuous successor representations

$$\mathcal{D}_{+} \sim p^{\pi}(\mathbf{s}_{\text{future}}|\mathbf{s}_{t}, \mathbf{a}_{t}) \qquad \mathcal{D}_{-} \sim p^{\pi}(\mathbf{s})$$

$$p^{\pi}(F = 1|\mathbf{s}_{t}, \mathbf{a}_{t}, \mathbf{s}_{\text{future}}) = \frac{p^{\pi}(\mathbf{s}_{\text{future}}|\mathbf{s}_{t}, \mathbf{a}_{t})}{p^{\pi}(\mathbf{s}_{\text{future}}|\mathbf{s}_{t}, \mathbf{a}_{t}) + p^{\pi}(\mathbf{s}_{\text{future}})}$$

$$p^{\pi}(F = 0|\mathbf{s}_{t}, \mathbf{a}_{t}, \mathbf{s}_{\text{future}}) = \frac{p^{\pi}(\mathbf{s}_{\text{future}}|\mathbf{s}_{t}, \mathbf{a}_{t}) + p^{\pi}(\mathbf{s}_{\text{future}})}{p^{\pi}(F = 0|\mathbf{s}_{t}, \mathbf{a}_{t}, \mathbf{s}_{\text{future}})} = \frac{p^{\pi}(\mathbf{s}_{\text{future}}|\mathbf{s}_{t}, \mathbf{a}_{t})}{p^{\pi}(\mathbf{s}_{\text{future}})}$$

$$\frac{p^{\pi}(F = 1|\mathbf{s}_{t}, \mathbf{a}_{t}, \mathbf{s}_{\text{future}})}{p^{\pi}(F = 0|\mathbf{s}_{t}, \mathbf{a}_{t}, \mathbf{s}_{\text{future}})} p^{\pi}(\mathbf{s}_{\text{future}}) = p^{\pi}(\mathbf{s}_{\text{future}}|\mathbf{s}_{t}, \mathbf{a}_{t})$$

$$\text{constant independent of } \mathbf{a}_{t}, \mathbf{s}_{t}$$

### The C-Learning algorithm

$$\mathcal{D}_{+} \sim p^{\pi}(\mathbf{s}_{\text{future}}|\mathbf{s}_{t}, \mathbf{a}_{t}) \qquad \mathcal{D}_{-} \sim p^{\pi}(\mathbf{s})$$

$$p^{\pi}(F = 1|\mathbf{s}_t, \mathbf{a}_t, \mathbf{s}_{\text{future}}) = \frac{p^{\pi}(F = 1|\mathbf{s}_t, \mathbf{a}_t, \mathbf{s}_{\text{future}})}{p^{\pi}(F = 1|\mathbf{s}_t, \mathbf{a}_t, \mathbf{s}_{\text{future}}) + p^{\pi}(\mathbf{s}_{\text{future}})}$$

#### To train:

- 1. Sample  $\mathbf{s} \sim p^{\pi}(\mathbf{s})$  (e.g., run policy, sample from trajectories)
- 2. Sample  $\mathbf{s} \sim p^{\pi}(\mathbf{s}_{\text{future}}|\mathbf{s}_t, \mathbf{a}_t)$  (e.g., pick  $\mathbf{s}_{t'}$  where  $t' = t + \Delta$ ,  $\Delta \sim \text{Geom}(\gamma)$ )
- 3. Update  $p^{\pi}(F=1|\mathbf{s}_t,\mathbf{a}_t,\mathbf{s})$  using SGD with cross entropy loss

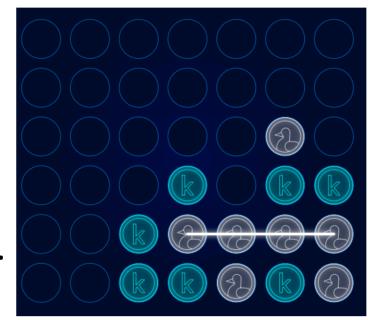
This is an **on policy** algorithm

Could also derive an off policy algorithm

#### Kaggle competition: Connect X



Submit your code on Moodle on **Sunday 10 March**. Presenting your solution on Wednesday 13 March. Graded on the stabilized version of March 18.



Don't submit after March 10 midnight Paris time, otherwise you will be disqualified.

20 points: 18 from your score and 2 from your oral presentation.

Good luck!