IASD M2 at Paris Dauphine

Deep Reinforcement Learning

21: Inverse Reinforcement Learning

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Acknowledgement

These materials are based on the seminal course of Sergey Levine CS285



Today's Lecture

- 1. So far: manually design reward function to define a task
- 2. What if we want to *learn* the reward function from observing an expert, and then use reinforcement learning?
- 3. Apply approximate optimality model from last time, but now learn the reward!
- Goals:
 - Understand the inverse reinforcement learning problem definition
 - Understand how probabilistic models of behavior can be used to derive inverse reinforcement learning algorithms
 - Understand a few practical inverse reinforcement learning algorithms we can use

Optimal Control as a Model of Human Behavior



Why should we worry about learning rewards?

The imitation learning perspective

Standard imitation learning:

- copy the *actions* performed by the expert
- no reasoning about outcomes of actions



Human imitation learning:

- copy the *intent* of the expert
- might take very different actions!



Why should we worry about learning rewards?

The reinforcement learning perspective





what is the reward?

Inverse reinforcement learning

Infer reward functions from demonstrations





by itself, this is an **underspecified** problem

many reward functions can explain the same behavior









A bit more formally

"forward" reinforcement learning

given:

states $\mathbf{s} \in \mathcal{S}$, actions $\mathbf{a} \in \mathcal{A}$ (sometimes) transitions $p(\mathbf{s'}|\mathbf{s}, \mathbf{a})$ reward function $r(\mathbf{s}, \mathbf{a})$

learn $\pi^{\star}(\mathbf{a}|\mathbf{s})$

inverse reinforcement learning given: states $\mathbf{s} \in \mathcal{S}$, actions $\mathbf{a} \in \mathcal{A}$ (sometimes) transitions $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$ samples $\{\tau_i\}$ sampled from $\pi^*(\tau)$

learn $r_{\psi}(\mathbf{s}, \mathbf{a})$ reward parameters

...and then use it to learn $\pi^*(\mathbf{a}|\mathbf{s})$

neural net reward function:

s a $r_{\psi}(\mathbf{s}, \mathbf{a})$ parameters ψ

linear reward function:

$$r_{\psi}(\mathbf{s}, \mathbf{a}) = \sum_{i} \psi_{i} f_{i}(\mathbf{s}, \mathbf{a}) = \psi^{T} \mathbf{f}(\mathbf{s}, \mathbf{a})$$

Feature matching IRL

linear reward function:

 $r_{\psi}(\mathbf{s}, \mathbf{a}) = \sum_{i} \psi_{i} f_{i}(\mathbf{s}, \mathbf{a}) = \psi^{T} \mathbf{f}(\mathbf{s}, \mathbf{a})$

if features \mathbf{f} are important, what if we match their expectations?

let $\pi^{r_{\psi}}$ be the optimal policy for r_{ψ}

pick ψ such that $E_{\pi^{r_{\psi}}}[\mathbf{f}(\mathbf{s}, \mathbf{a})] = E_{\pi^{\star}}[\mathbf{f}(\mathbf{s}, \mathbf{a})]$

state-action marginal under $\pi^{r_{\psi}}$

unknown optimal policy approximate using expert samples still ambiguous!

maximum margin principle:

$$\max_{\psi,m} m \qquad \text{such that } \psi^T E_{\pi^*}[\mathbf{f}(\mathbf{s}, \mathbf{a})] \ge \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s}, \mathbf{a})] + m$$

need to somehow "weight" by similarity between π^* and π

Feature matching IRL & maximum margin

remember the "SVM trick":

 $\max_{\psi,m} m \qquad \text{such that } \psi^T E_{\pi^\star}[\mathbf{f}(\mathbf{s},\mathbf{a})] \ge \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s},\mathbf{a})] + m$ $\prod_{\psi} \frac{1}{2} \|\psi\|^2 \qquad \text{such that } \psi^T E_{\pi^\star}[\mathbf{f}(\mathbf{s},\mathbf{a})] \ge \max_{\pi \in \Pi} \psi^T E_{\pi}[\mathbf{f}(\mathbf{s},\mathbf{a})] + D(\pi,\pi^\star)$ e.g., difference in feature expectations!

Issues:

- Maximizing the margin is a bit arbitrary
- No clear model of expert suboptimality (can add slack variables...)
- Messy constrained optimization problem not great for deep learning!

Further reading:

- Abbeel & Ng: Apprenticeship learning via inverse reinforcement learning
- Ratliff et al: Maximum margin planning

Optimal Control as a Model of Human Behavior



Muybridge (c. 1870)



Mombaur et al. '09



Li & Todorov '06



Ziebart '08





A probabilistic graphical model of decision making





Learning the Reward Function

Learning the optimality variable



The IRL partition function

$$\max_{\psi} \frac{1}{N} \sum_{i=1}^{N} r_{\psi}(\tau_i) - \log Z \qquad \qquad Z = \int p(\tau) \exp(r_{\psi}(\tau)) d\tau$$

$$\nabla_{\psi} \mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_i) - \frac{1}{Z} \int p(\tau) \exp(r_{\psi}(\tau)) \nabla_{\psi} r_{\psi}(\tau) d\tau$$
$$\underbrace{p(\tau | \mathcal{O}_{1:T}, \psi)}$$

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$

estimate with expert samples

soft optimal policy under current reward

Estimating the expectation

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau | \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$

$$E_{\tau \sim p(\tau | \mathcal{O}_{1:T}, \psi)} \left[\nabla_{\psi} \sum_{t=1}^{T} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}) \right]$$

$$= \sum_{t=1}^{T} E_{(\mathbf{s}_{t}, \mathbf{a}_{t}) \sim p(\mathbf{s}_{t}, \mathbf{a}_{t} | \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t})]$$

$$p(\mathbf{a}_{t} | \mathbf{s}_{t}, \mathcal{O}_{1:T}, \psi) p(\mathbf{s}_{t} | \mathcal{O}_{1:T}, \psi) \quad \text{where have we seen this before}?$$

$$= \frac{\beta(\mathbf{s}_{t}, \mathbf{a}_{t})}{\beta(\mathbf{s}_{t})} \propto \alpha(\mathbf{s}_{t})\beta(\mathbf{s}_{t})$$

$$p(\mathbf{a}_{t} | \mathbf{s}_{t}, \mathcal{O}_{1:T}, \psi) p(\mathbf{s}_{t} | \mathcal{O}_{1:T}, \psi) \propto \beta(\mathbf{s}_{t}, \mathbf{a}_{t})\alpha(\mathbf{s}_{t})$$

$$backward message \qquad \text{forward message}$$

Estimating the expectation

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$
$$\sum_{t=1}^{T} \int \int \mu_{t}(\mathbf{s}_{t}, \mathbf{a}_{t}) \nabla_{\psi} r_{\psi}(\mathbf{s}_{t}, \mathbf{a}_{t}) d\mathbf{s}_{t} d\mathbf{a}_{t}$$
$$= \sum_{t=1}^{T} \vec{\mu}_{t}^{T} \nabla_{\psi} \vec{r}_{\psi}$$

let $\mu_t(\mathbf{s}_t, \mathbf{a}_t) \propto \beta(\mathbf{s}_t, \mathbf{a}_t) \alpha(\mathbf{s}_t)$

state-action visitation probability for each $(\mathbf{s}_t, \mathbf{a}_t)$

The MaxEnt IRL algorithm

▶ 1. Given ψ , compute backward message $\beta(\mathbf{s}_t, \mathbf{a}_t)$ (see previous lecture)

2. Given ψ , compute forward message $\alpha(\mathbf{s}_t)$ (see previous lecture)

3. Compute
$$\mu_t(\mathbf{s}_t, \mathbf{a}_t) \propto \beta(\mathbf{s}_t, \mathbf{a}_t) \alpha(\mathbf{s}_t)$$

4. Evaluate $\nabla_{\psi} \mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\psi} r_{\psi}(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) - \sum_{t=1}^{T} \int \int \mu_t(\mathbf{s}_t, \mathbf{a}_t) \nabla_{\psi} r_{\psi}(\mathbf{s}_t, \mathbf{a}_t) d\mathbf{s}_t d\mathbf{a}_t$ 5. $\psi \leftarrow \psi + \eta \nabla_{\psi} \mathcal{L}$

Why MaxEnt?

in the case where $r_{\psi}(\mathbf{s}_t, \mathbf{a}_t) = \psi^T \mathbf{f}(\mathbf{s}_t, \mathbf{a}_t)$, we can show that it optimizes

$$\max_{\psi} \mathcal{H}(\pi^{r_{\psi}}) \text{ such that } E_{\pi^{r_{\psi}}}[\mathbf{f}] = E_{\pi^{\star}}[\mathbf{f}]$$
optimal max-ent policy under r^{ψ}
unknown expert policy
estimated with samples
Ziebart et al. 2008: Maximum Entropy Inverse Reinforcement Learning

as random as possible while matching features

Maximum Entropy Inverse Reinforcement Learning

Brian D. Ziebart, Andrew Maas, J.Andrew Bagnell, and Anind K. Dey School of Computer Science Carnegie Mellon University Pittsburgh, PA 15213

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Approximations in High Dimensions

What's missing so far?

- MaxEnt IRL so far requires...
 - Solving for (soft) optimal policy in the inner loop
 - Enumerating all state-action tuples for visitation frequency and gradient
- To apply this in practical problem settings, we need to handle...
 - Large and continuous state and action spaces
 - States obtained via sampling only
 - Unknown dynamics

Unknown dynamics & large state/action spaces

Assume we don't know the dynamics, but we can sample, like in standard RL

recall:

$$\nabla_{\psi} \mathcal{L} = E_{\tau \sim \pi^{\star}(\tau)} [\nabla_{\psi} r_{\psi}(\tau_{i})] - E_{\tau \sim p(\tau \mid \mathcal{O}_{1:T}, \psi)} [\nabla_{\psi} r_{\psi}(\tau)]$$

$$\swarrow \qquad \qquad \checkmark$$
estimate with expert samples soft optimal policy under current reward

idea: learn $p(\mathbf{a}_t | \mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm then run this policy to sample $\{\tau_j\}$ $I(\theta) - \nabla$

$$J(\theta) = \sum_{t} E_{\pi(\mathbf{s}_{t},\mathbf{a}_{t})} [r_{\psi}(\mathbf{s}_{t},\mathbf{a}_{t})] + E_{\pi(\mathbf{s}_{t})} [\mathcal{H}(\pi(\mathbf{a}|\mathbf{s}_{t}))]$$

$$\nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_i) - \frac{1}{M} \sum_{j=1}^{M} \nabla_{\psi} r_{\psi}(\tau_j)$$

sum over expert samples sum over policy samples

More efficient sample-based updates



sum over expert samples sum over policy samples

improve learn $p(\mathbf{a}_t | \mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm (a little) then run this policy to sample $\{\tau_i\}$

> looks expensive! what if we use "lazy" policy optimization? problem: estimator is now biased! wrong distribution! solution 1: use importance sampling

$$\nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_i) - \frac{1}{\sum_j w_j} \sum_{j=1}^{M} w_j \nabla_{\psi} r_{\psi}(\tau_j) \qquad w_j = \frac{p(\tau) \exp(r_{\psi}(\tau_j))}{\pi(\tau_j)}$$

Importance sampling

$$\nabla_{\psi} \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\psi} r_{\psi}(\tau_{i}) - \frac{1}{\sum_{j} w_{j}} \sum_{j=1}^{M} w_{j} \nabla_{\psi} r_{\psi}(\tau_{j}) \qquad w_{j} = \frac{p(\tau) \exp(r_{\psi}(\tau_{j}))}{\pi(\tau_{j})}$$

$$\frac{1}{\sqrt{1-\frac{1}{2}}} \sum_{i=1}^{N} w_{j} \nabla_{\psi} r_{\psi}(\tau_{j}) \qquad w_{j} = \frac{p(\tau) \exp(r_{\psi}(\tau_{j}))}{\pi(\tau_{j})}$$

$$\frac{1}{\sqrt{1-\frac{1}{2}}} \sum_{i=1}^{N} w_{j} \nabla_{\psi} r_{\psi}(\tau_{j}) \qquad w_{j} = \frac{p(\tau) \exp(r_{\psi}(\tau_{j}))}{\pi(\tau_{j})}$$

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$$\frac{1}{\sqrt{1-\frac{1}{2}}} \sum_{i=1}^{N} w_{j} = \frac{p(\tau) \exp(r_{\psi}(\tau_{j})}{\pi(\tau_{j})} \qquad w_{j} = \frac{p(\tau) \exp(r_{\psi}(\tau_{j})}{\pi(\tau_{j})} \qquad w_{j} = \frac{p(\tau) \exp(r_{\psi}(\tau_{j})}{\pi(\tau_{j})}} \qquad w_{j} = \frac{p(\tau) \exp(r_{\psi}(\tau_{j})}{\pi(\tau_{j})} \qquad w_{j} = \frac{p(\tau) \exp(r_{\psi}(\tau_{j})}{\pi(\tau_{j})} \qquad w_{j} = \frac{p(\tau) \exp($$

each policy update w.r.t. r_{ψ} brings us closer to the target distribution!



slides adapted from C. Finn

1x real-time

IRL and GANs

It looks a bit like a game...



policy changed to make it *harder* to distinguish from demos

Generative Adversarial Networks



Inverse RL as a GAN



which discriminator is best?

$$D^{\star}(\mathbf{x}) = \frac{p^{\star}(\mathbf{x})}{p_{\theta}(\mathbf{x}) + p^{\star}(\mathbf{x})}$$

for IRL, optimal policy approaches $\pi_{\theta}(\tau) \propto p(\tau) \exp(r_{\psi}(\tau))$



Finn*, Christiano* et al. "A Connection Between Generative Adversarial Networks, Inverse Reinforcement Learning, and Energy-Based Models."

Inverse RL as a GAN



policy changed to make it *harder* to distinguish from demos

Finn*, Christiano* et al. "A Connection Between Generative Adversarial Networks, Inverse Reinforcement Learning, and Energy-Based Models."

Generalization via inverse RL



demonstration

what can we learn from the demonstration to enable better transfer?

need to decouple the goal from the dynamics!

policy = reward + dynamics



reproduce behavior under different conditions

Fu et al. Learning Robust Rewards with Adversarial Inverse Reinforcement Learning

Can we just use a regular discriminator?



- + often simpler to set up optimization, fewer moving parts
- discriminator knows nothing at convergence
- generally cannot reoptimize the "reward"

Ho & Ermon. Generative adversarial imitation learning.

distinguish from demos

IRL as adversarial optimization



Guided Cost Learning

learns distribution $p(\tau)$ such that demos have max likelihood $p(\tau) \propto \exp(r(\tau))$ (MaxEnt model)



Generative Adversarial Imitation Learning Ho & Ermon, NIPS 2016



 $D(\tau) =$ probability τ is a demo

use $\log D(\tau)$ as "reward"

actually the $D(\tau) = \text{some classifier}$ same thing!



Hausman, Chebotar, Schaal, Sukhatme, Lim

Motion Imitation



(Mocap)



Learned Policy (Simulation)

the goal is to train a simulated character to imitate the motion.

Peng, Kanazawa, Toyer, Abbeel, Levine

Suggested Reading on Inverse RL

Classic Papers:

Abbeel & Ng ICML '04. Apprenticeship Learning via Inverse Reinforcement Learning. Good introduction to inverse reinforcement learning Ziebart et al. AAAI '08. Maximum Entropy Inverse Reinforcement Learning. Introduction to probabilistic method for inverse reinforcement learning

Modern Papers:

Finn et al. ICML '16. *Guided Cost Learning*. Sampling based method for MaxEnt IRL that handles unknown dynamics and deep reward functions Wulfmeier et al. arXiv '16. *Deep Maximum Entropy Inverse Reinforcement Learning*. MaxEnt inverse RL using deep reward functions Ho & Ermon NIPS '16. *Generative Adversarial Imitation Learning*. Inverse RL method using generative adversarial networks Fu, Luo, Levine ICLR '18. Learning Robust Rewards with Adversarial Inverse Reinforcement Learning