# Deep Reinforcement Learning 

15: Exploration (Part 2)

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## Acknowledgement

These materials are based on the seminal course of Sergey Levine CS285

## Sergey Levine

Advances in
Reinforcement Learning


## Recap: what's the problem?

this is easy (mostly)

this is impossible


## Unsupervised learning of diverse behaviors

What if we want to recover diverse behavior without any reward function at all?


Why?
$>$ Learn skills without supervision, then use them to accomplish goals
$>$ Learn sub-skills to use with
hierarchical reinforcement learning
$>$ Explore the space of possible behaviors

## An Example Scenario



## In this lecture...

$>$ Definitions \& concepts from information theory
$>$ Learning without a reward function by reaching goals
$>$ A state distribution-matching formulation of reinforcement learning
$>$ Is coverage of valid states a good exploration objective?
$>$ Beyond state covering: covering the space of skills

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## Some useful identities

$$
\begin{aligned}
& p(\mathbf{x}) \quad \text { distribution (e.g., over observations } \mathbf{x}) \\
& \mathcal{H}(p(\mathbf{x}))=-E_{\mathbf{x} \sim p(\mathbf{x})}[\log p(\mathbf{x})] \\
& \quad \text { entropy - how "broad" } p(\mathbf{x}) \text { is }
\end{aligned}
$$

## Some useful identities

entropy - how "broad" $p(\mathbf{x})$ is

$$
\begin{aligned}
\mathcal{H}(p(\mathbf{x})) & =-E_{\mathbf{x} \sim p(\mathbf{x})}[\log p(\mathbf{x})] \\
\mathcal{I}(\mathbf{x} ; \mathbf{y}) & =D_{\mathrm{KL}}(p(\mathbf{x}, \mathbf{y}) \| p(\mathbf{x}) p(\mathbf{y})) \\
& =E_{(\mathbf{x}, \mathbf{y}) \sim p(\mathbf{x}, \mathbf{y})}\left[\log \frac{p(\mathbf{x}, \mathbf{y})}{p(\mathbf{x}) p(\mathbf{y})}\right] \\
& =\mathcal{H}(p(\mathbf{y}))-\mathcal{H}(p(\mathbf{y} \mid \mathbf{x}))
\end{aligned}
$$

## Information theoretic quantities in RL

$\pi(\mathbf{s}) \quad$ state marginal distribution of policy $\pi$
$\mathcal{H}(\pi(\mathbf{s})) \quad$ state marginal entropy of policy $\pi$

example of mutual information: "empowerment" (Polani et al.)
$\mathcal{I}\left(\mathbf{s}_{t+1} ; \mathbf{a}_{t}\right)=\mathcal{H}\left(\mathbf{s}_{t+1}\right)-\mathcal{H}\left(\mathbf{s}_{t+1} \mid \mathbf{a}_{t}\right)$
can be viewed as quantifying "control authority" in an information-theoretic way

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## An Example Scenario



## Learn without any rewards at all



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$\longrightarrow$

1. Propose goal: $z_{g} \sim p(z), x_{g} \sim p_{\theta}\left(x_{g} \mid z_{g}\right)$
2. Attempt to reach goal using $\pi\left(a \mid x, x_{g}\right)$, reach $\bar{x}$
3. Use data to update $\pi$
4. Use data to update $p_{\theta}\left(x_{g} \mid z_{g}\right), q_{\phi}\left(z_{g} \mid x_{g}\right)$


## How do we get diverse goals?



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4. Use data to update $p_{\theta}\left(x_{g} \mid z_{g}\right), q_{\phi}\left(z_{g} \mid x_{g}\right)$
standard MLE: $\theta, \phi \leftarrow \arg \max _{\theta, \phi} E[\log p(\bar{x})]$
weighted MLE: $\theta, \phi \leftarrow \arg \max _{\theta, \phi} E[w(\bar{x}) \log p(\bar{x})]$ $w(\bar{x})=p_{\theta}(\bar{x})^{\alpha}$


## How do we get diverse goals?

what is the objective?

goals get higher entropy due to Skew-Fit

$$
\begin{gathered}
w(\bar{x})=p_{\theta}(\bar{x})^{\alpha} \\
\alpha \in[-1,0)
\end{gathered}
$$


what does RL do?
$\pi(a \mid S, G)$ trained to reach goal $G$
as $\pi$ gets better, final state $S$ gets close to $G$
that means $p(G \mid S)$ becomes more deterministic!


## How do we get diverse goals?

what is the objective?

$$
\max \mathcal{H}(p(G))-\mathcal{H}(p(G \mid S))=\max \mathcal{I}(S ; G)
$$

maximizing mutual information between $S$ and $G$ leads to good exploration (state coverage) $-\mathcal{H}(p(G))$ effective goal reaching - $\mathcal{H}(p(G \mid S))$


## Reinforcement learning with imagined goals



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## Aside: exploration with intrinsic motivation


common method for exploration:
incentivize policy $\pi(\mathbf{a} \mid \mathbf{s})$ to explore diverse states
...before seeing any reward
reward visiting novel states
if a state is visited often, it is not novel
$\Rightarrow$ add an exploration bonus to reward: $\tilde{r}(\mathbf{s})=r(\mathbf{s})-\log p_{\pi}(\mathbf{s})$


1. update $\pi(\mathbf{a} \mid \mathbf{s})$ to maximize $E_{\pi}[\tilde{r}(\mathbf{s})]$
2. update $p_{\pi}(\mathbf{s})$ to fit state marginal

## Can we use this for state marginal matching?

the state marginal matching problem: learn $\pi(\mathbf{a} \mid \mathbf{s})$ so as to minimze $D_{\mathrm{KL}}\left(p_{\pi}(\mathbf{s}) \| p^{\star}(\mathbf{s})\right)$ idea: can we use intrinsic motivation?
$\tilde{r}(\mathbf{s})=\log p^{\star}(\mathbf{s})-\log p_{\pi}(\mathbf{s})$
this does not perform marginal matching!

1. learn $\pi^{k}(\mathbf{a} \mid \mathbf{s})$ to maximize $E_{\pi}\left[\tilde{r}^{k}(\mathbf{s})\right]$
2. update $p_{\pi^{k}}(\mathrm{~s})$ to fit state marginat
3. update $p_{\pi^{k}}(\mathbf{s})$ to fit all states seen so far
target state density

4. return $\pi^{\star}(\mathbf{a} \mid \mathbf{s})=\sum_{k} \pi^{k}(\mathbf{a} \mid \mathbf{s})$
special case: $\log p^{\star}(\mathbf{s})=C \Rightarrow$ uniform target

$$
D_{\mathrm{KL}}\left(p_{\pi}(\mathbf{s}) \| U(\mathbf{s})\right)=\mathcal{H}\left(p_{\pi}(\mathbf{s})\right)
$$

this does perform marginal matching!
$p_{\pi}(\mathbf{s})=p^{\star}(\mathbf{s})$ is Nash equilibrium of two player game between $\pi^{k}$ and $p_{\pi^{k}}$

## State marginal matching for exploration

the state marginal matching problem: learn $\pi(\mathbf{a} \mid \mathbf{s})$ so as to minimze $D_{\mathrm{KL}}\left(p_{\pi}(\mathbf{s}) \| p^{\star}(\mathbf{s})\right)$


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## Is state entropy really a good objective?

Skew-Fit: $\quad \max \mathcal{H}(p(G))-\mathcal{H}(p(G \mid S))=\max \mathcal{I}(S ; G)$
SMM (special case where $\left.p^{\star}(\mathbf{s})=C\right): \max \mathcal{H}\left(p_{\pi}(S)\right)$ more or less the same thing

When is this a good idea?
"Eysenbach's Theorem" (not really what it's called)
(follows trivially from classic maximum entropy modeling)
at test time, an adversary will choose the worst goal $G$
which goal distribution should you use for training?
answer: choose $p(G)=\arg \max _{p} \mathcal{H}(p(G))$

See also: Hazan, Kakade, Singh, Van Soest. Provably Efficient Maximum Entropy Exploration

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## Learning diverse skills



Reaching diverse goals is not the same as performing diverse tasks not all behaviors can be captured by goal-reaching


Intuition: different skills should visit different state-space regions

## Diversity-promoting reward function

$$
\pi(\mathbf{a} \mid \mathbf{s}, z)=\arg \max _{\pi} \sum_{z} E_{\mathbf{s} \sim \pi(\mathbf{s} \mid z]}[r(\mathbf{s}, z)]
$$

reward states that are unlikely for other $z^{\prime} \neq z$

$$
r(\mathbf{s}, z)=\log p(z \mid \mathbf{s})
$$



## Examples of learned tasks



Cheetah


Ant


Mountain car

## A connection to mutual information

$$
\pi(\mathbf{a} \mid \mathbf{s}, z)=\arg \max _{\pi} \sum_{z} E_{\mathbf{s} \sim \pi(\mathbf{s} \mid z)}[r(\mathbf{s}, z)]
$$

$$
r(\mathbf{s}, z)=\log p(z \mid \mathbf{s})
$$


maximized by using uniform prior $p(z)$
minimized by maximizing $\log p(z \mid \mathbf{s})$

