IASD M2 at Paris Dauphine

### Deep Reinforcement Learning

#### 16: Offline Reinforcement Learning

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### Acknowledgement

These materials are based on the seminal course of Sergey Levine CS285



## The generalization gap



Mnih et al. '13



Schulman et al. '14 & '15



Levine\*, Finn\*, et al. '16





enormous gulf







### What makes modern machine learning work?









### Can we develop data-driven RL methods?



Levine, Kumar, Tucker, Fu. Offline Reinforcement Learning: Tutorial, Review, and Perspectives on Open Problems. '20

## What does offline RL mean?



off-policy RL



offline reinforcement learning



Formally:

$$\mathcal{D} = \{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$$
  

$$\mathbf{s} \sim d^{\pi_{\beta}}(\mathbf{s})$$

$$\mathbf{a} \sim \pi_{\beta}(\mathbf{a}|\mathbf{s})$$

$$\mathbf{s}' \sim p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$$

$$r \leftarrow r(\mathbf{s}, \mathbf{a})$$

RL objective: 
$$\max_{\pi} \sum_{t=0}^{T} E_{\mathbf{s}_{t} \sim d^{\pi}(\mathbf{s}), \mathbf{a}_{t} \sim \pi(\mathbf{a}|\mathbf{s})} [\gamma^{t} r(\mathbf{s}_{t}, \mathbf{a}_{t})]$$

## Types of offline RL problems

off-policy evaluation (OPE):

given 
$$\mathcal{D}$$
, estimate  $J(\pi) = E_{\pi} \left[ \sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t) \right]$ 

$$\mathcal{D} = \{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$$
$$\mathbf{s} \sim d^{\pi_\beta}(\mathbf{s})$$
$$\mathbf{a} \sim \pi_\beta(\mathbf{a}|\mathbf{s})$$
$$\mathbf{s}' \sim p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$$
$$r \leftarrow r(\mathbf{s}, \mathbf{a})$$

offline reinforcement learning: (a.k.a. batch RL, sometimes fully off-policy RL)

given  $\mathcal{D}$ , learn the best possible policy  $\pi_{\theta}$ 

not necessarily obvious what this means

## How is this even possible?

1. Find the "good stuff" in a dataset full of good and bad behaviors

2. Generalization: good behavior in one place may suggest good behavior in another place

3. "Stitching": parts of good behaviors can be recombined





# What do we expect offline RL methods to do?

#### Bad intuition: it's like imitation learning

Though it can be shown to be **provably** better than imitation learning even with optimal data, under some structural assumptions!

See: Kumar, Hong, Singh, Levine. Should I Run Offline Reinforcement Learning or Behavioral Cloning?



#### Better intuition: get order from chaos



"Macro-scale" stitching

But this is just the clearest example!

If we have algorithms that properly perform dynamic programming, we can take this idea much further and get near-optimal policies from highly suboptimal data

 $\mathcal{D}$ 

"Micro-scale" stitching:

## A vivid example



Singh, Yu, Yang, Zhang, Kumar, Levine. COG: Connecting New Skills to Past Experience with Offline Reinforcement Learning. '20

## Why should we care?













Kalashnikov, Irpan, Pastor, Ibarz, Herzong, Jang, Quillen, Holly, Kalakrishnan, Vanhoucke, Levine. QT-Opt: Scalable Deep Reinforcement Learning of Vision-Based Robotic Manipulation Skills

## Does it work?







Method	Dataset	Success	Failure
Offline QT-Opt	580k offline	87%	13%
Finetuned QT-Opt	580k offline + 28k online	96%	4%

Kalashnikov, Irpan, Pastor, Ibarz, Herzong, Jang, Quillen, Holly, Kalakrishnan, Vanhoucke, Levine. QT-Opt: Scalable Deep Reinforcement Learning of Vision-Based Robotic Manipulation Skills

## Why is offline RL hard?



## Why is offline RL hard?

Fundamental problem: counterfactual queries



**Online RL** algorithms don't have to handle this, because they can simply **try** this action and see what happens

Offline RL methods must somehow account for these unseen ("out-of-distribution") actions, ideally in a safe way ...while still making use of generalization to come up with behaviors that are better than the best thing seen in the data!

Levine, Kumar, Tucker, Fu. Offline Reinforcement Learning: Tutorial, Review, and Perspectives on Open Problems. '20

## Distribution shift in a nutshell

Example empirical risk minimization (ERM) problem:

$$\theta \leftarrow \arg\min_{\theta} E_{\mathbf{x} \sim p(\mathbf{x}), y \sim p(y|\mathbf{x})} \left[ (f_{\theta}(\mathbf{x}) - y)^2 \right]$$

given some  $\mathbf{x}^*$ , is  $f_{\theta}(\mathbf{x}^*)$  correct?

$$E_{\mathbf{x} \sim p(\mathbf{x}), y \sim p(y|\mathbf{x})} \left[ (f_{\theta}(\mathbf{x}) - y)^2 \right]$$
 is low  
 $E_{\mathbf{x} \sim p(\mathbf{x}), y \sim p(y|\mathbf{x})} \left[ (f_{\theta}(\mathbf{x}) - y)^2 \right]$  is not, for general  $\bar{p}(\mathbf{x}) \neq p(\mathbf{x})$   
what if  $\mathbf{x}^* \sim p(\mathbf{x})$ ? not necessarily...

usually we are not worried – neural nets generalize well!

what if we pick  $\mathbf{x}^{\star} \leftarrow \arg \max_{\mathbf{x}} f_{\theta}(\mathbf{x})$ ?



## Where do we suffer from distribution shift?

$$Q(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + \max_{\mathbf{a}'} Q(\mathbf{s}', \mathbf{a}')$$

$$Q(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + E_{\mathbf{a}' \sim \pi_{new}}[Q(\mathbf{s}', \mathbf{a}')]$$

$$y(\mathbf{s}, \mathbf{a})$$

what is the objective?

Denavior policy



how well it *thinks* it does (Q-values)

Kumar, Fu, Tucker, Levine. Stabilizing Off-Policy Q-Learning via Bootstrapping Error Reduction. NeurIPS '19

#### Issues with generalization are not corrected

online RL setting

offline RL setting



Existing challenges with sampling error and function approximation error in standard RL become **much more severe** in offline RL

### Batch RL via Importance Sampling

Offline RL with policy gradients  
RL objective: 
$$\max_{\pi} \sum_{t=0}^{T} E_{\mathbf{s}_{t} \sim d^{\pi}(\mathbf{s}), \mathbf{a}_{t} \sim \pi(\mathbf{a}|\mathbf{s})} [\gamma^{t} r(\mathbf{s}_{t}, \mathbf{a}_{t})]$$

$$\nabla_{\theta} J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)} \left[ \sum_{t=0}^{T} \nabla_{\theta} \gamma^{t} \log \pi_{\theta}(\mathbf{a}_{t}|\mathbf{s}_{t}) \hat{Q}(\mathbf{s}_{t}, \mathbf{a}_{t}) \right]$$

$$\approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{T} \nabla_{\theta} \gamma^{t} \log \pi_{\theta}(\mathbf{a}_{t,i}|\mathbf{s}_{t,i}) \hat{Q}(\mathbf{s}_{t,i}, \mathbf{a}_{t,i})$$
requires sampling from  $\pi_{\theta}$ ! what if

what if we only have samples from  $\pi_{\beta}$ ?

importance sampling:

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{\pi_{\theta}(\tau_i)}{\pi_{\beta}(\tau_i)} \sum_{t=0}^{T} \nabla_{\theta} \gamma^t \log \pi_{\theta}(\mathbf{a}_{t,i} | \mathbf{s}_{t,i}) \hat{Q}(\mathbf{s}_{t,i}, \mathbf{a}_{t,i})$$
importance weight

Offline RL with policy gradients  

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{\pi_{\theta}(\tau_{i})}{\pi_{\beta}(\tau_{i})} \sum_{t=0}^{T} \nabla_{\theta} \gamma^{t} \log \pi_{\theta}(\mathbf{a}_{t,i} | \mathbf{s}_{t,i}) \hat{Q}(\mathbf{s}_{t,i}, \mathbf{a}_{t,i}) \checkmark E_{\pi_{\theta}} \left[ \sum_{t'=t}^{T} \gamma^{t'-t} r_{t'} \right] \approx \sum_{t'=t}^{T} \gamma^{t'-t} r_{t',i}$$

$$\frac{\pi_{\theta}(\tau)}{\pi_{\beta}(\tau)} = \frac{p(\mathbf{s}_{1}) \prod_{t} p(\mathbf{s}_{t+1} | \mathbf{s}_{t}, \mathbf{a}_{t}) \pi_{\theta}(\mathbf{a}_{t} | \mathbf{s}_{t})}{p(\mathbf{s}_{1}) \prod_{t} p(\mathbf{s}_{t+1} | \mathbf{s}_{t}, \mathbf{a}_{t}) \pi_{\beta}(\mathbf{a}_{t} | \mathbf{s}_{t})}$$

this is exponential in Tweights likely to be degenerate as T becomes large

can we fix this?

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{T} \left( \prod_{t'=0}^{t-1} \frac{\pi_{\theta}(\mathbf{a}_{t',i} | \mathbf{s}_{t',i})}{\pi_{\beta}(\mathbf{a}_{t',i} | \mathbf{s}_{t',i})} \right) \nabla_{\theta} \gamma^{t} \log \pi_{\theta}(\mathbf{a}_{t,i} | \mathbf{s}_{t,i}) \left( \prod_{t'=t}^{T} \frac{\pi_{\theta}(\mathbf{a}_{t',i} | \mathbf{s}_{t',i})}{\pi_{\beta}(\mathbf{a}_{t',i} | \mathbf{s}_{t',i})} \right) \hat{Q}(\mathbf{s}_{t,i}, \mathbf{a}_{t,i})$$

accounts for difference in probability of landing in  $\mathbf{s}_{t,i}$ we have  $\mathbf{s}_t \sim d^{\pi_{\beta}}(\mathbf{s}_t)$ , but want  $\mathbf{s}_t \sim d^{\pi_{\theta}}(\mathbf{s}_t)$ 

why is this a reasonable approximation?

accounts for having the incorrect  $\hat{Q}(\mathbf{s}_{t,i}, \mathbf{a}_{t,i})$ 

Estimating the returns  

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{T} \nabla_{\theta} \gamma^{t} \log \pi_{\theta}(\mathbf{a}_{t,i} | \mathbf{s}_{t,i}) \left( \prod_{t'=t}^{T} \frac{\pi_{\theta}(\mathbf{a}_{t',i} | \mathbf{s}_{t',i})}{\pi_{\beta}(\mathbf{a}_{t',i} | \mathbf{s}_{t',i})} \right) \hat{Q}(\mathbf{s}_{t,i}, \mathbf{a}_{t,i}) \stackrel{\mathsf{E}_{\pi_{\theta}}}{=} \left[ \sum_{t'=t}^{T} \gamma^{t'-t} r_{t'} \right] \approx \sum_{t'=t}^{T} \gamma^{t'-t} r_{t',i}$$

$$\sum_{t'=t}^{T} \left( \prod_{t''=t}^{T} \frac{\pi_{\theta}(\mathbf{a}_{t'',i} | \mathbf{s}_{t'',i})}{\pi_{\beta}(\mathbf{a}_{t'',i} | \mathbf{s}_{t'',i})} \right) \gamma^{t'-t} r_{t',i}$$

but this is *still* exponential!

To avoid exponentially exploding importance weights, we **must** use value function estimation!

imagine we knew  $Q^{\pi_{\theta}}(\mathbf{s}, \mathbf{a})$ 

We'll see how to do this shortly, but first let's conclude our discussion of importance sampling

### The doubly robust estimator

$$V^{\pi_{\theta}}(\mathbf{s}_{\theta}) \approx \sum_{tt'=0}^{T} \left( \prod_{t'=0}^{t'} \frac{\pi_{\theta}((\mathbf{a}_{tt'}|\mathbf{s}_{tt}|\mathbf{s}_{t'}), i)}{\pi_{\beta}((\mathbf{a}_{tt''}|\mathbf{s}_{tt}|\mathbf{s}_{t'}), i)} \right)^{t} \gamma^{t'-t} r_{t',i}$$
$$= \sum_{t=0}^{T} \left( \prod_{t'=0}^{t} \rho_{t'} \right) \gamma^{t} r_{t}$$

$$= \rho_0 r_0 + \rho_0 \gamma \rho_1 r_1 + \rho_0 \gamma \rho_1 \gamma \rho_2 r_2 + \dots$$

$$= \rho_0(r_0 + \gamma(\rho_1(r_1 + \gamma(\rho_2(r_2 + \gamma...)))))$$

$$= \bar{V}^T \qquad \text{where } \bar{V}^{T+1-t} = \rho_t (r_t + \gamma \bar{V}^{T-t})$$

 $\bar{V}_{\mathrm{DR}}^{T+1-t} = \hat{V}(\mathbf{s}_t) + \rho_t (r_t + \gamma \bar{V}_{\mathrm{DR}}^{T-t} - \hat{Q}(\mathbf{s}_t, \mathbf{a}_t))$ 

doubly robust estimation (bandit case)

$$V_{\rm DR}(s) = \hat{V}(s) + \rho(s,a)(r_{s,a} - \hat{Q}(s,a))$$

model or function approximator

1

Jiang, N. and Li, L. (2015). Doubly robust off-policy value evaluation for reinforcement learning.

### Marginalized importance sampling

Main idea: instead of using  $\prod_t \frac{\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)}{\pi_{\beta}(\mathbf{a}_t | \mathbf{s}_t)}$ , estimate  $w(\mathbf{s}, \mathbf{a}) = \frac{d^{\pi_{\theta}}(\mathbf{s}, \mathbf{a})}{d^{\pi_{\beta}}(\mathbf{s}, \mathbf{a})}$ 

if we can do this, we can estimate  $J(\theta) \approx \frac{1}{N} \sum_{i} w(\mathbf{s}_i, \mathbf{a}_i) r_i$ 

typically this is done for off-policy evaluation, rather than policy learning

how to determine  $w(\mathbf{s}, \mathbf{a})$ ? typically solve some kind of consistency condition example (Zhang et al., GenDICE):

$$d^{\pi_{\beta}}(\mathbf{s}', \mathbf{a}')w(\mathbf{s}', \mathbf{a}') = (1 - \gamma)p_0(\mathbf{s}')\pi_{\theta}(\mathbf{a}'|\mathbf{s}') + \gamma \sum_{\mathbf{s}, \mathbf{a}} \pi_{\theta}(\mathbf{a}'|\mathbf{s}')p(\mathbf{s}'|\mathbf{s}, \mathbf{a})d^{\pi_{\beta}}(\mathbf{s}, \mathbf{a})w(\mathbf{s}, \mathbf{a})$$
  
probability of starting in  $(\mathbf{s}', \mathbf{a}')$  probability of transitioning into  $(\mathbf{s}', \mathbf{a}')$ 

solving for  $w(\mathbf{s}, \mathbf{a})$  typically involves some fixed point problem

## Additional readings: importance sampling

#### **Classic work on importance sampled policy gradients and return estimation:**

Precup, D. (2000). Eligibility traces for off-policy policy evaluation. Peshkin, L. and Shelton, C. R. (2002). Learning from scarce experience.

#### Doubly robust estimators and other improved importance-sampling estimators:

Jiang, N. and Li, L. (2015). Doubly robust off-policy value evaluation for reinforcement learning. Thomas, P. and Brunskill, E. (2016). Data-efficient off-policy policy evaluation for reinforcement learning.

#### Analysis and theory:

Thomas, P. S., Theocharous, G., and Ghavamzadeh, M. (2015). High-confidence off-policy evaluation.

#### Marginalized importance sampling:

Hallak, A. and Mannor, S. (2017). Consistent on-line off-policy evaluation. Liu, Y., Swaminathan, A., Agarwal, A., and Brunskill, E. (2019). Off-policy policy gradient with state distribution correction.

#### Additional readings in our offline RL survey: Section 3.1, 3.2, 3.3, 3.4: <u>https://arxiv.org/abs/2005.01643</u>

#### Batch RL via Linear Fitted Value Functions

## Offline value function estimation

#### How have people thought about it before?

- Extend existing ideas for approximate dynamic programming and Q-learning to offline setting
- Derive tractable solutions with simple (e.g., linear) function approximators

#### How are people thinking about it now?

- Derive approximate solutions with highly expressive function approximators (e.g., deep nets)
- The primary challenge turns out to be distributional shift

We'll discuss some older offline/batch RL methods next for completeness generally not concerned with distributional shift before

(maybe it was not such a big problem with linear models)

## Warmup: linear models

 $\Phi$  – feature matrix,  $|S| \times K$ 

could also think of as a vector-valued function  $\Phi(\mathbf{s})$ 

Can we do (offline) model-based RL in feature space?

- 1. Estimate the reward
- 2. Estimate the transitions
- 3. Recover the value function
- 4. Improve the policy
- 1. Reward model:  $\Phi \mathbf{w}_r \approx r$
- 2. Transition model:  $\Phi \mathbf{P}_{\Phi} \approx \mathbf{P}^{\pi} \Phi$

estimated feature-space transition matrix K imes K

real transition matrix (on states)



vector of rewards for all state-action tuples but we'll talk about sample-based setting soon!

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all of this is for a fixed policy \pi
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least squares:  $\mathbf{w}_r = (\Phi^T \Phi)^{-1} \Phi^T \vec{\mathbf{r}}$ 

least squares:  $\mathbf{P}_{\Phi} = (\Phi^T \Phi)^{-1} \Phi^T \mathbf{P}^{\pi} \Phi$ 

material adapted from Ron Parr

## Recovering the value function





Aside: solving for  $V^{\pi}$  in terms of  $\mathbf{P}^{\pi}$  and  $\mathbf{r}$ :

$$V^{\pi} = \mathbf{r} + \gamma \mathbf{P}^{\pi} V^{\pi}$$
$$(\mathbf{I} - \gamma \mathbf{P}^{\pi}) V^{\pi} = \mathbf{r}$$
$$V^{\pi} = (\mathbf{I} - \gamma \mathbf{P}^{\pi})^{-1} \mathbf{r}$$

this is called least-squares temporal difference (LSTD)

material adapted from Ron Parr

 $\mathbf{w}_V = (\Phi^T \Phi - \gamma \Phi^T \mathbf{P}^{\pi} \Phi)^{-1} \Phi^T \vec{\mathbf{r}}$ 



Everything else works **exactly** the same way, only now we have some sampling error

material adapted from Ron Parr

## Improving the policy

- 1. Estimate the reward
- 2. Estimate the transitions

- or just do these together with LSTD!
- 3. Recover the value function
- 4. Improve the policy



That's not going to work for offline RL!

this requires samples from  $\pi$ !

### Least-squares policy iteration (LSPI)

**Main idea:** replace LSTD with LSTDQ – LSTD but for Q-functions

LSPI:

⇒1. compute  $\mathbf{w}_Q$  for  $\pi_k$ 

encode the action  $\pi$  would take not the action in the data

2. 
$$\pi_{k+1}(\mathbf{s}) = \arg \max_{\mathbf{a}} \phi(\mathbf{s}, \mathbf{a}) \mathbf{w}_Q$$

**3.** Set 
$$\Phi'_i = \phi(\mathbf{s}'_i, \pi_{k+1}(\mathbf{s}'_i))$$



 $\Phi$ |S||A|total # of states-action tuples



### What's the issue?



how well it does

how well it *thinks* it does (Q-values) In general, all approximate dynamic programming (e.g., fitted value/Q iteration) methods will suffer from action distributional shift, and we **must** fix it!

$$Q(\mathbf{s}, \mathbf{a}) \leftarrow r(\mathbf{s}, \mathbf{a}) + E_{\mathbf{a}' \sim \pi_{\text{new}}}[Q(\mathbf{s}', \mathbf{a}')]$$
$$y(\mathbf{s}, \mathbf{a})$$

 $\min_{Q} E_{(\mathbf{s},\mathbf{a})\sim\pi_{\beta}(\mathbf{s},\mathbf{a})} \left[ (Q(\mathbf{s},\mathbf{a}) - y(\mathbf{s},\mathbf{a}))^{2} \right]$   $\int \\ \text{target value}$ behavior policy

expect good accuracy when  $\pi_{\beta}(\mathbf{a}|\mathbf{s}) = \pi_{\text{new}}(\mathbf{a}|\mathbf{s})$  how often does that happen?

even worse:  $\pi_{\text{new}} = \arg \max_{\pi} E_{\mathbf{a} \sim \pi(\mathbf{a}|\mathbf{s})}[Q(\mathbf{s}, \mathbf{a})]$