IASD M2 at Paris Dauphine

Deep Reinforcement Learning

13: Model-Based Policy Learning

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Acknowledgement

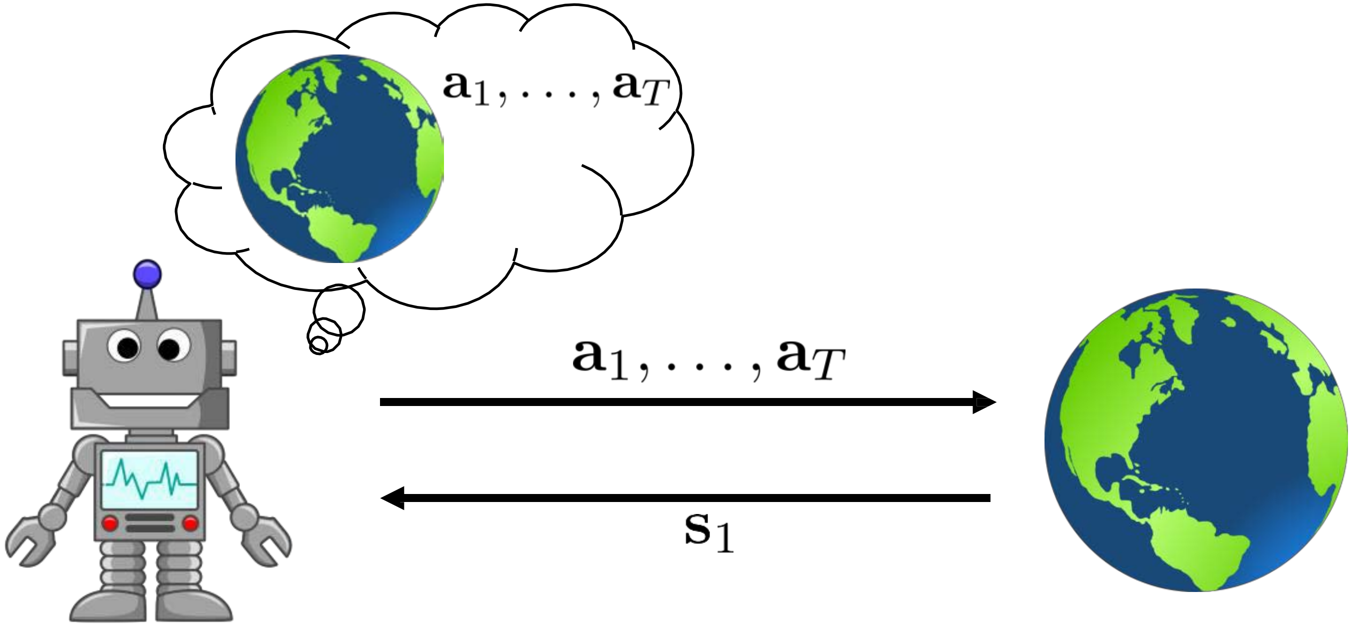
These materials are based on the seminal course of Sergey Levine CS285

Last time: model-based RL with MPC

model-based reinforcement learning version 1.5:

1. run base policy $\pi_0(\mathbf{a}_t|\mathbf{s}_t)$ (e.g., random policy) to collect $\mathcal{D} = \{(\mathbf{s}, \mathbf{a}, \mathbf{s}')_i\}$
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. plan through $f(\mathbf{s}, \mathbf{a})$ to choose actions
4. execute the first planned action, observe resulting state $\mathbf{s}'$ (MPC)
5. append $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to dataset $\mathcal{D}$

The stochastic open-loop case

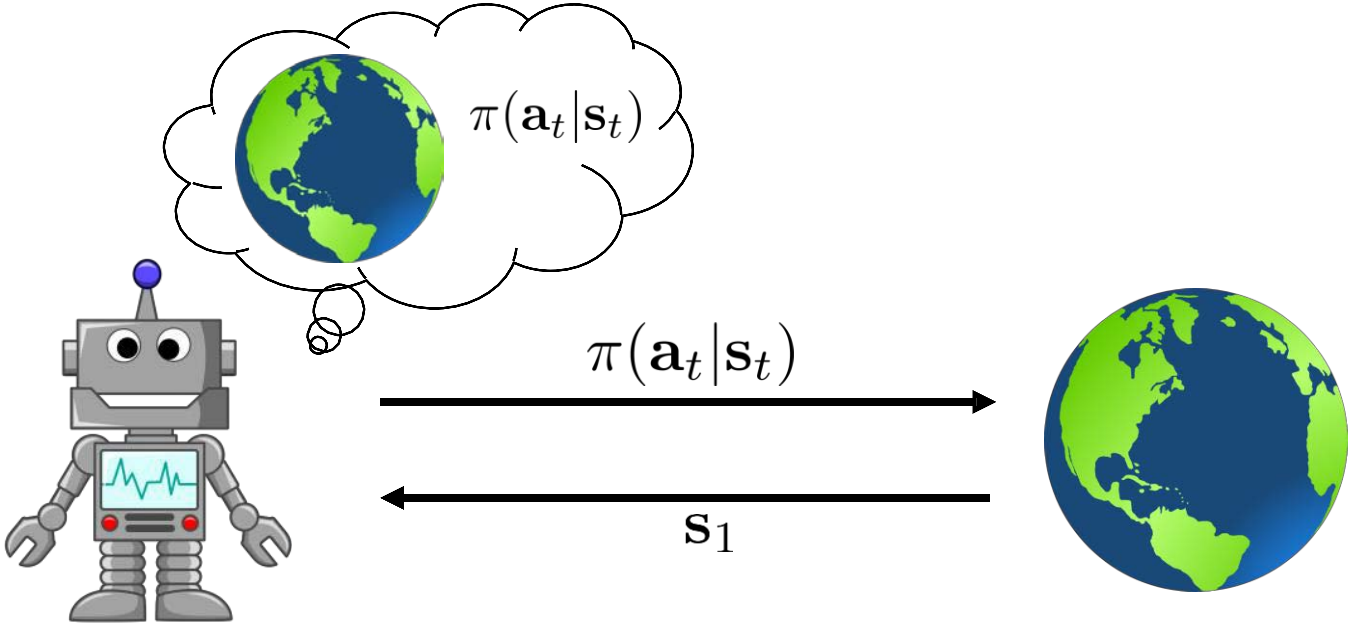


$$p_\theta(\mathbf{s}_1, \ldots, \mathbf{s}_T | \mathbf{a}_1, \ldots, \mathbf{a}_T) = p(\mathbf{s}_1) \prod_{t=1}^{T} p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$

$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} E\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) | \mathbf{a}_1, \ldots, \mathbf{a}_T\right]$$

why is this suboptimal?

The stochastic closed-loop case



form of π?

neural net global

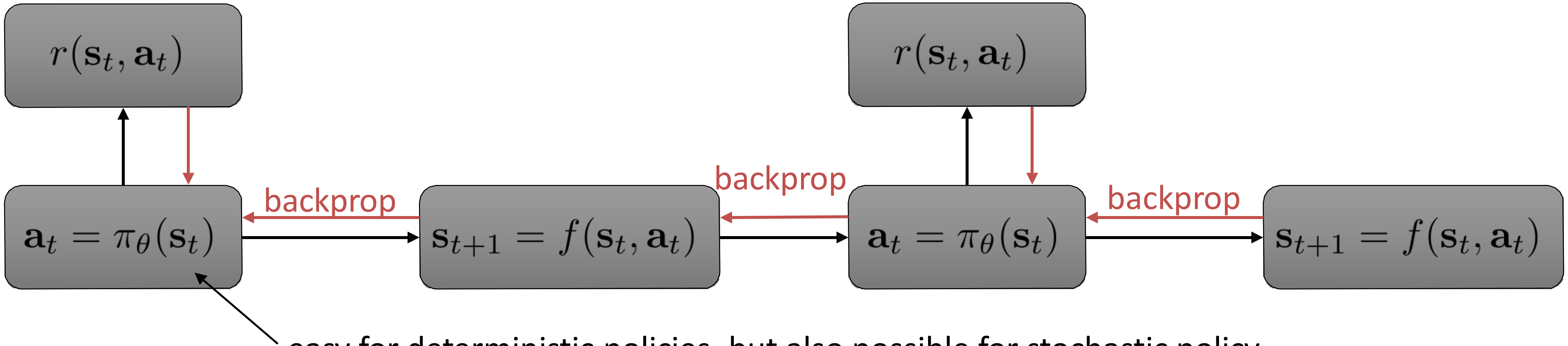
time-varying linear

$\mathbf{K}_t\mathbf{s}_t + \mathbf{k}_t$ local

$$p(\mathbf{s}_1, \mathbf{a}_1, \ldots, \mathbf{s}_T, \mathbf{a}_T) = p(\mathbf{s}_1)\prod_{t=1}^{T}\pi(\mathbf{a}_t|\mathbf{s}_t)p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\pi = \arg\max_{\pi} E_{\tau\sim p(\tau)}\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right]$$

Backpropagate directly into the policy?



easy for deterministic policies, but also possible for stochastic policy

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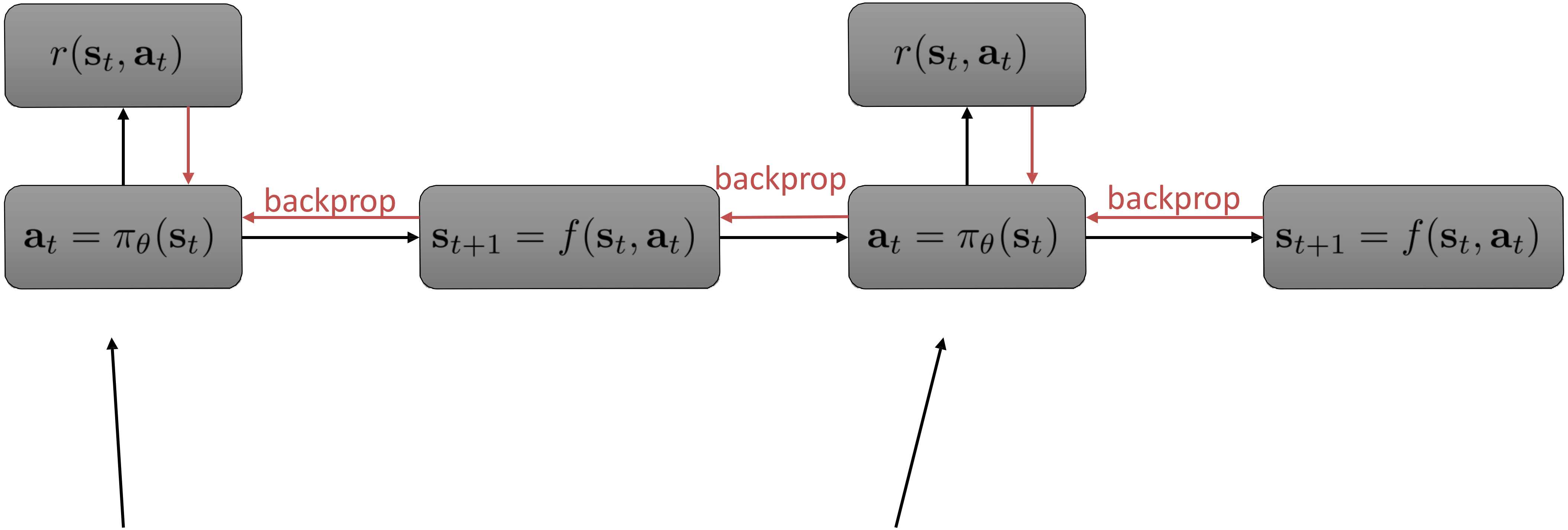
2. learn dynamics model $f(\mathbf{s}, \mathbf{a})$ to minimize $\sum_i \|f(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{s}'_i\|^2$
3. backpropagate through $f(\mathbf{s}, \mathbf{a})$ into the policy to optimize $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
4. run $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$, appending the visited tuples $(\mathbf{s}, \mathbf{a}, \mathbf{s}')$ to $\mathcal{D}$

What's the problem with backprop into policy?



big gradients here

small gradients here

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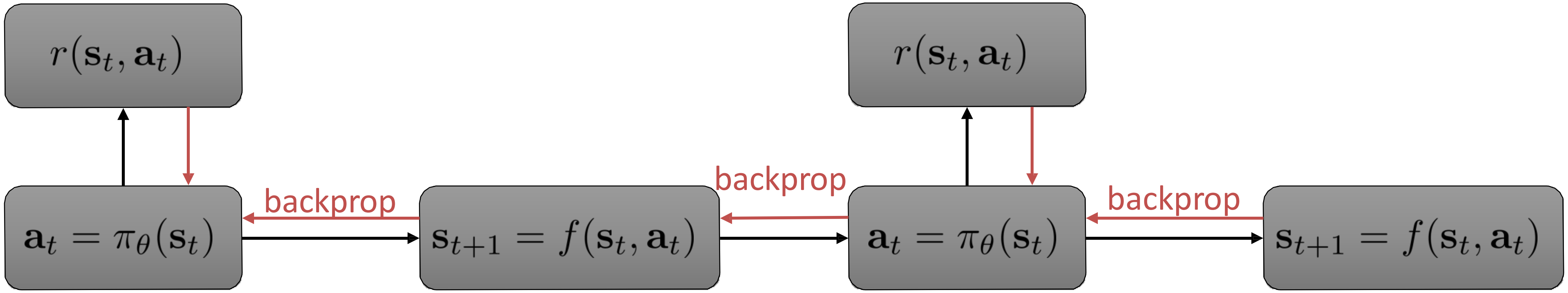

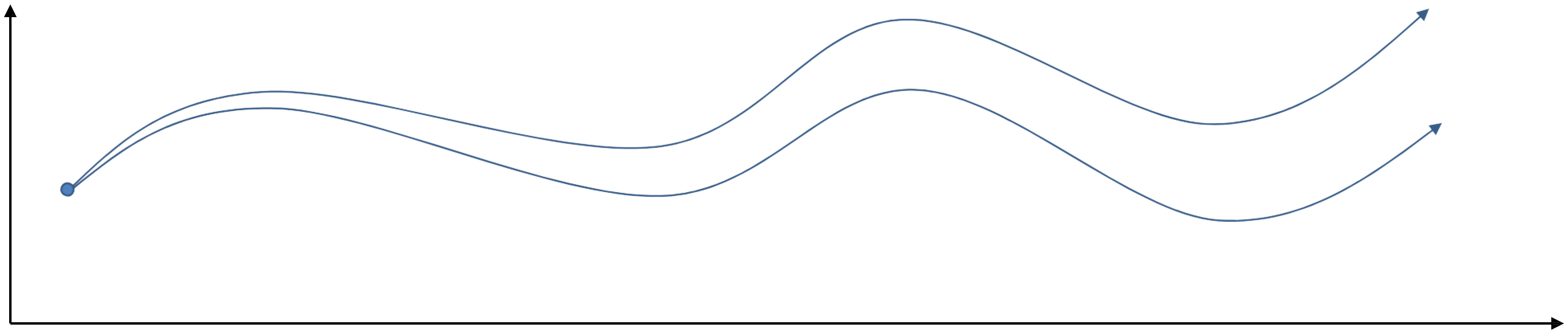

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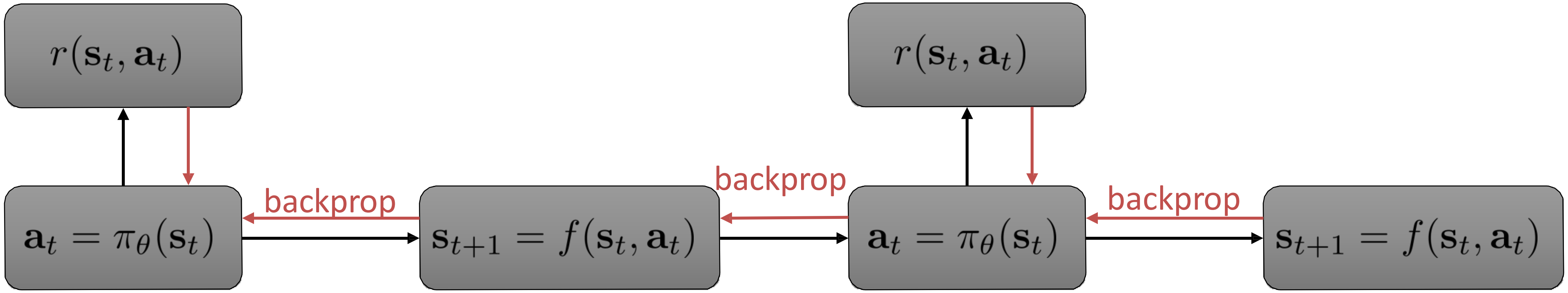

- Similar parameter sensitivity problems as shooting methods
  - But no longer have convenient second order LQR-like method, because policy parameters couple all the time steps, so no dynamic programming
- Similar problems to training long RNNs with BPTT
  - Vanishing and exploding gradients
  - Unlike LSTM, we can't just "choose" a simple dynamics, dynamics are chosen by nature

What’s the solution?

- Use derivative-free (“model-free”) RL algorithms, with the model used to generate synthetic samples
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Model-Free Learning With a Model

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Model-free optimization with a model

Policy gradient: $$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \hat{Q}^{\pi}_{i,t}$$

Backprop (pathwise) gradient: $$\nabla_\theta J(\theta) = \sum_{t=1}^{T} \frac{dr_t}{d\mathbf{s}_t} \prod_{t'=2}^{t} \frac{d\mathbf{s}_{t'}}{d\mathbf{a}_{t'-1}} \frac{d\mathbf{a}_{t'-1}}{d\mathbf{s}_{t'-1}}$$

- Policy gradient might be more *stable* (if enough samples are used) because it does not require multiplying many Jacobians
- See a recent analysis here:
  - Parmas et al. '18: PIPP: Flexible Model-Based Policy Search Robust to the Curse of Chaos

Model-free optimization with a model

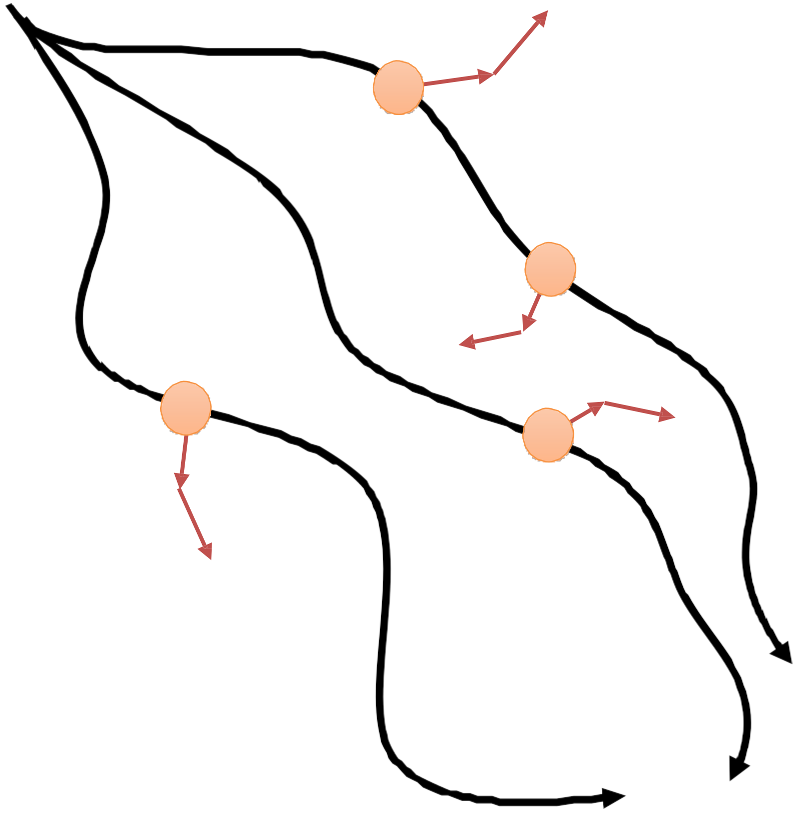
Dyna online Q-learning algorithm that performs model-free RL with a model

1. given state s, pick action a using exploration policy
2. observe s' and r, to get transition (s, a, s', r)
3. update model $\hat{p}(s'|s, a)$ and $\hat{r}(s, a)$ using (s, a, s')
4. Q-update: $Q(s, a) \leftarrow Q(s, a) + \alpha E_{s',r}[r + \max_{a'} Q(s', a') - Q(s, a)]$
5. repeat K times:
   6. sample $(s, a) \sim \mathcal{B}$ from buffer of past states and actions
   7. Q-update: $Q(s, a) \leftarrow Q(s, a) + \alpha E_{s',r}[r + \max_{a'} Q(s', a') - Q(s, a)]$

Richard S. Sutton. Integrated architectures for learning, planning, and reacting based on approximating dynamic programming.

General “Dyna-style” model-based RL recipe

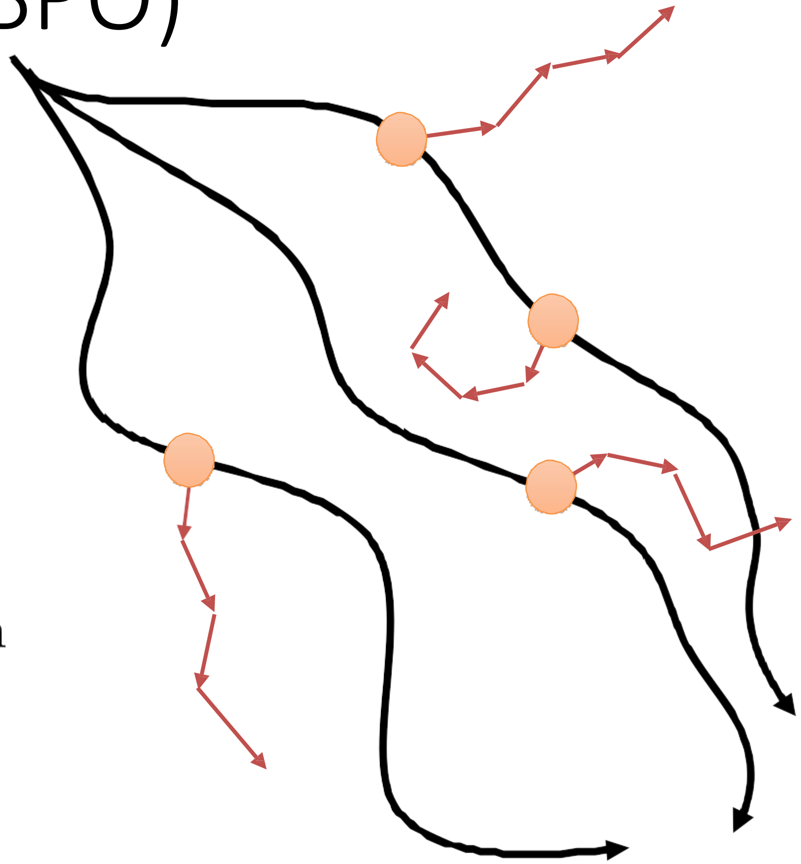
1. collect some data, consisting of transitions (s, a, s', r)
2. learn model $\hat{p}(s'|s, a)$ (and optionally, $\hat{r}(s, a)$)
3. repeat K times:
    4. sample $s \sim \mathcal{B}$ from buffer
    5. choose action a (from $\mathcal{B}$, from π, or random)
    6. simulate $s' \sim \hat{p}(s'|s, a)$ (and $r = \hat{r}(s, a)$)
    7. train on (s, a, s', r) with model-free RL
    8. (optional) take N more model-based steps



+ only requires short (as few as one step) rollouts from model

+ still sees diverse states

Model-Based Acceleration (MBA)
Model-Based Value Expansion (MVE)
Model-Based Policy Optimization (MBPO)



1. take some action $\mathbf{a}_i$ and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to $\mathcal{B}$
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from $\mathcal{B}$ uniformly
3. use $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j\}$ to update model $\hat{p}(\mathbf{s}'|\mathbf{s}, \mathbf{a})$
4. sample $\{\mathbf{s}_j\}$ from $\mathcal{B}$
5. for each $\mathbf{s}_j$, perform model-based rollout with $\mathbf{a} = \pi(\mathbf{s})$
6. use all transitions $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$ along rollout to update Q-function

+ why is this a *good* idea?

- why is this a *bad* idea?

Gu et al. Continuous deep Q-learning with model-based acceleration. '16
Feinberg et al. Model-based value expansion. '18
Janner et al. When to trust your model: model-based policy optimization. '19

Local Models

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Local models

$$\min_{\mathbf{u}_1,\dots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \text{ s.t. } \mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$$

$$\min_{\mathbf{u}_1,\dots,\mathbf{u}_T} c(\mathbf{x}_1, \mathbf{u}_1) + c(f(\mathbf{x}_1, \mathbf{u}_1), \mathbf{u}_2) + \cdots + c(f(f(\dots)\dots), \mathbf{u}_T)$$

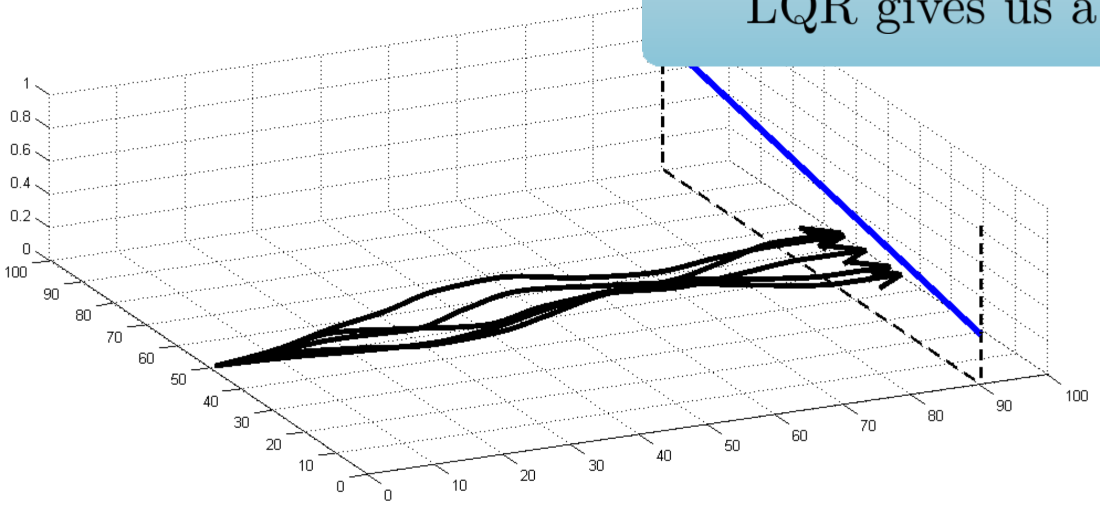
usual story: differentiate via backpropagation and optimize!

need $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}, \frac{dc}{d\mathbf{x}_t}, \frac{dc}{d\mathbf{u}_t}$

Local models

need $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}, \frac{dc}{d\mathbf{x}_t}, \frac{dc}{d\mathbf{u}_t}$

idea: just fit $\frac{df}{d\mathbf{x}_t}, \frac{df}{d\mathbf{u}_t}$ around current trajectory or policy!



LQR gives us a linear feedback controller

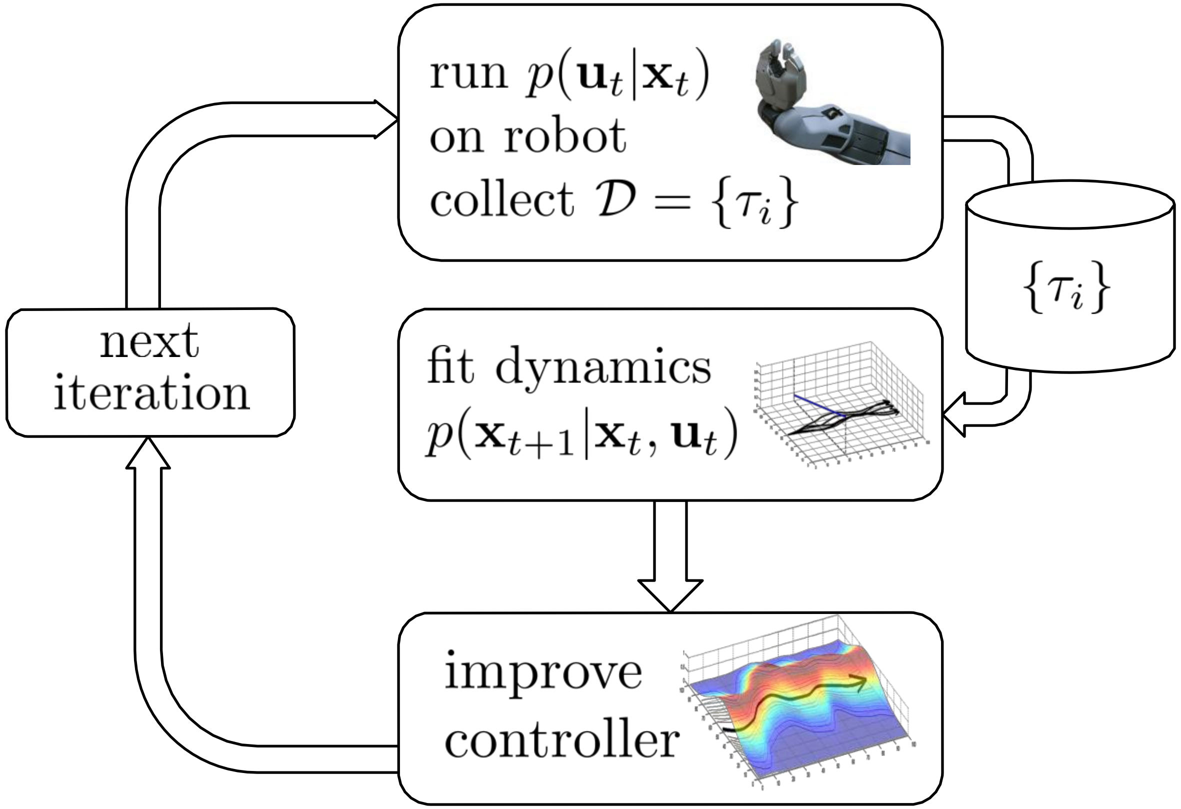
can **execute** in the real world!

Local models

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(f(\mathbf{x}_t, \mathbf{u}_t), \Sigma)$$

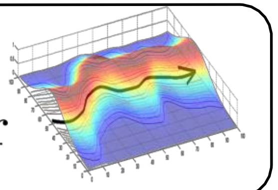
$$f(\mathbf{x}_t, \mathbf{u}_t) \approx \mathbf{A}_t \mathbf{x}_t + \mathbf{B}_t \mathbf{u}_t$$

$$\mathbf{A}_t = \frac{df}{d\mathbf{x}_t} \qquad \mathbf{B}_t = \frac{df}{d\mathbf{u}_t}$$



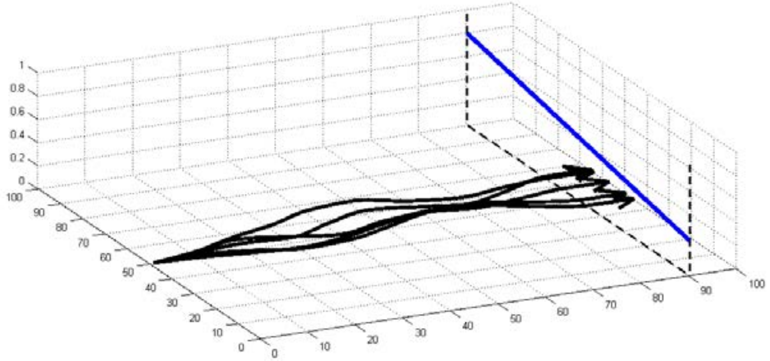
What controller to execute?

improve controller



iLQR produces: $\hat{\mathbf{x}}_t, \hat{\mathbf{u}}_t, \mathbf{K}_t, \mathbf{k}_t$

$$\mathbf{u}_t = \mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t$$



Version 0.5: $p(\mathbf{u}_t|\mathbf{x}_t) = \delta(\mathbf{u}_t = \hat{\mathbf{u}}_t)$

Doesn't correct deviations or drift

Version 1.0: $p(\mathbf{u}_t|\mathbf{x}_t) = \delta(\mathbf{u}_t = \mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t)$

Better, but maybe a little too good?

Version 2.0: $p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$

Add noise so that all samples don't look the same!

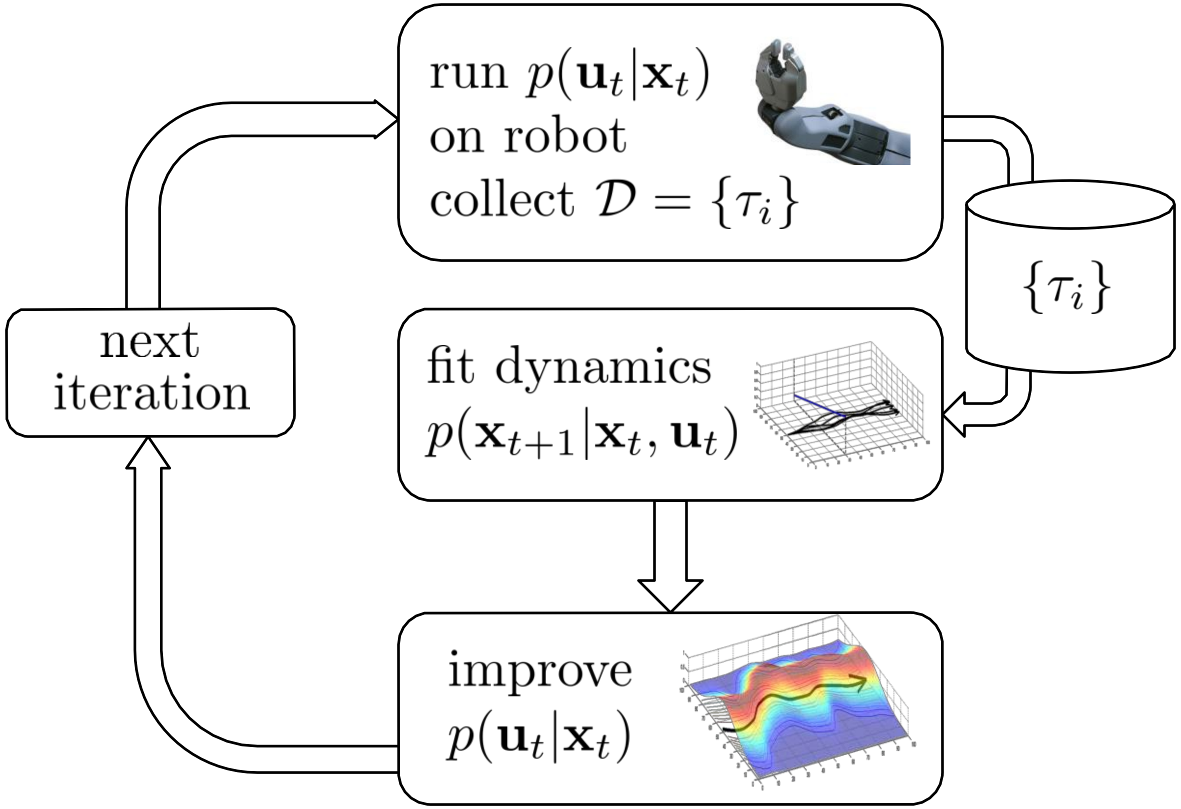
Set $\Sigma_t = \mathbf{Q}_{\mathbf{u}_t,\mathbf{u}_t}^{-1}$

Local models

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(f(\mathbf{x}_t, \mathbf{u}_t), \Sigma)$$

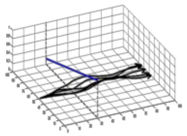
$$f(\mathbf{x}_t, \mathbf{u}_t) \approx \mathbf{A}_t \mathbf{x}_t + \mathbf{B}_t \mathbf{u}_t$$

$$\mathbf{A}_t = \frac{df}{d\mathbf{x}_t} \qquad \mathbf{B}_t = \frac{df}{d\mathbf{u}_t}$$



How to fit the dynamics?

fit dynamics
$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$

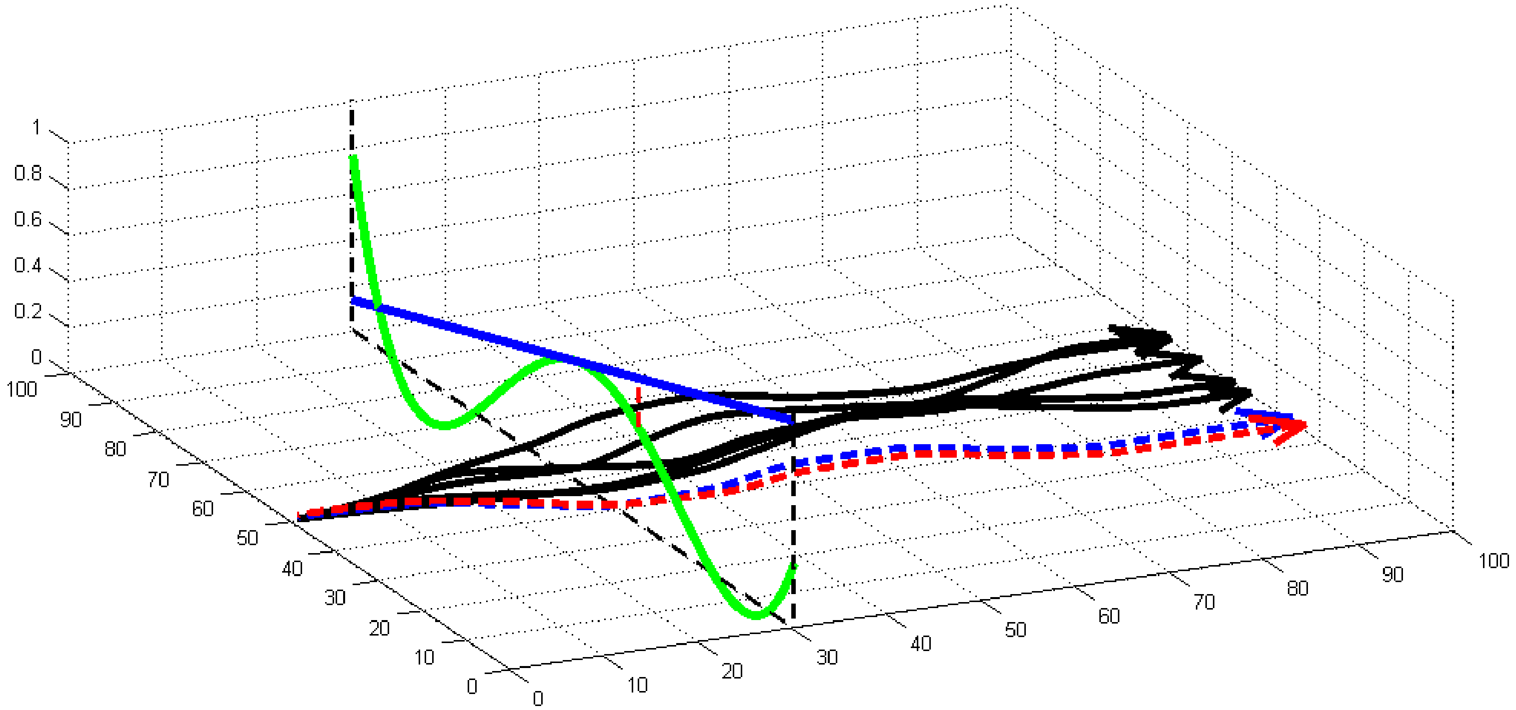


$\{(\mathbf{x}_t, \mathbf{u}_t, \mathbf{x}_{t+1})_i\}$

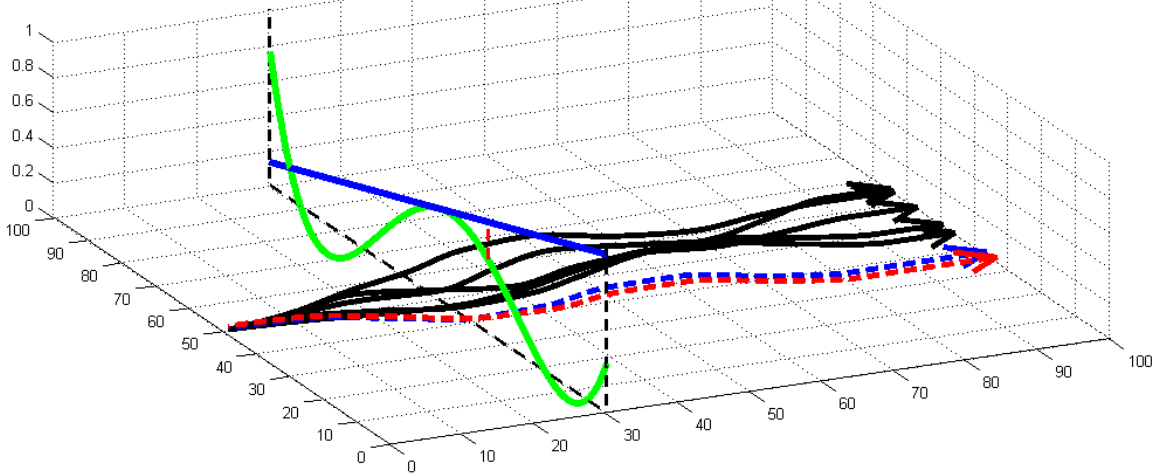
fit $p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$ at each time step using linear regression

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(\mathbf{A}_t\mathbf{x}_t + \mathbf{B}_t\mathbf{u}_t + \mathbf{c}, \mathbf{N}_t) \qquad \mathbf{A}_t \approx \frac{df}{d\mathbf{x}_t} \quad \mathbf{B}_t \approx \frac{df}{d\mathbf{u}_t}$$

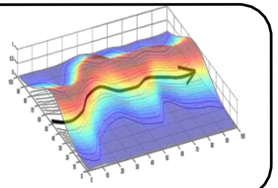
What if we go too far?



How to stay close to old controller?



improve
$p(\mathbf{u}_t|\mathbf{x}_t)$



$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \Sigma_t)$$

$$p(\tau) = p(\mathbf{x}_1) \prod_{t=1}^{T} p(\mathbf{u}_t|\mathbf{x}_t) p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$$

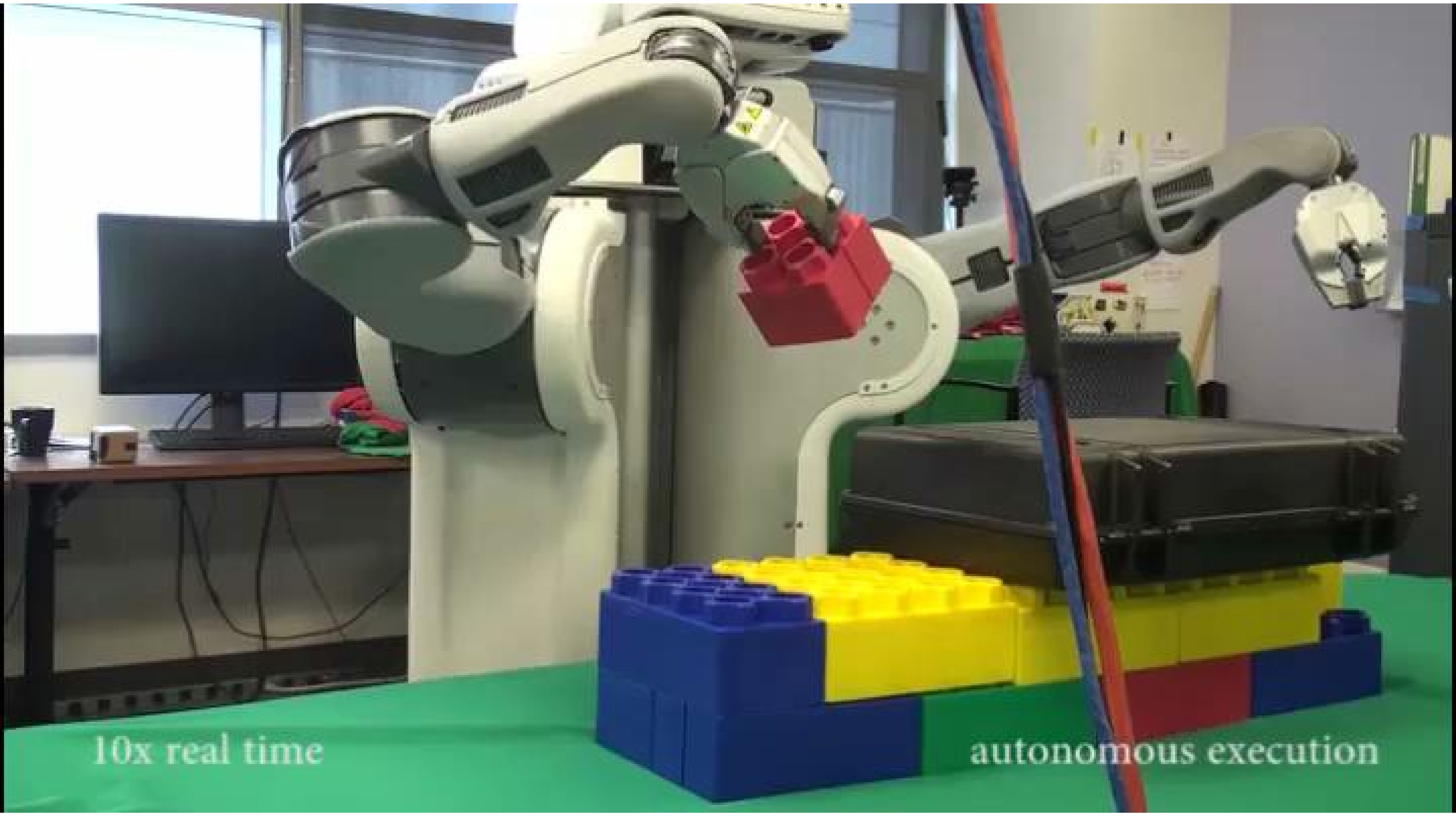
What if the new $p(\tau)$ is "close" to the old one $\bar{p}(\tau)$?

If trajectory distribution is close, then dynamics will be close too!

What does "close" mean? $D_{\text{KL}}(p(\tau)\|\bar{p}(\tau)) \leq \epsilon$

This is easy to do if $\bar{p}(\tau)$ also came from linear controller!

For details, see: "**Learning Neural Network Policies with Guided Policy Search under Unknown Dynamics**"


10x real time
autonomous execution

Global Policies from Local Models

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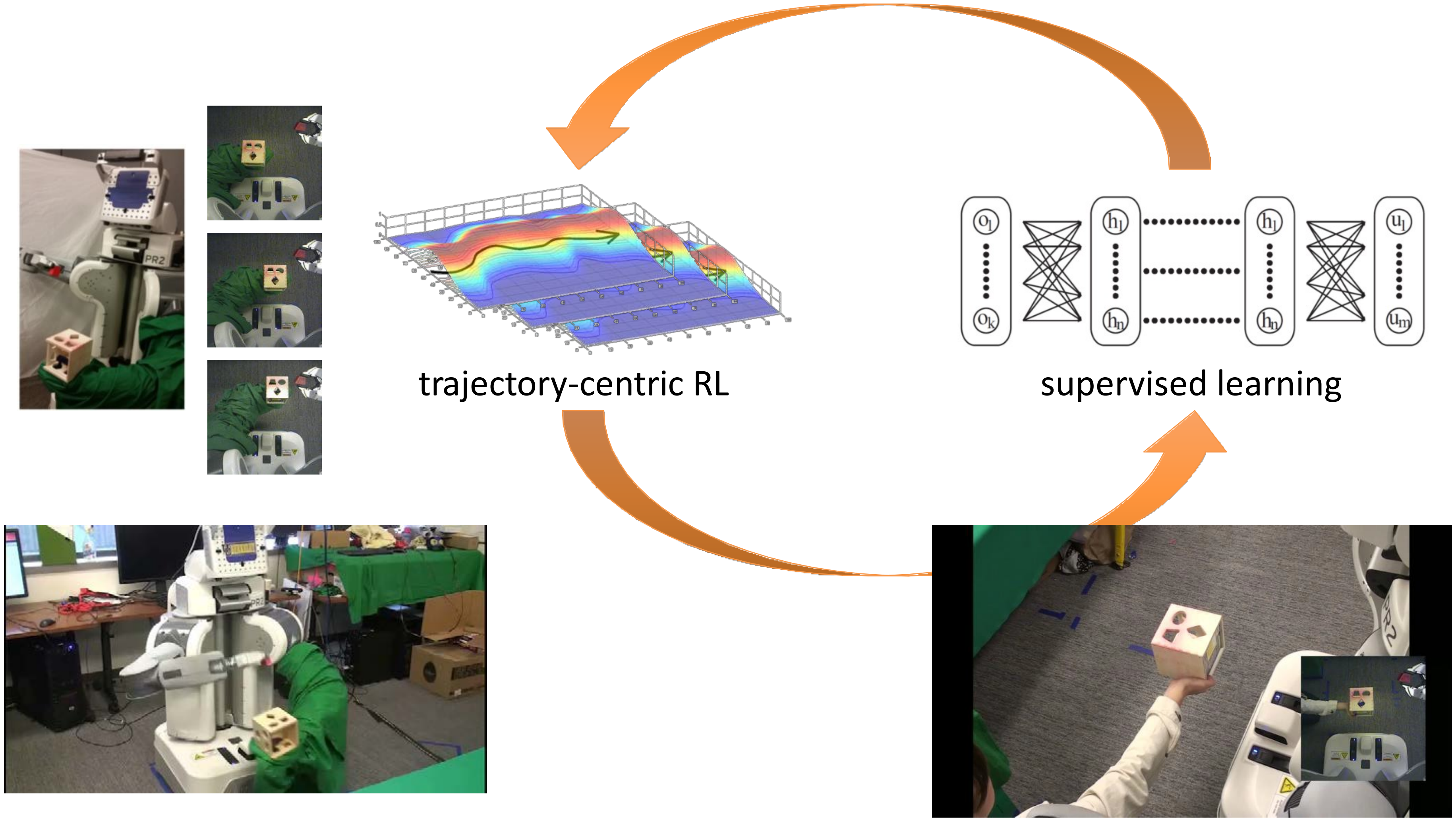
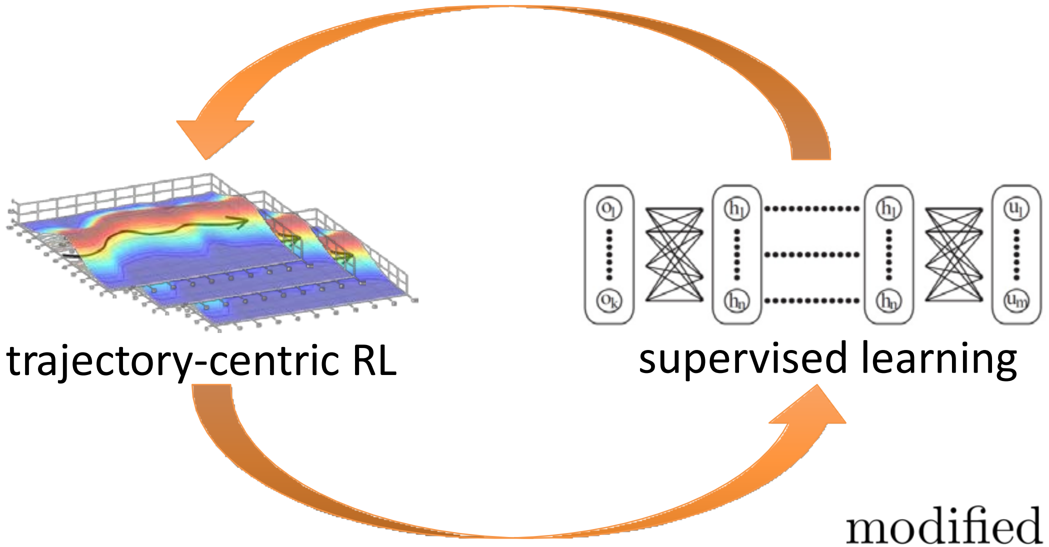
Guided policy search: high-level idea



Guided policy search: algorithm sketch



modified cost to keep $\pi_{\text{LQR},i}$ close to π_θ

1. optimize each local policy $\pi_{\text{LQR},i}(\mathbf{u}_t|\mathbf{x}_t)$ on initial state $\mathbf{x}_{0,i}$ w.r.t. $\tilde{c}_{k,i}(\mathbf{x}_t, \mathbf{u}_t)$
2. use samples from step (1) to train $\pi_\theta(\mathbf{u}_t|\mathbf{x}_t)$ to mimic each $\pi_{\text{LQR},i}(\mathbf{u}_t|\mathbf{x}_t)$
3. update cost function $\tilde{c}_{k+1,i}(\mathbf{x}_t, \mathbf{u}_t) = c(\mathbf{x}_t, \mathbf{u}_t) + \lambda_{k+1,i} \log \pi_\theta(\mathbf{u}_t|\mathbf{x}_t)$

Lagrange multiplier

For details, see: “End-to-End Training of Deep Visuomotor Policies”

Underlying principle: distillation

Ensemble models: single models are often not the most robust – instead train many models and average their predictions

this is how most ML competitions (e.g., Kaggle) are won

this is very expensive at test time

Can we make a single model that is as good as an ensemble?

Distillation: train on the ensemble's predictions as "soft" targets

logit

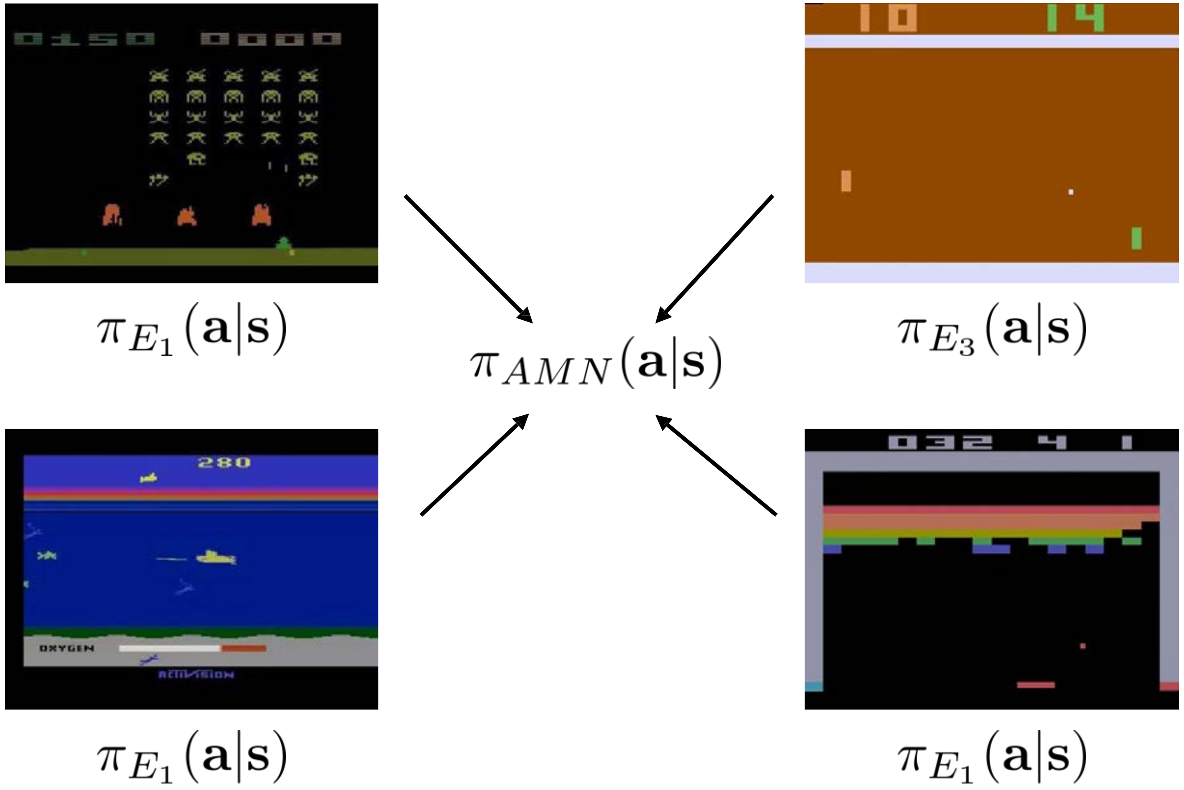
$$p_i = \frac{\exp(z_i/T)}{\sum_j \exp(z_j/T)}$$

temperature

Intuition: more knowledge in soft targets than hard labels!

Slide adapted from G. Hinton, see also Hinton et al. "Distilling the Knowledge in a Neural Network"

Distillation for Multi-Task Transfer



$$\mathcal{L} = \sum_{\mathbf{a}} \pi_{E_i}(\mathbf{a}|\mathbf{s}) \log \pi_{AMN}(\mathbf{a}|\mathbf{s})$$

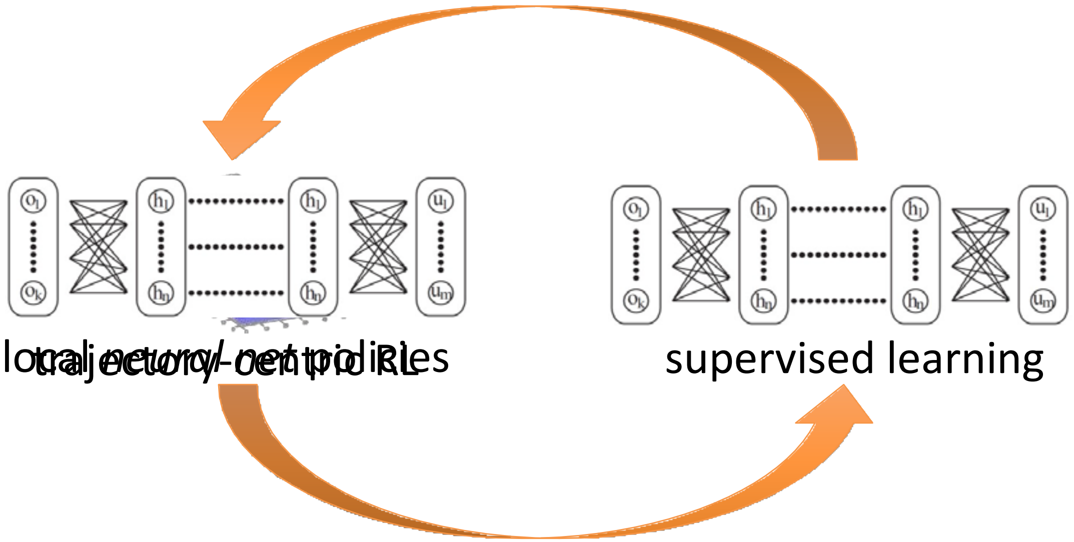
(just supervised learning/distillation)

analogous to guided policy search, but for multi-task learning

some other details
(e.g., feature regression objective)
– see paper

Parisotto et al. "Actor-Mimic: Deep Multitask and Transfer Reinforcement Learning"

Combining weak policies into a strong policy



Divide and Conquer Reinforcement Learning

Divide and conquer reinforcement learning algorithm sketch:

1. optimize each local policy $\pi_{\theta_i}(\mathbf{a}_t|\mathbf{s}_t)$ on initial state $\mathbf{s}_{0,i}$ w.r.t. $\tilde{r}_{k,i}(\mathbf{s}_t, \mathbf{a}_t)$
2. use samples from step (1) to train $\pi_\theta(\mathbf{u}_t|\mathbf{x}_t)$ to mimic each $\pi_{\theta_i}(\mathbf{u}_t|\mathbf{x}_t)$
3. update reward function $\tilde{r}_{k+1,i}(\mathbf{x}_t, \mathbf{u}_t) = r(\mathbf{x}_t, \mathbf{u}_t) + \lambda_{k+1,i} \log \pi_\theta(\mathbf{u}_t|\mathbf{x}_t)$

For details, see: “Divide and Conquer Reinforcement Learning”

Readings: guided policy search & distillation

- L.*, Finn*, et al. End-to-End Training of Deep Visuomotor Policies. 2015.
- Rusu et al. Policy Distillation. 2015.
- Parisotto et al. Actor-Mimic: Deep Multitask and Transfer Reinforcement Learning. 2015.
- Ghosh et al. Divide-and-Conquer Reinforcement Learning. 2017.
- Teh et al. Distral: Robust Multitask Reinforcement Learning. 2017.