IASD M2 at Paris Dauphine

Deep Reinforcement Learning

6: Actor-Critic Algorithms

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Acknowledgement

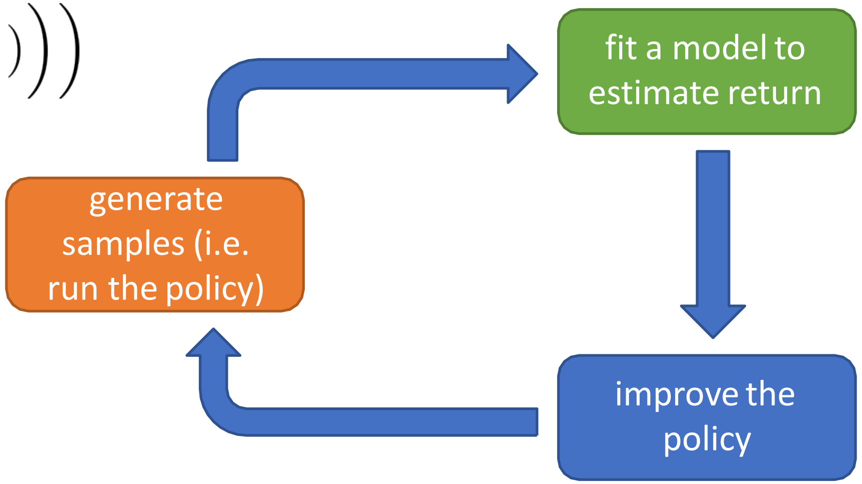
Most of the materials of this course is based on the seminal course of Sergey Levine CS285

Recap: policy gradients

REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run the policy)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i) \left(\sum_{t'=t}^T r(\mathbf{s}_{t'}^i, \mathbf{a}_{t'}^i) \right) \right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$$\hat{Q}^\pi(\mathbf{x}_t, \mathbf{u}_t) = \sum_{t'=t}^T r(\mathbf{x}_{t'}, \mathbf{u}_{t'})$$



$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

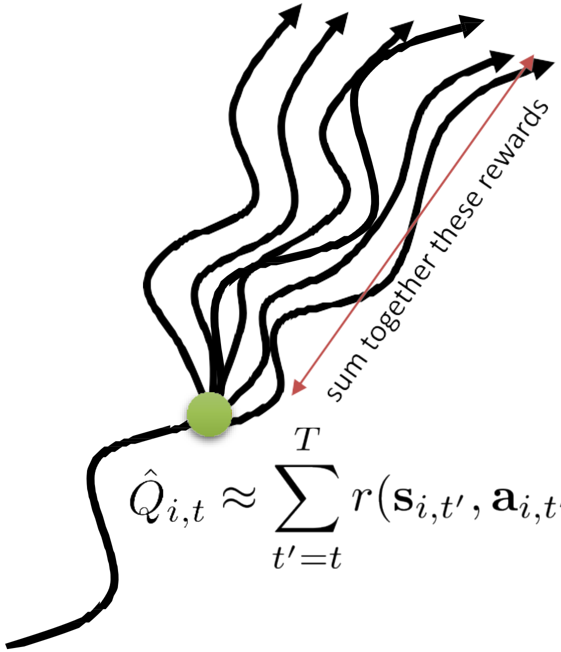
$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \underbrace{\hat{Q}_{i,t}^\pi}_{\text{“reward to go”}}$$

“reward to go”

Improving the policy gradient

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\underbrace{\left(\sum_{t'=1}^{T} r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right)}_{\text{"reward to go"}}$$

$$\hat{Q}_{i,t}$$



$$\hat{Q}_{i,t} \approx \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})$$

$\hat{Q}_{i,t}$: estimate of expected reward if we take action $\mathbf{a}_{i,t}$ in state $\mathbf{s}_{i,t}$

can we get a better estimate?

$Q(\mathbf{s}_t,\mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'},\mathbf{a}_{t'})|\mathbf{s}_t,\mathbf{a}_t\right]$: true *expected* reward-to-go

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) Q(\mathbf{s}_{i,t},\mathbf{a}_{i,t})$$

What about the baseline?

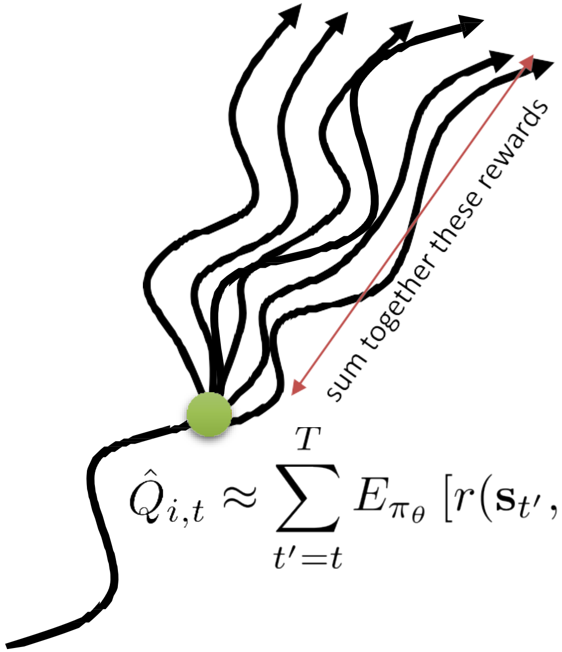
$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$: true *expected* reward-to-go

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) - V(\mathbf{s}_{i,t})\right)$$

$$b_t = \frac{1}{N} \sum_i Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

average what?

$$V(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q(\mathbf{s}_t, \mathbf{a}_t)]$$



$$\hat{Q}_{i,t} \approx \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

State & state-action value functions

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$: total reward from taking $\mathbf{a}_t$ in $\mathbf{s}_t$

$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$: total reward from $\mathbf{s}_t$

$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$: how much better $\mathbf{a}_t$ is

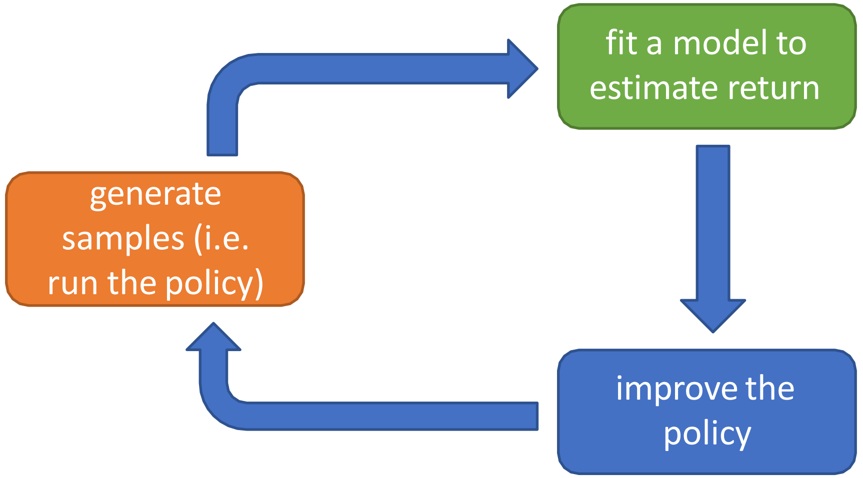
$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

the better this estimate, the lower the variance

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\sum_{t'=1}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) - b\right)$$

unbiased, but high variance single-sample estimate

fit Q^π, V^π, or A^π



$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

fit *what* to *what*?

Q^π, V^π, A^π?

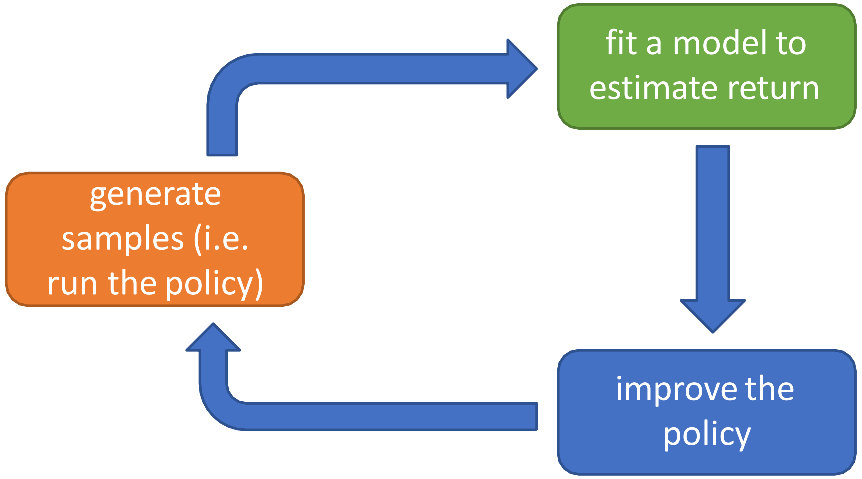
$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \sum_{t'=t+1}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t\right]$$

$V^\pi(\mathbf{s}_{t+1})$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1}) - V^\pi(\mathbf{s}_t)$$

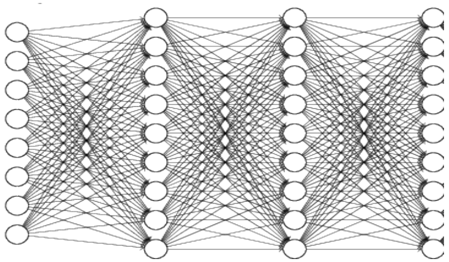
let's just fit $V^\pi(\mathbf{s})$!

fit Q^π, V^π, or A^π



$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

$\mathbf{s}$

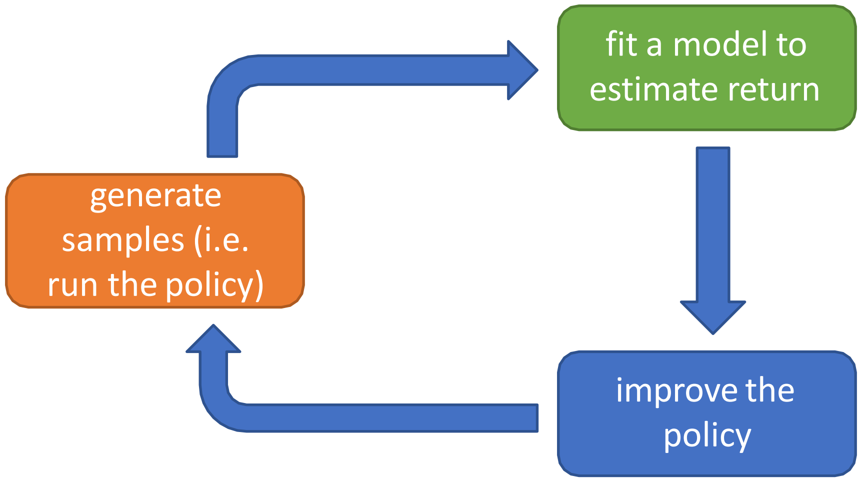


$\hat{V}^\pi(\mathbf{s})$

parameters ϕ

Policy evaluation

$\text{fit } V^\pi$



$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t\right]$

$J(\theta) = E_{\mathbf{s}_1 \sim p(\mathbf{s}_1)}[V^\pi(\mathbf{s}_1)]$

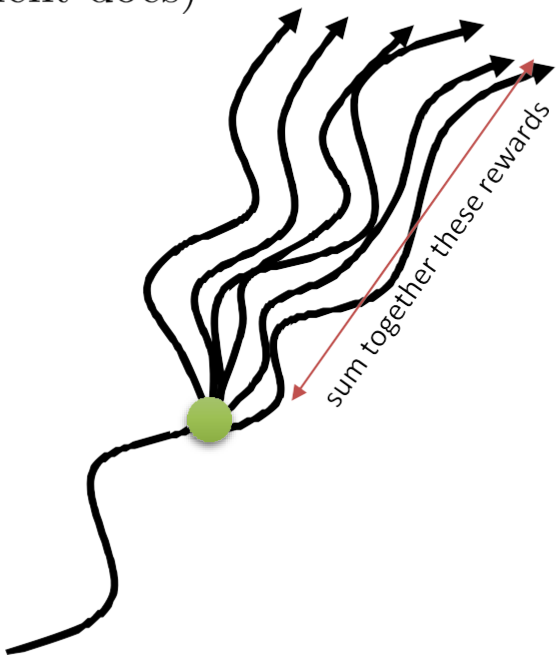
how can we perform policy evaluation?

Monte Carlo policy evaluation (this is what policy gradient does)

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$$

$$V^\pi(\mathbf{s}_t) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$$

(requires us to reset the simulator)

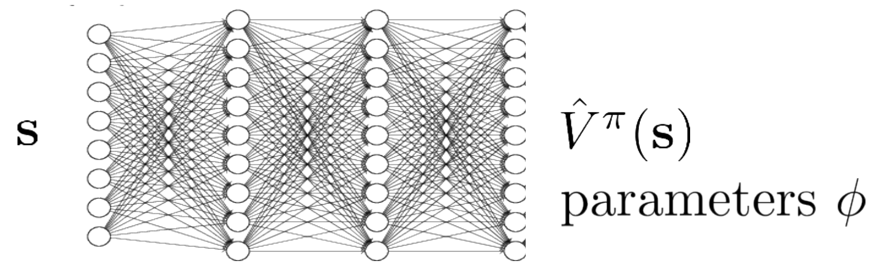


Monte Carlo evaluation with function approximation

$$V^\pi(\mathbf{s}_t) \approx \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$$

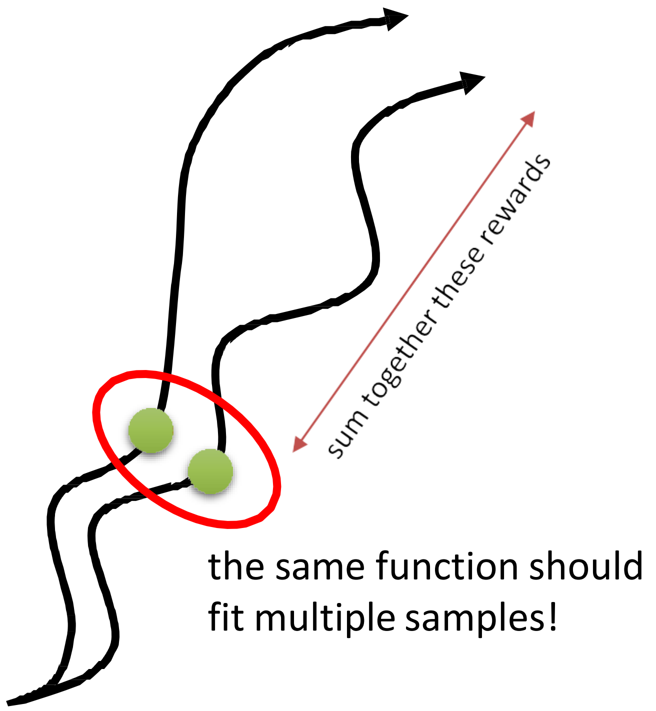
not as good as this: $V^\pi(\mathbf{s}_t) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'})$

but still pretty good!



training data: $\left\{ \left(\mathbf{s}_{i,t}, \underbrace{\sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})}_{y_{i,t}} \right) \right\}$

supervised regression: $\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}_\phi^\pi(\mathbf{s}_i) - y_i \right\|^2$



Can we do better?

ideal target: $y_{i,t} = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_{i,t}\right] \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + V^\pi(\mathbf{s}_{i,t+1}) \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \underbrace{\hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})}$

Monte Carlo target: $y_{i,t} = \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$

directly use previous fitted value function!

training data: $\left\{\left(\mathbf{s}_{i,t}, \underbrace{r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})}_{y_{i,t}}\right)\right\}$

supervised regression: $\mathcal{L}(\phi) = \frac{1}{2}\sum_i \left\|\hat{V}_\phi^\pi(\mathbf{s}_i) - y_i\right\|^2$

sometimes referred to as a “bootstrapped” estimate

Policy evaluation examples

TD-Gammon, Gerald Tesauro 1992

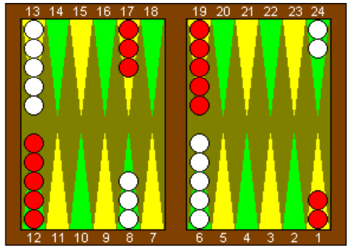


Figure 2. An illustration of the normal opening position in backgammon. TD-Gammon has sparked a near-universal conversion in the way experts play certain opening rolls. For example, with an opening roll of 4-1, most players have now switched from the traditional move of 13-9, 6-5, to TD-Gammon's preference, 13-9, 24-23. TD-Gammon's analysis is given in Table 2.

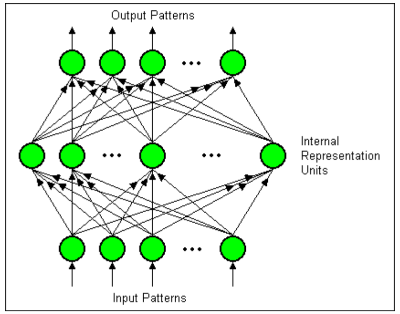


Figure 1. An illustration of the multilayer perception architecture used in TD-Gammon's neural network. This architecture is also used in the popular backpropagation learning procedure. Figure reproduced from [9].

reward: game outcome

value function $\hat{V}^{\pi}_{\phi}(\mathbf{s}_t)$:

expected outcome given board state

AlphaGo, Silver et al. 2016

reward: game outcome

value function $\hat{V}^{\pi}_{\phi}(\mathbf{s}_t)$:

expected outcome given board state

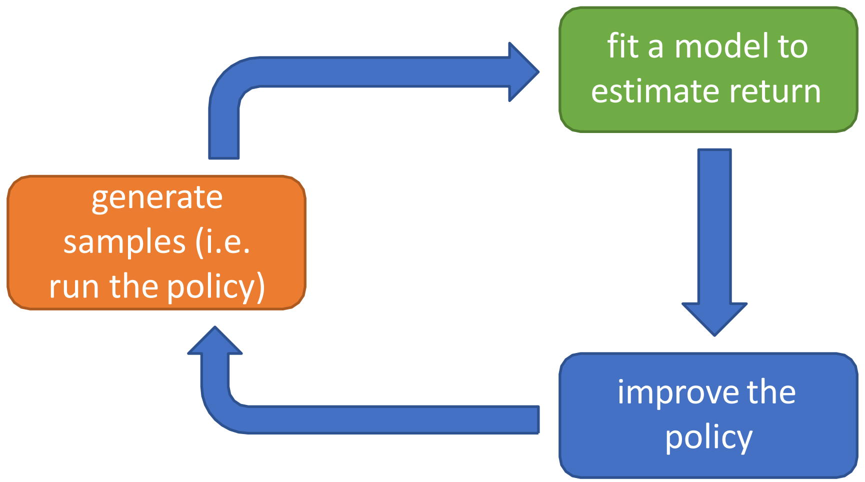
From Evaluation to Actor Critic

An actor-critic algorithm

batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i)\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

fit $\hat{V}^\pi$

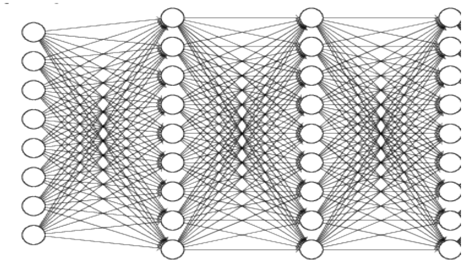


$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$y_{i,t} \approx \sum_{t'=t}^{T} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$

$$\mathcal{L}(\phi) = \frac{1}{2}\sum_i \left\| \hat{V}^\pi_\phi(\mathbf{s}_i) - y_i \right\|^2$$

$\mathbf{s}$



$\hat{V}^\pi(\mathbf{s})$

parameters ϕ

$$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t\right]$$

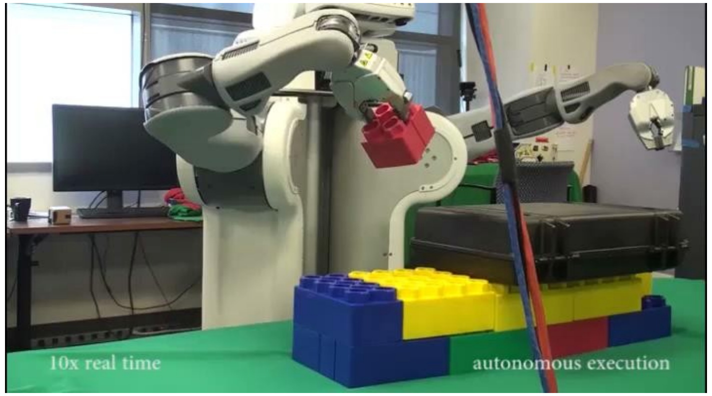
Aside: discount factors

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}^\pi_\phi(\mathbf{s}_{i,t+1})$$

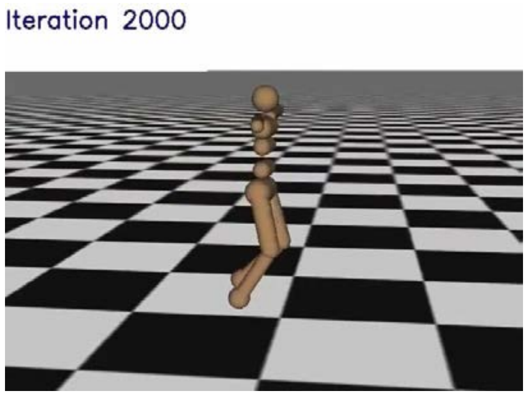
$$\mathcal{L}(\phi) = \frac{1}{2}\sum_i \left\| \hat{V}^\pi_\phi(\mathbf{s}_i) - y_i \right\|^2$$

what if T (episode length) is ∞?

$\hat{V}^\pi_\phi$ can get infinitely large in many cases



episodic tasks



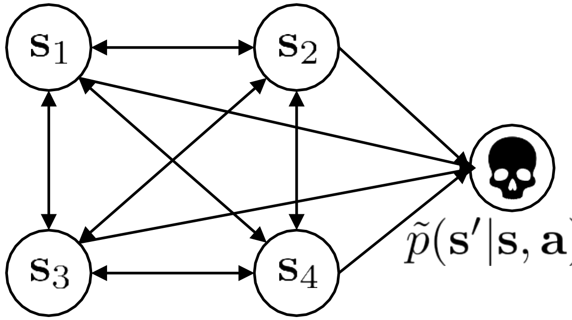
continuous/cyclical tasks

simple trick: better to get rewards sooner than later

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma\hat{V}^\pi_\phi(\mathbf{s}_{i,t+1})$$

discount factor $\gamma \in [0, 1]$ (0.99 works well)

γ changes the MDP:



$\tilde{p}(\mathbf{s}'|\mathbf{s}, \mathbf{a}) = (1 - \gamma)$

$\tilde{p}(\mathbf{s}'|\mathbf{s}, \mathbf{a}) = \gamma p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$

Aside: discount factors for policy gradients

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}_{i,t+1})$$

$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}^\pi_\phi(\mathbf{s}_i) - y_i \right\|^2$$

with critic:

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \underbrace{\left(r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}_{i,t+1}) - \hat{V}^\pi_\phi(\mathbf{s}_{i,t}) \right)}_{\hat{A}^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})}$$

what about (Monte Carlo) policy gradients?

option 1: $$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\sum_{t'=t}^T \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$

option 2: $$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left(\sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \right) \left(\sum_{t=1}^T \gamma^{t-1} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$

not the same!

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\sum_{t'=t}^T \gamma^{t'-1} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \gamma^{t-1} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\sum_{t'=t}^T \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$

(later steps matter less)

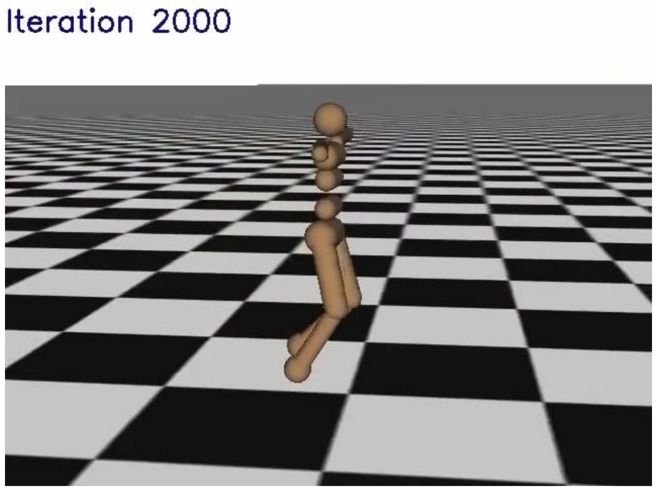
Which version is the right one?

option 1: $$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\sum_{t'=t}^{T} \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$

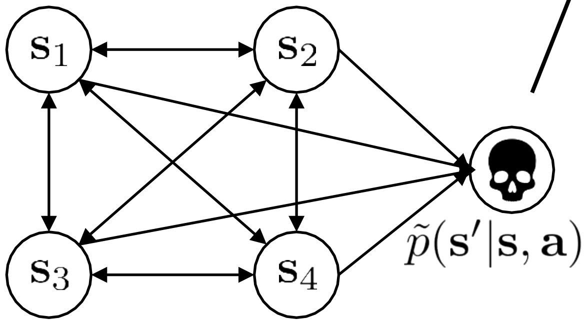
this is what we actually use...

why?

option 2: $$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \gamma^{t-1} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\sum_{t'=t}^{T} \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$



later steps don't matter if you're dead!



$\tilde{p}(\mathbf{s}'|\mathbf{s}, \mathbf{a}) = (1 - \gamma)$

$\tilde{p}(\mathbf{s}'|\mathbf{s}, \mathbf{a}) = \gamma p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$

Further reading: Philip Thomas, Bias in natural actor-critic algorithms. ICML 2014

Actor-critic algorithms (with discount)

batch actor-critic algorithm:

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma\hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i)\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha\nabla_\theta J(\theta)$

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}^\pi_\phi$ using target $r + \gamma\hat{V}^\pi_\phi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma\hat{V}^\pi_\phi(\mathbf{s}') - \hat{V}^\pi_\phi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s})\hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha\nabla_\theta J(\theta)$

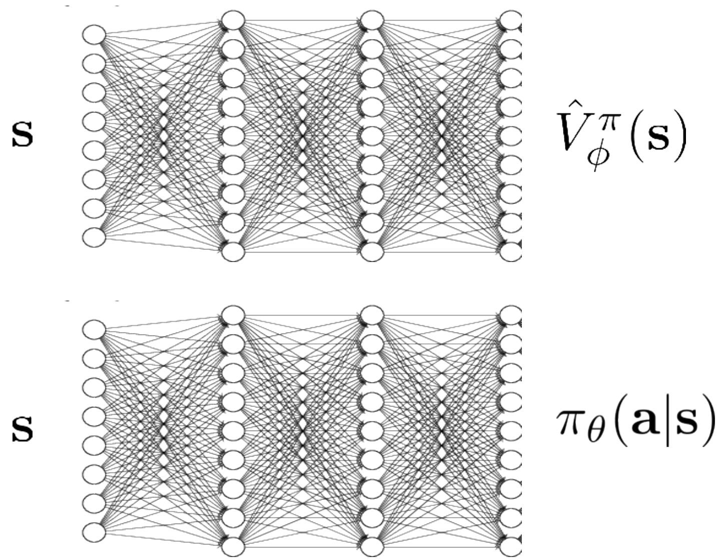
Actor-Critic Design Decisions

Architecture design

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}_\phi^\pi$ using target $r + \gamma \hat{V}_\phi^\pi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}') - \hat{V}_\phi^\pi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

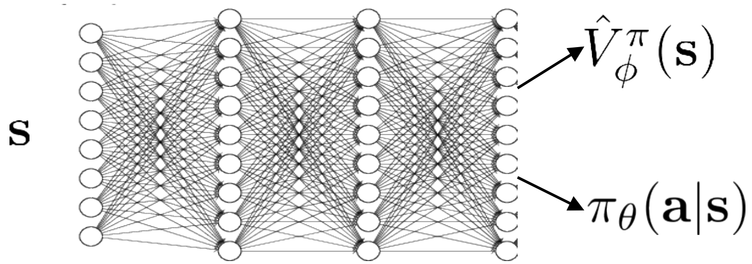
two network design



+ simple & stable

- no shared features between actor & critic

shared network design



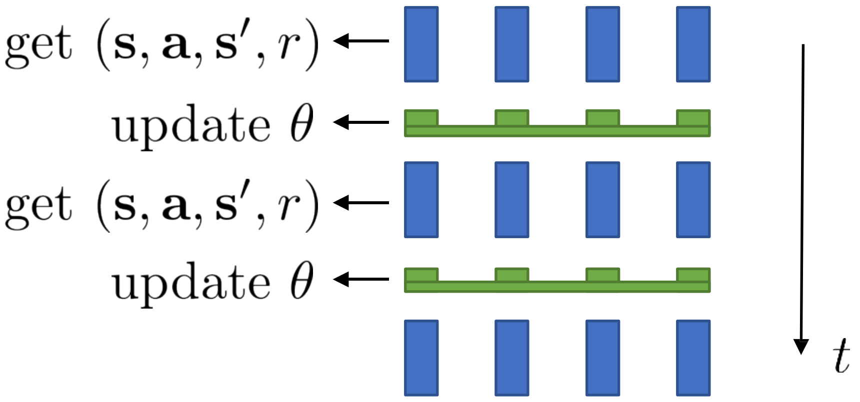
Online actor-critic in practice

online actor-critic algorithm:

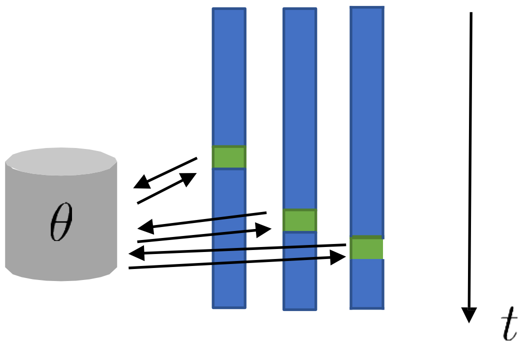
1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}^\pi_\phi$ using target $r + \gamma \hat{V}^\pi_\phi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}') - \hat{V}^\pi_\phi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

works best with a batch (e.g., parallel workers)

synchronized parallel actor-critic



asynchronous parallel actor-critic



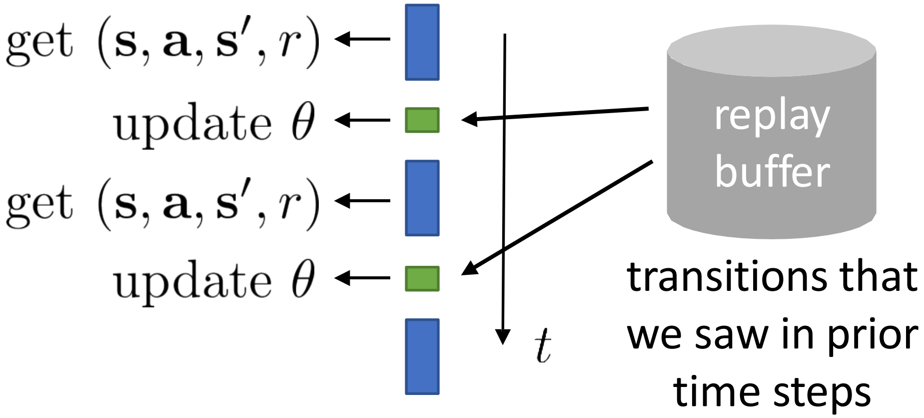
Can we remove the on-policy assumption entirely?

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
2. update $\hat{V}^\pi_\phi$ using target $r + \gamma \hat{V}^\pi_\phi(\mathbf{s}')$
3. evaluate $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}^\pi_\phi(\mathbf{s}') - \hat{V}^\pi_\phi(\mathbf{s})$
4. $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

form a **batch** by using old previously seen transitions

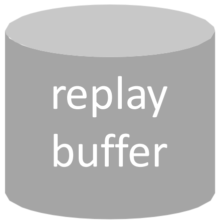
off-policy actor-critic



Let's see what that looks like

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$, store in $\mathcal{R}$
2. sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}$ from buffer $\mathcal{R}$
3. update $\hat{V}^\pi_\phi$ using targets $y_i = r_i + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i)$ for each $\mathbf{s}_i$
4. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
5. $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
6. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$



$$\mathcal{L}(\phi) = \frac{1}{N} \sum_i \left\| \hat{V}^\pi_\phi(\mathbf{s}_i) - y_i \right\|^2$$

batch size

not the right target value

not the action π_θ would have taken!

This algorithm is broken!

Can you spot the problems?

Fixing the value function

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$, store in $\mathcal{R}$
2. sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}$ from buffer $\mathcal{R}$
3. update $\hat{V}^\pi_\phi$ using targets $y_i = r_i + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i)$ for each $\mathbf{s}_i$
4. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
5. $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
6. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

~~$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^T E_{\pi_\theta}[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t]$~~

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta}[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t]$

"total reward we get if we take $\mathbf{a}_t$ in $\mathbf{s}_t$… … and then follow the policy π"

not the right target value

not the action π_θ would have taken!

where does this come from?

3. update $\hat{Q}^\pi_\phi$ using targets $y_i = r_i + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i)$ for each $\mathbf{s}_i, \mathbf{a}_i$

$$= r_i + \gamma \hat{Q}^\pi_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$$

$$\mathcal{L}(\phi) = \frac{1}{N} \sum_i \left\| \hat{Q}^\pi_\phi(\mathbf{s}_i, \mathbf{a}_i) - y_i \right\|^2$$

not from replay buffer $\mathcal{R}$!

$$\mathbf{a}'_i \sim \pi_\theta(\mathbf{a}'_i|\mathbf{s}'_i)$$

$$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^T E_{\pi_\theta}[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t] = E_{\mathbf{a} \sim \pi(\mathbf{a}_t|\mathbf{s}_t)}[Q(\mathbf{s}_t, \mathbf{a}_t)]$$

Fixing the policy update

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$, store in $\mathcal{R}$
2. sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}$ from buffer $\mathcal{R}$
3. update $\hat{Q}^\pi_\phi$ using targets $y_i = r_i + \gamma \hat{Q}^\pi_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$ for each $\mathbf{s}_i, \mathbf{a}_i$
4. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = Q(\mathbf{s}_i, \mathbf{a}_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
5. $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
6. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

not the action π_θ would have taken!

use the same trick, but this time for $\mathbf{a}_i$ rather than $\mathbf{a}'_i$!

sample $\mathbf{a}^\pi_i \sim \pi_\theta(\mathbf{a}|\mathbf{s}_i)$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}^\pi_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}^\pi_i)$$

not from replay buffer $\mathcal{R}$!

higher variance, but convenient

why is higher variance OK here?

in practice: $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}^\pi_i|\mathbf{s}_i) \hat{Q}^\pi(\mathbf{s}_i, \mathbf{a}^\pi_i)$

What else is left?

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$, store in $\mathcal{R}$
2. sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}$ from buffer $\mathcal{R}$
3. update $\hat{Q}^\pi_\phi$ using targets $y_i = r_i + \gamma \hat{Q}^\pi_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$ for each $\mathbf{s}_i, \mathbf{a}_i$
4. $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}^\pi_i|\mathbf{s}_i) \hat{Q}^\pi(\mathbf{s}_i, \mathbf{a}^\pi_i)$ where $\mathbf{a}^\pi_i \sim \pi_\theta(\mathbf{a}|\mathbf{s}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Is there any remaining problem?

$\mathbf{s}_i$ didn't come from $p_\theta(\mathbf{s})$

nothing we can do here, just accept it

intuition: we want optimal policy on $p_\theta(\mathbf{s})$
but we get optimal policy on a *broader* distribution

Some implementation details

online actor-critic algorithm:

1. take action $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$, get $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$, store in $\mathcal{R}$
2. sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}$ from buffer $\mathcal{R}$
3. update $\hat{Q}^\pi_\phi$ using targets $y_i = r_i + \gamma \hat{Q}^\pi_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$ for each $\mathbf{s}_i, \mathbf{a}_i$
4. $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}^\pi_i|\mathbf{s}_i) \hat{Q}^\pi(\mathbf{s}_i, \mathbf{a}^\pi_i)$ where $\mathbf{a}^\pi_i \sim \pi_\theta(\mathbf{a}|\mathbf{s}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

lots of fancier ways to fit Q-functions (more on this in next two lectures)

could also use **reparameterization trick** to better estimate the integral

Example practical algorithm:
Tuomas Haarnoja, Aurick Zhou, Pieter Abbeel, Sergey Levine. Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor. 2018.

We'll also learn about algorithms that do this with deterministic policies later!

Critics as Baselines

Critics as state-dependent baselines

Actor-critic: $\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(r(\mathbf{s}_{i,t},\mathbf{a}_{i,t}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1}) - \hat{V}_\phi^\pi(\mathbf{s}_{i,t})\right)$

+ lower variance (due to critic)

- not unbiased (if the critic is not perfect)

Policy gradient: $\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(\left(\sum_{t'=t}^{T}\gamma^{t'-t} r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right) - b\right)$

+ no bias

- higher variance (because single-sample estimate)

can we use $\hat{V}_\phi^\pi$ and still keep the estimator unbiased?

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\left(\left(\sum_{t'=t}^{T}\gamma^{t'-t} r(\mathbf{s}_{i,t'},\mathbf{a}_{i,t'})\right) - \hat{V}_\phi^\pi(\mathbf{s}_{i,t})\right)$$

+ no bias

+ lower variance (baseline is closer to rewards)

Control variates: action-dependent baselines

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t, \mathbf{a}_t\right]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)}[Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$$

$$\hat{A}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^\infty \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - V_\phi^\pi(\mathbf{s}_t)$$

+ no bias
- higher variance (because single-sample estimate)

$$\hat{A}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^\infty \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - Q_\phi^\pi(\mathbf{s}_t, \mathbf{a}_t)$$

+ goes to zero in expectation if critic is correct!
- not correct

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\hat{Q}_{i,t} - Q_\phi^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})\right) + \frac{1}{N}\sum_{i=1}^N \sum_{t=1}^T \nabla_\theta E_{\mathbf{a} \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_{i,t})}\left[Q_\phi^\pi(\mathbf{s}_{i,t}, \mathbf{a}_t)\right]$$

use a critic *without* the bias (still unbiased),
provided second term can be evaluated

Gu et al. 2016 (Q-Prop)

Eligibility traces & n-step returns

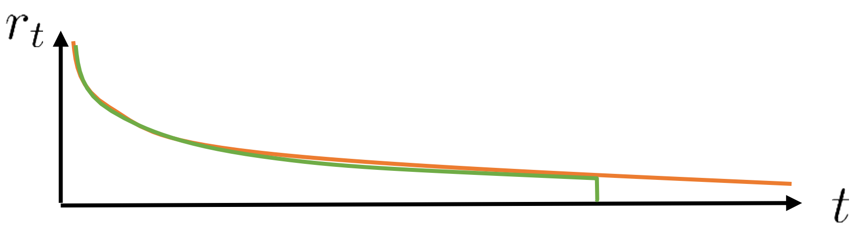
$$\hat{A}^{\pi}_{\mathrm{C}}(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \hat{V}^{\pi}_{\phi}(\mathbf{s}_{t+1}) - \hat{V}^{\pi}_{\phi}(\mathbf{s}_t)$$

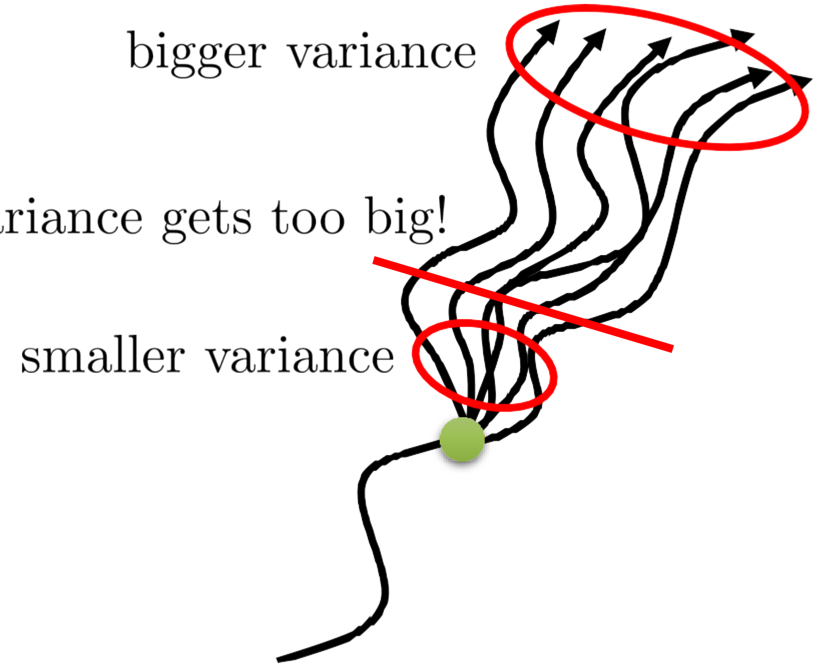
+ lower variance
- higher bias if value is wrong (it always is)

$$\hat{A}^{\pi}_{\mathrm{MC}}(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{\infty} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}^{\pi}_{\phi}(\mathbf{s}_t)$$

+ no bias
- higher variance (because single-sample estimate)

Can we combine these two, to control bias/variance tradeoff?

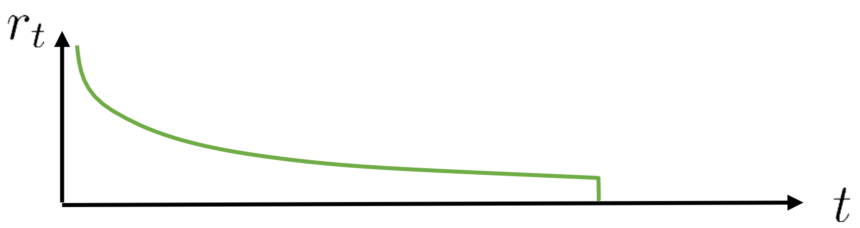




$$\hat{A}^{\pi}_{n}(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{t+n} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}^{\pi}_{\phi}(\mathbf{s}_t) + \gamma^n \hat{V}^{\pi}_{\phi}(\mathbf{s}_{t+n})$$

choosing $n > 1$ often works better!

Generalized advantage estimation



Do we have to choose just one n?

Cut everywhere all at once!

$$\hat{A}_n^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{t+n} \gamma^{t'-t} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) - \hat{V}_\phi^\pi(\mathbf{s}_t) + \gamma^n \hat{V}_\phi^\pi(\mathbf{s}_{t+n})$$

$$\hat{A}_{\mathrm{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{n=1}^{\infty} w_n \hat{A}_n^\pi(\mathbf{s}_t, \mathbf{a}_t)$$

weighted combination of n-step returns

How to weight? Mostly prefer cutting earlier (less variance) $w_n \propto \lambda^{n-1}$ exponential falloff

$$\hat{A}_{\mathrm{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \gamma((1-\lambda)\hat{V}_\phi^\pi(\mathbf{s}_{t+1}) + \lambda(r(\mathbf{s}_{t+1}, \mathbf{a}_{t+1}) + \gamma((1-\lambda)\hat{V}_\phi^\pi(\mathbf{s}_{t+2}) + \lambda r(\mathbf{s}_{t+2}, \mathbf{a}_{t+2}) + \dots)$$

$$\hat{A}_{\mathrm{GAE}}^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^{\infty} (\gamma\lambda)^{t'-t} \delta_{t'} \qquad \delta_{t'} = r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{t'+1}) - \hat{V}_\phi^\pi(\mathbf{s}_{t'})$$

similar effect as discount!

$$\text{option 1:} \quad \nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \left(\sum_{t'=t}^{T} \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right)$$

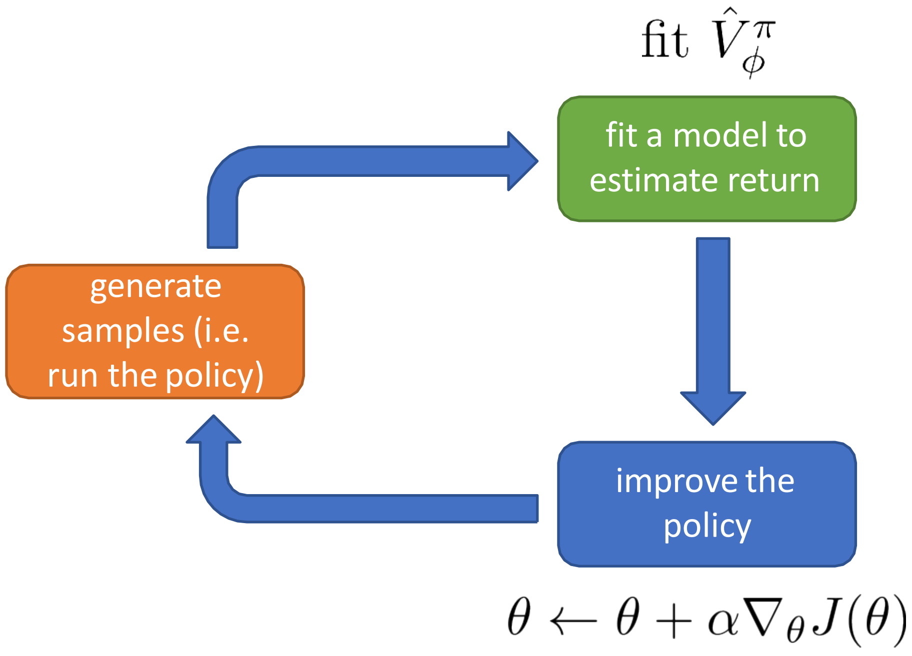
remember this?
discount = variance reduction!

Schulman, Moritz, Levine, Jordan, Abbeel '16

Review, Examples, and Additional Readings

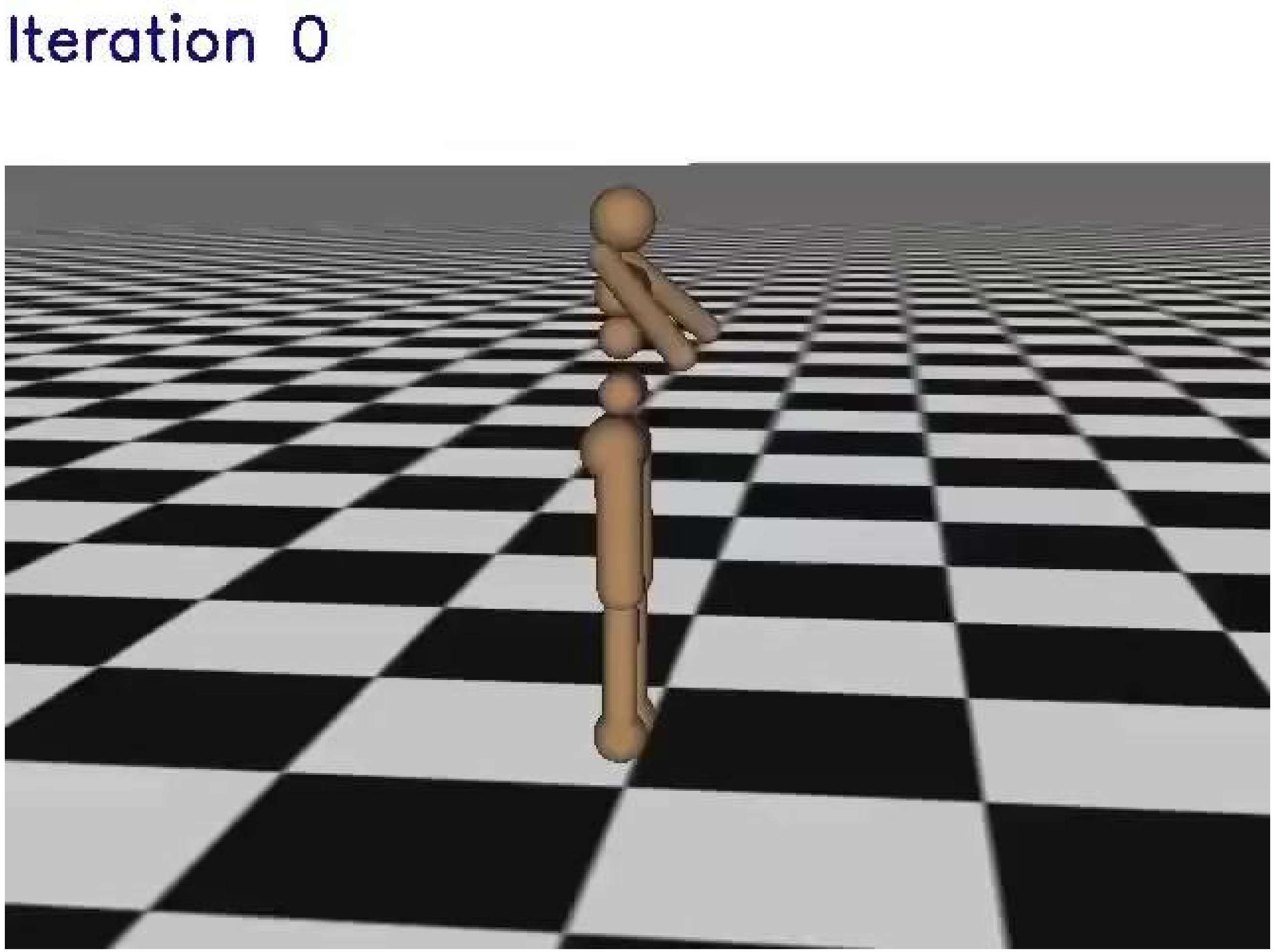
Review

- Actor-critic algorithms:
  - Actor: the policy
  - Critic: value function
  - Reduce variance of policy gradient
- Policy evaluation
  - Fitting value function to policy
- Discount factors
  - Carpe diem Mr. Robot
  - …but also a variance reduction trick
- Actor-critic algorithm design
  - One network (with two heads) or two networks
  - Batch-mode, or online (+ parallel)
- State-dependent baselines
  - Another way to use the critic
  - Can combine: n-step returns or GAE



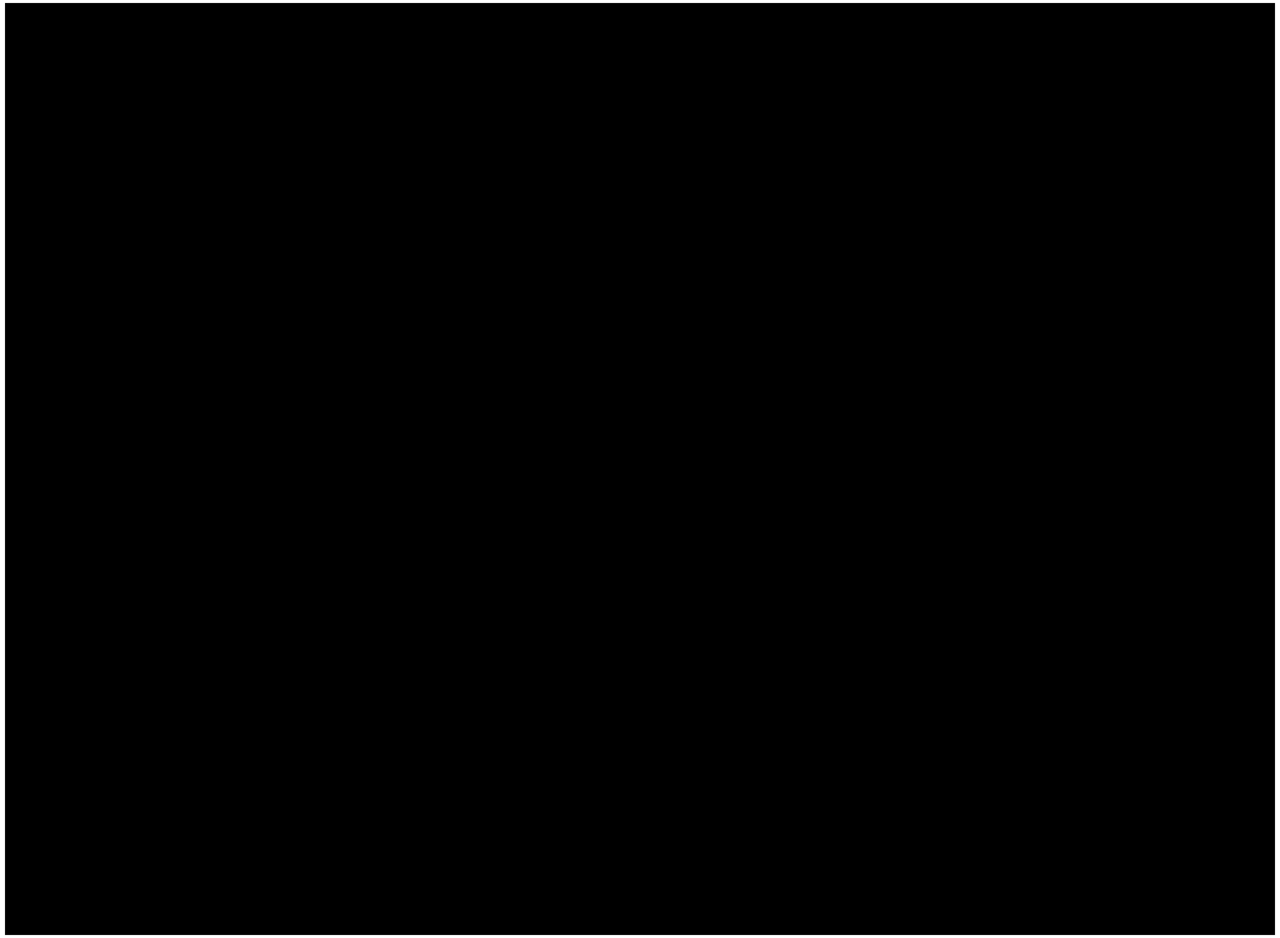
Actor-critic examples

- High dimensional continuous control with generalized advantage estimation (Schulman, Moritz, L., Jordan, Abbeel ‘16)
- Batch-mode actor-critic
- Blends Monte Carlo and function approximator estimators (GAE)



Actor-critic examples

- Asynchronous methods for deep reinforcement learning (Mnih, Badia, Mirza, Graves, Lillicrap, Harley, Silver, Kavukcuoglu ‘16)
- Online actor-critic, parallelized batch
- N-step returns with N = 4
- Single network for actor and critic



Actor-critic suggested readings

- Classic papers
  - Sutton, McAllester, Singh, Mansour (1999). Policy gradient methods for reinforcement learning with function approximation: actor-critic algorithms with value function approximation
- Deep reinforcement learning actor-critic papers
  - Mnih, Badia, Mirza, Graves, Lillicrap, Harley, Silver, Kavukcuoglu (2016). Asynchronous methods for deep reinforcement learning: A3C -- parallel online actor-critic
  - Schulman, Moritz, L., Jordan, Abbeel (2016). High-dimensional continuous control using generalized advantage estimation: batch-mode actor-critic with blended Monte Carlo and function approximator returns
  - Gu, Lillicrap, Ghahramani, Turner, L. (2017). Q-Prop: sample-efficient policy-gradient with an off-policy critic: policy gradient with Q-function control variate