Mathematical Modelling – Linear Programming

Exercise

A production company has $n$ warehouses ($i = 1, 2, \ldots, n$) and it has to serve $m$ shops ($j = 1, 2, \ldots, m$). Each warehouses $i$ can deliver a determined amount of product per week; $a_i$ is called the capacity of the warehouse. Each shop $j$ has a known weekly demand of $b_j$. The cost of shipping one unit of product from warehouse $i$ to the shop $j$ has been estimated in $c_{ij}$ (dollars per unit).

- Write a LP model to minimize the total cost to satisfy the demands of the shops. Identify the decisions that must be taken and the corresponding decision variables that have to be used. Identify and comment the objective function of the problem and the constraints.

- Now consider the follow case in which two are the warehouses ($n = 2$) with weekly capacities $a_1 = 1200.5$ and $a_2 = 1100.5$. Three are the shops ($m = 3$), the corresponding weekly demands are $b_1 = 500.5, b_2 = 600.5$ and $b_3 = 1000.5$. The following are the estimated shipping costs:

<table>
<thead>
<tr>
<th></th>
<th>$j = 1$</th>
<th>$j = 2$</th>
<th>$j = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i = 1$</td>
<td>300</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>$i = 2$</td>
<td>250</td>
<td>400</td>
<td>80</td>
</tr>
</tbody>
</table>

Write the LP model for this specific case.

- Try to find a feasible solution of the problem which satisfies the needs of the 3 shops and do not exceed the capacity of the 2 warehouses. What is the total cost of the proposed solution? Try to justify your choices.
Decision variables:

- \( x_{ij} = \) units delivered from warehouse \( i \) to shop \( j \)

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\begin{align*}
i = 1, \ldots, n, j = 1, \ldots, m
\end{align*}
\]

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\begin{align*}
Z(\text{LP}) &= \min \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \\
&\quad \sum_{j=1}^{m} x_{ij} \leq a_i \quad i = 1, \ldots, n \\
&\quad \sum_{i=1}^{n} x_{ij} \geq b_j \quad j = 1, \ldots, m \\
&\quad x_{ij} \geq 0 \quad i = 1, \ldots, n, j = 1, \ldots, m
\end{align*}
\]

\[
\begin{align*}
Z(\text{LP}) &= \min 300x_{11} + 200x_{12} + 100x_{13} + \\
&\quad 250x_{21} + 400x_{22} + 80x_{23} \\
&\quad x_{11} + x_{12} + x_{13} \leq 1200.5 \\
&\quad x_{21} + x_{22} + x_{23} \leq 1100.5 \\
&\quad x_{11} + x_{21} \geq 500.5 \\
&\quad x_{12} + x_{22} \geq 600.5 \\
&\quad x_{13} + x_{23} \geq 1000.5 \\
&\quad x_{11}, x_{12}, x_{13}, \\
&\quad x_{21}, x_{22}, x_{23} \geq 0
\end{align*}
\]