Stochastic FAP Using SDP

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Outline

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Semidefinite matrices

$A \in \mathbb{S}_n(\mathbb{R}), \forall\lambda$ eigenvalue of A , $\lambda \geq 0,$ Notation $A \succeq 0$,

1. The set of semidefinite matrices SDP_n is a cone
2. If A is semidefinite matrix (sdp), then
   * its associated quadratic form x^tAx is positive for all $x \in \mathbb{R}^n$.
   * it has Cholesky factorization LU such that $U = L^t$
3. $A \bullet B$ is the inner product
4. The quadratic form x^tAx can be written as $A \bullet xx^t$.
5. Since the extreme rays of SDP_n are of the form xx^t, we derive $A \bullet B \geq 0, \forall A \succeq 0, \forall B \succeq 0$.

What is a Semidefinite programming problem ?

- A LP can be defined as

$$z = \inf\{c^t x : Ax = b, x \in K\} \text{ where } K \text{ is a convex set}$$

- A semidefinite program is an LP over the positive semidefinite Cone :

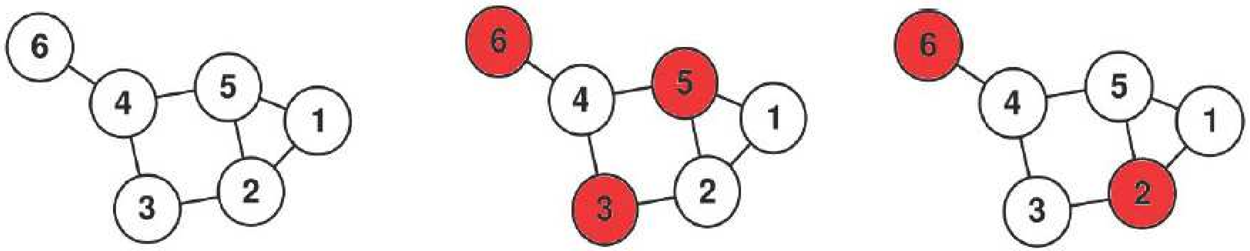
$$\inf\{C \bullet Y : A_i \bullet Y = b_i,\ \forall i \in \{i, \ldots, m\}, Y \succeq 0.\}$$

- and its dual is : $\sup\{\sum_{i=1}^{m} b_i y_i : \sum_i y_i A_i \preceq C\}$

- Polynomial algorithms exist for solving SDP problems e.g. IPM and ellipsoid methods

Lovasz's Theta function

Definition 1. *Let a graph $G = (V, E)$, a stable or (independent) set is subset S of vertices such that no two vertices of S are adjacent.*



Definition 2. *The maximum cardinality of stable set is the stability number of G and is denoted $\alpha(G)$*

In this example, $\alpha(G) = 3$.

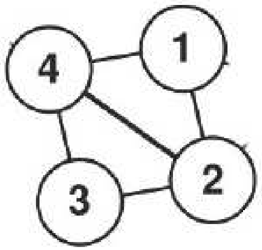
Lovasz's Theta function

- Lovasz (1979) proposed an upper bound on $\alpha(G)$ known as the theta function $\vartheta(G)$.

- The theta function can be expressed in many equivalent ways:
  - as an eigenvalue bound,
  - as semidefinite program,
  - or in terms of orthogonal representations.

Lovasz's Theta function : An eigenvalue Bound

- Consider $P = \{A \in S_n : a_{ij} = 1 \text{ if } (i,j) \notin E \text{ (or } i = j)\}$



$$P = \left\{ \begin{pmatrix} 1 & x & 1 & y \\ x & 1 & z & t \\ 1 & z & 1 & u \\ y & t & u & 1 \end{pmatrix} / (x,y,z,t,u) \in \{0,1\}^5 \right\}$$

- If there exists a stable set of size k, the corresponding principal submatrix of any $A \in P$ will be J_k, the all ones matrix of size k.
- By a classical results on symmetric matrices, we derive $\forall A \in P, \lambda_{max}(A) \geq \lambda_{max}(J_k)$.
- As a result, $\vartheta(G) = min_{A \in P} \lambda_{max}(A)$ is an upper bound on $\alpha(G)$ and then an equivalent formulation for the theta function.

Lovasz's Theta function : A SDP formulation

- A largest eigenvalue of a matrix can be formulated as semidefinite program i.e. $\lambda_{max}(A) = \min\{t : tI - A \succeq 0\}$. (the eigenvalues of $tI - A$ are $t - \lambda_i$ where λ_i are the eigenvalues of A).

- To express $\vartheta(G)$ as SDP, observe that $A \in P$ is equivalent to $A - J$ generated by $E_{ij}, (i,j) \in E$ where all entries are zero except for (i,j) and (j,i).

- $\vartheta(G) = min\{t : tI + \sum_{(i,j)\in E} x_{ij}E_{ij} \succeq J\}$.

- By strong duality, we can also write the dual

$$\vartheta(G) = \max\{J \bullet Y : y_{ij} = 0 \text{ for } (i,j) \in E, I \bullet Y = 1, Y \succeq 0\}.$$

- $\alpha(G) \leq \vartheta(G)$

Lovasz's Theta function : Orthonormal representation

- An orthonormal representation of G is a system $(v_1, \ldots, v_n) \in \mathbb{R}^n$ such that v_i and v_j are orthogonal (i.e. $v_i^t v_j = 0$) whenever i and j are not adjacent.

- The value of the orthonormal representation of G is $z = min_{c:\|c\|=1} max_{i \in V} \frac{1}{(c^t v_i)^2}$.

- z is an upper bound on $\alpha(G)$ i.e. $z \geq \alpha(G)$ (since $\|c\|^2 \geq \sum_{i \in S} (c^t v_i)^2 \geq |S|/z$)

- $\vartheta(G) = \inf\{z :$ orthonormal representation$\}$ (See Lovasz).

Lovasz's Theta function : Orthonormal representation

- Let $x = (x_1, \ldots, x_n)$ denotes the incidence vector of stable set, then we have

$$\sum_i (c^t v_i)^2 x_i \leq 1, \forall c : \|c\| = 1, \forall \text{ orthonormal representation } V \qquad (1)$$

- The orthonormal representation constraints (1) are valid inequalities for $STAB(G)$ where $STAB(G)$ is the convex hull of incidence vectors of stable sets of G.

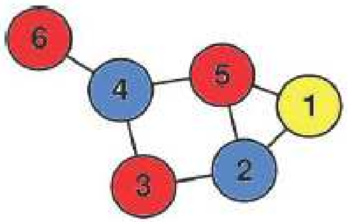
- Grotchel et al. show that if we let $TH(G) = \{x : x \text{ satisfies } (1) \text{ and } x \geq 0\}$ then $\vartheta(G) = \max\{\sum_i x_i : x \in TH(G)\}$.

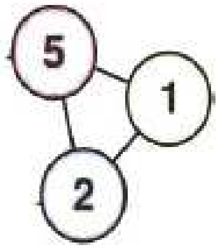
Schrijver's strenghthening of Lovasz's Theta function

- $\vartheta(G) = \min\{\lambda_{max}(A) : A \in S_n, a_{ij} = 1 \text{ for } (i,j) \notin E \text{ (or } i = j)\}$
- $\vartheta'(G) = \min\{\lambda_{max}(A) : A \in S_n, a_{ij} \geq 1 \text{ for } (i,j) \notin E \text{ (or } i = j)\}$
- $\vartheta(G) = \max\{J \bullet Y : y_{ij} = 0 \text{ for } (i,j) \in E, I \bullet Y = 1, Y \succeq 0\}$.
- $\vartheta'(G) = \max\{J \bullet Y : y_{ij} = 0 \text{ for } (i,j) \in E, y_{ij} \geq 0 \text{ for } (i,j) \notin E, I \bullet Y = 1, Y \succeq 0\}$.

$$\boxed{\alpha(G) \leq \vartheta'(G) \leq \vartheta(G)}$$

Lovasz's Theta function and perfect graphs





- Chromatic Number and Clique
- A graph G is perfect if, for every induced sub-graph G', its chromatic number is equal to the size of the largest clique in G'.
- Theta function gives some important characterizations of perfect graphs.

Lovasz's Theta function and perfect graphs

Theorem 1. *The following are equivalent:*

- G *is perfect,*
- $TH(G) = \{x \geq 0 : \sum_{i \in C} x_i \leq 1$ *for all cliques* $C\}$
- $TH(G)$ *is polyhedral.*

- We can find a largest stable set in perfect graph in polynomial time by computing the theta function using semidefinite programming.

- For perfect graphs, we have $\alpha(G) = \vartheta(G)$.

Max Cut Problem

Definition 3. *Given a graph G, the cut $\delta(S)$ induced by vertex set S is the set of edges with exactly one endpoint in S.*

- *The Max Cut Problem consists in finding a cut of maximum weight in a weighted undirected graph.*
- *The weight of $\delta(S)$ is $\omega(\delta(S)) = \sum_{e \in \delta(S)} \omega_e$.*
- $OPT = Max(\omega(\delta(S)))$.

Max Cut Problem

- Delorme and Poljak conjecture : $\frac{OPT}{SDP} \sim 0.88445$.
- Goemans and Williamson randomized approximation algorithm: $\frac{OPT}{SDP} \geq 0.87856$.

FAP

FAP problem : Assign n frequencies to m sites in order to satisfy given demands for frequencies and minimize the interferences between different frequencies.

The frequencies are represented as a set of positive integers $i = 1, .., n$.

For every pair (i, j) of frequencies the distance ρ_{ij} is defined:

$$\rho_{ij} = |i - j|$$

Deterministic FAP. contd

Let d_i be the demand for frequencies for the site i,

w_{ij}^{kl} is the interference attained if frequency k is assigned to site i and frequency l is assigned to site j,

and x_i^k is a decision binary variable which equals 1 if frequency k is assigned to site i and zero otherwise;

Let N be the set of sites and M the set of frequencies.

The FAP can be written as :

FAP

$$\min_{x_i^k \in \{0,1\}} \sum_{i,j,k,l} w_{ij}^{kl} x_i^k x_j^l \tag{2}$$

subject to :

$$\sum_k x_i^k = d_i, \ \forall i \tag{3}$$

$$x_i^k + x_j^l \leq 1, \ \forall i,j,k,l : i,j \text{ adjacent}, \ |k-l| \leq c_1 \tag{4}$$

$$x_i^k + x_i^l \leq 1, \ \forall i,k,l : \ |k-l| \leq c_2{}^1 \tag{5}$$

[1] It is common to set $c_1 = 2$ and $c_2 = 3$ in practice. We will use such constants both for our modelling or numerical experiments.

FAP Semidefinite relaxation

Let $y_{i,j}^{l,k} = x_i^l x_j^k$. The FAP can be written as :

$$\sum_{l=1}^{m} y_{i,i}^{l,l} = d_i, \forall i \in N$$

$$y_{i,i}^{l,k} = 0, \forall i \in N, \text{ and } |l-k| \leq 3.$$

or

$$\begin{cases} y_{i,i}^{l,l+1} = 0 \\ y_{i,i}^{l,l+2} = 0 \quad \forall i \in N, \forall l \in M \\ y_{i,i}^{l,l+3} = 0 \end{cases}$$

FAP Semidefinite relaxation

and

$$y_{i,j}^{l,k} = 0, \forall i \in N, \text{ and } |l-k| \leq 2.$$

or

$$\begin{cases} y_{i,j}^{l,l} = 0 \\ y_{i,j}^{l,l+1} = 0 \quad 1 \leq i,j \leq n, \; j \text{ co-site of } i \text{ and } l \in M \\ y_{i,j}^{l,l+2} = 0 \end{cases}$$

FAP Semidefinite relaxation

Let the matrices Y and W defined as :

$$Y = \begin{bmatrix} Y_{1,1} & \cdots & Y_{1,n} \\ \vdots & \ddots & \vdots \\ Y_{n,1} & \cdots & Y_{n,n} \end{bmatrix}, \text{ where } Y_{i,j} = \begin{bmatrix} y_{i,j}^{1,1} & \cdots & y_{i,j}^{1,m} \\ \vdots & \ddots & \vdots \\ y_{i,j}^{m,1} & \cdots & y_{i,j}^{m,m} \end{bmatrix},$$

$$\text{and } W = \begin{bmatrix} W_{1,1} & \cdots & W_{1,n} \\ \vdots & \ddots & \vdots \\ W_{n,1} & \cdots & W_{n,n} \end{bmatrix}, \text{ where } W_{i,j} = \begin{bmatrix} w_{i,j}^{1,1} & \cdots & w_{i,j}^{1,m} \\ \vdots & \ddots & \vdots \\ w_{i,j}^{m,1} & \cdots & w_{i,j}^{m,m} \end{bmatrix}$$

The SDP relaxed FAP can be written as :

$$(SDPFAP)\left\{\begin{array}{lll} Min & Trace(W*Y) & \\ s.c & Trace(Y_{ii}) = d_i & i \in N \\ & \left\{\begin{array}{l} y_{i,i}^{l,l+1} = 0 \\ y_{i,i}^{l,l+2} = 0 \\ y_{i,i}^{l,l+3} = 0 \end{array}\right. & i \in N,\ l \in M \\ & & \\ & \left\{\begin{array}{l} y_{i,j}^{l,l} = 0 \\ y_{i,j}^{l,l+1} = 0 \\ y_{i,j}^{l,l+2} = 0 \end{array}\right. & 1 \leq i,j \leq n, \text{ j co-site of i and } l \in M \\ & diag(Y) = y & \\ & Y - yy^t \succeq 0. & \end{array}\right. \tag{6}$$

where $y = (x_i^l)_{1\leq l\leq m, 1\leq i\leq n}$ is the decision variable vector.

FAP SDP relaxation

Let Z_{il} and Z_{ijl} be a determined matrices for each co-site and co-station constraint respectively. The (6) can be also written as :

$$(SDPFAP1)\begin{cases} Min & Trace(W * Y) \\ s.c & \\ & Trace(Y_{ii}) = d_i \qquad i \in N \\ & Trace(Z_{il}Y_{il}) = 0 \qquad \forall i, l \\ & Trace(Z_{ijl}Y_{ijl}) = 0 \quad \forall i, j, l, \text{ i and j are cosite} \\ & diag(Y) = y \\ & Y - yy^t \succeq 0 \end{cases} \tag{7}$$

Stochastic problem

The main uncertainty sources in the FAP are the following :

- Interference may change due to changing atmospheric conditions, time of the day and of the year, etc.

  It is more realistic to assume that interferences are described by joint probabilistic distribution $H(w)$.

- Assume that demand at sites $i = 1,,,m$ is a random vector with joint distribution $P(d)$.

We consider the case where the frequency assignment is based on the probabilistic description of demand and interference patterns *i.e.* non adaptive case.

Non adaptive FAP

Frequency assignment decision is made before any specific demand realization becomes known and is not changed afterwards.

The objective function can be written as:

$$\min_{x_i^k} \mathbb{E}_w \sum_{i,j,k,l} w_{ij}^{kl} x_i^k x_j^l + c\mathbb{E}_d \sum_i \max \left\{ 0, d_i - \sum_k x_i^k \right\} \tag{8}$$

As

$$\mathbb{E}_w \sum_{i,j,k,l} w_{ij}^{kl} x_i^k x_j^l = \sum_{i,j,k,l} \left(\mathbb{E}_w w_{ij}^{kl} \right) x_i^k x_j^l = \sum_{i,j,k,l} \bar{w}_{ij}^{kl} x_i^k x_j^l$$

Non adaptive Case

- The demand distribution $P(d)$ is concentrated in a finite number of points $d^r = (d_1^r, ..., d_m^r)$ with weights p_r, $r = 1, ..., R$.
- These points will be called *demand scenarios*.

The problem (8) is equivalent to the following:

$$\min_{x_i^k} \sum_{i,j,k,l} \bar{w}_{ij}^{kl} x_i^k x_j^l + c \sum_r \sum_i p_r \max \left\{ 0, d_i - \sum_k x_i^k \right\} \tag{9}$$

Non adaptive Case

After introducing auxiliary variables v_i^r for each site $i = 1, ..., m$ and for each scenario $r = 1, ..., R$, we obtain the following familiar quadratic problem:

$$\min_{x_i^k, v_i^r} \sum_{i,j,k,l} \bar{w}_{ij}^{kl} x_i^k x_j^l + c \sum_r \sum_i p_r v_i^r \tag{10}$$

$$\sum_k x_i^k + v_i^r = d_i^r, \quad \forall i, r \tag{11}$$

where v_i^r is integer and nonnegative. Constraints (4)-(5) can be added to this formulation.

Semidefinite relaxations for stochastic FAP

Assume that $v_i^r \in \{0,1\}$. As for the deterministic FAP, we introduce $y_{i,j}^{l,k} = x_i^l x_j^k$ together with the following notations:

$$Y = \begin{bmatrix} Y_{1,1} & \cdots & Y_{1,n} \\ \vdots & \ddots & \vdots \\ Y_{n,1} & \cdots & Y_{n,n} \end{bmatrix}, \text{ with } Y_{i,j} = \begin{bmatrix} y_{i,j}^{1,1} & \cdots & y_{i,j}^{1,m} \\ \vdots & \ddots & \vdots \\ y_{i,j}^{m,1} & \cdots & y_{i,j}^{m,m} \end{bmatrix},$$

$$\tilde{W} = \begin{bmatrix} \tilde{W}_{1,1} & \cdots & \tilde{W}_{1,n} \\ \vdots & \ddots & \vdots \\ \tilde{W}_{n,1} & \cdots & \tilde{W}_{n,n} \end{bmatrix}, \text{ with } \tilde{W}_{i,j} = \begin{bmatrix} \tilde{w}_{i,j}^{1,1} & \cdots & \tilde{w}_{i,j}^{1,m} \\ \vdots & \ddots & \vdots \\ \tilde{w}_{i,j}^{m,1} & \cdots & \tilde{w}_{i,j}^{m,m} \end{bmatrix}$$

$$V_r = (v_1^r, ..., v_N^r),\ U = \begin{bmatrix} Y & 0 & \cdots & 0 \\ 0 & V_1V_1^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & V_RV_R^T \end{bmatrix}$$

$$D = \begin{bmatrix} \tilde{W} & 0 & \cdots & 0 \\ 0 & cp_1 I_N & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & cp_N I_N \end{bmatrix}$$ where I_N is NxN unit matrix.

To represent demand constraints, we consider a submatrix U_{ir} of matrix U defined by:

$$U_{ir} = \begin{bmatrix} Y_{ii} & 0 \\ 0 & (v_i^r)^2 \end{bmatrix}$$

Then the corresponding constraint is

$$trace(U_{ir}) \geq d_i^r, \forall i, r \tag{12}$$

Then SDP lifting of (10)-(11) is the following:

$$\min_{U} trace(D * U) \tag{13}$$

subject to

$$trace(U_{ir}) \geq d_i^r, \forall i, r \tag{14}$$

$$U - diag(U)diag(U)^T \succeq 0 \tag{15}$$

Thus, positive semidefinite relaxation of the stochastic FAP is the problem (13)-(15) where co-cite and co-station constraints should be added similarly to the deterministic FAP.

SDP relaxation within decomposition algorithm

Generic scheme for the Benders decomposition algorithm is:

- ***Initialization***.
  Select a feasible frequency assignment x_i^{k0} and let $u^{+0} = +\infty$, $u^{-0} = -\infty$ be the current upper and lower bound respectively.

- ***Generic step***.
  At step s, let x_i^{ks} be the current frequency assignment and u^{+s}, u^{-s} the upper and lower bounds resp. Then at step s we do the following:

  – Solve subproblem,
  – Add new cut to the master program.

Benders decomposition steps

Solve the subproblem

$$\min_{v_i^r} cp_r v_i^r \tag{16}$$

$$\sum_k x_i^{ks} + v_i^r \geq d_i^r, \quad \forall i, r \tag{17}$$

whose dual

$$\max_{\mu_i^r \geq 0} \sum_r \sum_i \mu_i^r \left(d_i^r - \sum_k x_i^{ks} \right)$$

$$0 \leq \mu_i^r \leq cp_r, \quad \forall i, r$$

has an explicit solution

$$\mu_i^{rs} = \begin{cases} cp_r & \text{if} \quad d_i^r \geq \sum_k x_i^{ks} \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

Then, **Add the cut**

$$z_0 \geq \sum_{i,j,k,l} \bar{w}_{ij}^{kl} x_i^k x_j^l + \sum_i \sum_r \left(d_i^r - \sum_k x_i^k \right) \mu_i^{rs}$$

to the master problem and compute the upper bound by

$$u^{+0} = \min \left\{ u^{+0}, \sum_{i,j,k,l} \bar{w}_{ij}^{kl} x_i^{ks} x_j^{ls} + \sum_i \sum_r \left(d_i^r - \sum_k x_i^{ks} \right) \mu_i^{rs} \right\}$$

Solve the master problem using SDP relaxation :

$$\bar{z}_0 = \min_{z_0, x_i^k} z_0 \tag{19}$$

$$z_0 - \sum_{i,j,k,l} \bar{w}_{ij}^{kl} x_i^k x_j^l + \sum_i \sum_k x_i^k \sum_r \mu_i^{rq} \geq \sum_i \sum_r d_i^r \mu_i^{rq}, \; q = 1, ..., s \tag{20}$$

and

- let $u^{-0} = \bar{z}_0$.
- Stop if $u^{-0} - u^{-0} < \epsilon$ where ϵ is some prespecified tolerance.
- Otherwise, let $x_i^{k,s+1}$ be the solution of (19)-(20) and go to the step $s+1$.

We have to lift inequality (20) into the cone of positive semidefinite matrices. Observe that this inequality can be expressed as follows:

$$trace\left(\begin{bmatrix} 1 & b_q^T \\ b_q & -\tilde{W} \end{bmatrix} * \begin{bmatrix} z_0 & x^T \\ x & Y \end{bmatrix}\right) \geq \sum_i \sum_r d_i^r \mu_i^{rq}$$

with

$$trace\left(\begin{bmatrix} 0 & -\frac{1}{2}1_{nm}^T \\ -\frac{1}{2}1_{nm} & I_{nm} \end{bmatrix} * \begin{bmatrix} z_0 & x^T \\ x & Y \end{bmatrix}\right) = 0$$

where

$$x = (x_1^1, ..., x_1^m, ..., x_n^1, ..., x_n^m),\ b_q = \frac{1}{2}\left(\overbrace{b_{1q}, ..., b_{1q}}^{n \text{ times}}, ..., \overbrace{b_{nq}, ..., b_{nq}}^{n \text{ times}}\right),\ b_{iq} = \sum_r \mu_i^{rq}$$

Denoting now

$$U = \begin{bmatrix} z_0 & x^T \\ x & Y \end{bmatrix}, \; A = \begin{bmatrix} 1 & 0_{nm}^T \\ 0_{nm} & 0_{nmxnm} \end{bmatrix},$$

$$B_q = \begin{bmatrix} 1 & b_q^T \\ b_q & -\tilde{W} \end{bmatrix}, \; C = \begin{bmatrix} 0 & -\frac{1}{2}1_{nm}^T \\ -\frac{1}{2}1_{nm} & I_{nm} \end{bmatrix}, \; D = \begin{bmatrix} 0 & 0_{nm} \\ 0_{nm} & I_{nm} \end{bmatrix}$$

we obtain the following relaxation of the master problem:

$$\min_U trace(AU) \tag{21}$$

$$trace(B_q U) \geq \sum_i \sum_r d_i^r \mu_i^{rq}, \; q = 1, ..., s \tag{22}$$

$$trace(CU) = 0 \tag{23}$$

$$DU - diag(DU)diag(DU)^T \succeq 0 \tag{24}$$

Numerical Results

Our first numerical experiments concerned small instances.

Table 1: Instances testés

Instances	$\#sites$	$\#frequencies$
Fap1	3	12
Fap2	5	14
Fap3	6	16
Fap4	8	20

We solved 4 variants of each instance with $5, 10, 15$ and 30 scenarios respectively.

LP equivalent problems

Table 2: LP equivalent sizes

Instances	S=5		S=10		S=15		S=30	
	$\#var$	$\#const$	$\#var$	$\#const$	$\#var$	$\#const$	$\#var$	$\#const$
Fap1	717	2076	732	2091	747	2106	792	2151
Fap2	2580	7765	2605	7790	2630	7815	2705	7890
Fap3	4782	14172	4812	14202	4842	14232	4932	14322
Fap4	13080	38976	13120	39016	13160	39056	13280	39176

- Linearization leads to a large number of binary variables and constraints,
- CPLEX solved to optimality only the first two instances,
- Weak lower bounds of the LP relaxation.

Benders results

Table 3: Test SDP Sizes

Instances	Master SDP Program					
	#var	#const	MP iterations number			
			S=5	S=10	S=15	S=30
Fap1	666	100	4	3	7	4
Fap2	2484	356	7	7	10	9
Fap3	4656	271	7	9	6	7
Fap4	12880	457	15	19	16	17

- The number of constraints is related to the last master program solved,
- SDP master program iterations are less than 20 for all instances.

Numerical Results

Table 4: Lower and Upper Bounds

Data	S=5				S=10			
	LB	Opt	UB	Gap	LB	Opt	UB	Gap
Fap1	312	320	324	3.7	292	301	310	5.8
Fap2	766	776	803	4.6	741	759	807	8.1
Fap3	915	940†	955	4.1	820	863†	907	9.5
Fap4	9772	11157†	10186	4.0	9850	‡	10250	3.9

Table 5: Lower and Upper Bounds

Data	S=15				S=30			
	LB	Opt	UB	Gap	LB	Opt	UB	Gap
Fap1	296	302	304	2.6	299	299	299	0
Fap2	932	965	1002	6.9	970	983	1004	3.3
Fap3	875	903†	910	3.6	996	1017†	1029	3.2
Fap4	10336	‡	11036	6.3	9428	‡	9837	4.1

†: Best solution given by CPLEX ‡: No solution given by CPLEX

Numerical Results

Table 6: Costs vs penalties

Data	S=5			S=10			S=15			S=30		
	UB	cost	Penalty	UB	cost	Penalty	UB	cost	Penalty	UB	cost	Penalty
Fap1	324	164	160	310	145	165	304	122	182	299	80	219
Fap2	803	223	620	807	267	540	1002	214	978	832	200	632
Fap3	955	279	676	907	237	570	910	250	660	1029	199	830
Fap4	10186	2286	7900	10250	3050	7200	11036	1122	9914	9837	1016	8821

Conclusion

- SDP helps better solving the stochastic FAP than LP approaches,
- Size limit depends of the ability of SDP packages,
- New stochastic FAP variants are under tests.