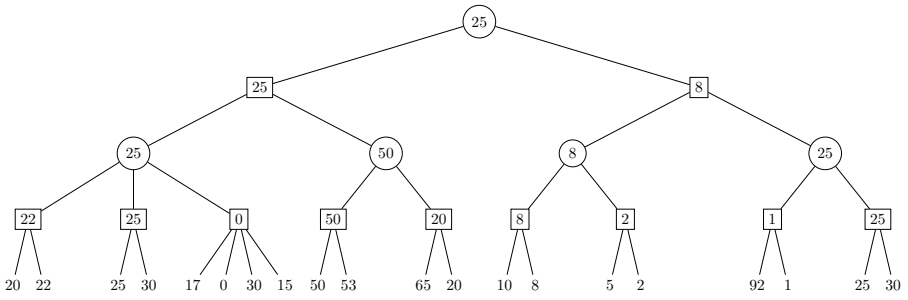
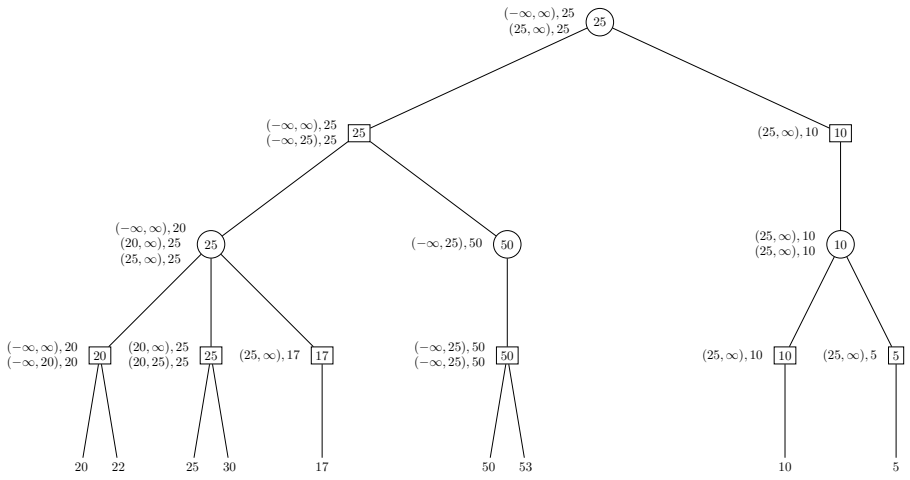
IA Solutions 4: Minimax and $\alpha - \beta$ pruning

Exercise 1

The minmax algorithm places the following values at the nodes.



The `ALPHA_BETA` algorithm will always return the same value as minimax (i.e. 25). However, it reduces the amount of computation required. The tree below shows some of the procedure involved. The numbers inside the nodes are the returned values for each `MAX_VALUE` or `MIN_VALUE` (if we are respectively at a max or min node). The numbers on the left side of each node record the values of (α, β) and v at line 5 for each iteration of the 'for' loop in these functions.



Exercise 2

One solution is to set $a = b = j = k = l = 0$, $c = e = f = g = 1$, $h = 2$, and $d = i = m = n = o = p = q = 3$.

In fact, any solution that satisfies the following conditions works:

1. $c \geq a, b$
2. $\max\{e, f, g\} \geq a, b$
3. $h \geq e, f, g$
4. $\max\{e, f, g\} > j, k, l$

It is easy to check that 1-4. are *necessary*. If 1. is not satisfied, then the branch leading to d will not be cut. If 2. is not satisfied, then the branch leading to G will be cut. If 3. is not satisfied, then the branch leading to i will not be cut. If 4. is not satisfied, then the two rightmost branches from H will not be cut.

(It is harder to see that the conditions 1-4. are *sufficient* for exactly the indicated branches to be cut. If anyone finds a counterexample, let me know.)