Exercise 1

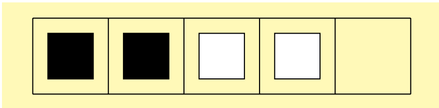
We have five cells in a line, each can host a black chip (B), a white chip (W) or no chip. The following actions are available :

— a chip that is next to the empty cell can move there for a cost of 1

— a chip that is two steps away from the empty cell can jump there for a cost of 2

— a chip that is three steps away from the empty cell can jump there for a cost of 3

We start from a state where there are two black chips on the left, then two white chips in the middle, and finally an empty cell on the right. We would like to move to a state where the black chips are located to the right of the white chips.



Define an admissible heuristic h and find a solution using $A^\star$.

Exercise 2

Suppose n vehicles occupy squares $(1,1)$ through $(n,1)$ (i.e., the bottom row) of an $n \times n$ grid. The vehicles must be moved to the top row but in reverse order ; so the vehicle i that starts in $(i,1)$ must end up in $(n-i+1,n)$. At each time step, every one of the n vehicles can move one square up, down, left, or right, or stay put ; but if a vehicle stays put, one other adjacent vehicle (but not more than one) can hop over it. Two vehicles cannot occupy the same square.

1. The size of the state space is roughly
   (a) n^2 (b) n^3 (c) n^{2n}
2. The branching factor is roughly
   (a) 5 (b) $5n$ (c) 5^n
3. Suppose that vehicle i is at (x_i, y_i) ; write a nontrivial admissible heuristic h_i for the number of moves it will require to get to its goal location $(n-i+1,n)$, assuming there are no other vehicles on the grid.
4. Which of the following heuristics are admissible for the problem of moving all n vehicles to their destinations ?
   (a) $\sum_{i=1}^{n} h_i$ (b) $\max\{h_1, \ldots, h_n\}$ (c) $\min\{h_1, \ldots, h_n\}$
   Explain your answer.

Exercise 3

Show that a monotone heuristic is admissible.