## Game Theory

## Exercise 1 : Pure Equilibria

Find all the Nash equilibria in the following games:

|  | G | D |
| :---: | :---: | :---: |
| H | 2,2 | 0,1 |
| B | 1,0 | 1,1 |


|  | G | D |
| :---: | :---: | :---: |
| H | 1,1 | 0,0 |
| B | 0,0 | 1,1 |


|  | G | D |
| :---: | :---: | :---: |
| H | 2,2 | 0,0 |
| B | 0,0 | 1,1 |

G D
H $\quad 0,0 \quad 3,1$
B 1,3 2,2

## Exercise 2 : Mixed Equilibria

Explain why in the following game the pair of mixed strategies where

- the row player plays H with probability $\frac{3}{4}, \mathrm{M}$ with probability 0 and B with probability $\frac{1}{4}$
- the column player plays $G$ with probability $0, C$ with probability $\frac{1}{3}$ and $D$ with probability $\frac{2}{3}$ is a Nash equilibrium (each $\star$ represents an unknown utility).

|  | G | C | D |
| :---: | :---: | :---: | :---: |
| H | $\star, 2$ | 3,3 | 1,1 |
| M | $\star, \star$ | $1, \star$ | $2, \star$ |
| B | $\star, 4$ | 5,1 | 0,7 |

## Exercise 3



We consider $n$ farmers who can each produce at no cost as much wheat as they want. If the $k$ th farmer produces $q_{k}$, the total quantity produced is $Q=q_{1}+q_{2}+\ldots+q_{n}$. The price of wheat will then be $p=e^{-Q}$.

1. Show that the individual strategy of producing one unit of wheat is dominant for each farmer. From this deduce that the profit for each farmer is $e^{-n}$.
2. Suppose that the farmers reach an explicit agreement where in total 1 unit of wheat is produced. Show that in this case the total profit is maximal. Would this happen in the absence of an explicit contract?

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