Fall if it Lifts your Teammate: A Novel Type of Candidate Manipulation

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ABSTRACT

We present a new interpretation of the traditional computational social choice framework, where what are traditionally the candidates are construed as the agents. The particular implementation in mind is the proposed system for determining the medal winners for sports climbing in the 2020 Olympic games. We consider the issues of ties and of potential manipulation with respect to this interpretation. Simulation results suggest that for the proposed system ties are unlikely to be a problem, but that there is at least potential for manipulation, of a novel type. We formalise this conception of manipulation axiomatically. The strongest axioms lead to an impossibility along the lines of Arrow's impossibility, while a small weakening leads to a possibility. We also provide a hardness result concerning the determination of possible manipulation.

KEYWORDS

Social choice theory; Game Theory for practical applications

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1 INTRODUCTION

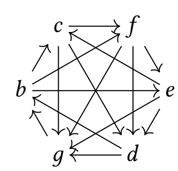
The 2020 Olympic Games in Tokyo will inaugurate ten new gold medals; one male and one female in each of five new events: karate, skateboarding, surfing, baseball and sports climbing. Of these, sports climbing did not exist in a unique competition format before its introduction as an Olympic event. Instead there are three types of competitive climbing: bouldering, lead-climbing and speed-climbing. Each requires different skills and measures the performance of athletes using different methods. Thus sports climbing is to be a composite event, similar to the pentathlon, however the novel event will have its own novel system for determining the medal winners.

For Tokyo 2020 the International Federation of Sports Climbing (IFSC) has devised a combined format for sports climbing [15]. Twenty athletes—each country can have up to two representatives—will be involved in the main event. A qualification round reduces this to six, who then compete in a final round to determine the medal winners. Both rounds proceed in the same manner: the athletes compete in all three disciplines, producing three linear orders over the athletes. Each athlete is assigned a score corresponding to the product of their rank in each discipline, where the rank of an athlete is the number of other athletes that defeat her plus one. The products are used to determine a final, overall ranking: athletes with lower product scores are ranked better than those with higher. If two athletes receive the same score ties are broken in favour of the athlete that performs better in more disciplines.

Unfortunately, the combined Olympic format may lead to tied situations if more than two athletes have the same product score and the pairwise comparisons form a cycle. Ties are problematic in both rounds of the competition: in the final round, it is desirable to have a single gold winner; in the qualifying round, ties may necessitate an extra method to determine which athletes progress. An example demonstrates the problem. The following table shows potential ranks of seven of the athletes and bounds on the ranks of the other thirteen athletes after the qualification round.

Athlete	Ranking in discipline Speed	Bouldering	Lead	Product
a	3	3	6	54
b	1	5	12	60
c	4	15	1	60
d	15	1	4	60
e	2	10	3	60
f	5	6	2	60
g	6	2	5	60
others	≥ 7	≥ 4	≥ 7	≥ 196

Clearly a progresses to the final round and none of the unnamed athletes progress, thus one of the athletes b–g must be eliminated. We here display a directed graph over these athletes such that arcs give the pairwise comparisons, e.g. b ranks better than c in speed and bouldering, so we draw an arrow from b to c. Amongst the six tied athletes there is no *Condorcet loser*—no athlete defeated by all the others. So it is not immediately obvious who should be eliminated.



We continue developing our example to demonstrate another potential problem with the combined Olympic format: it may prompt deliberate bad performances. Suppose that b is eliminated.[1] The final round can be predicted by the performances in the qualifying round, giving the following ranks and products.

Athlete	Ranking in discipline Speed	Bouldering	Lead	Product
a	2	3	6	36
c	3	6	1	18
d	6	1	4	24
e	1	5	3	15
f	4	4	2	32
g	5	2	5	50

[1]The IFSC uses a "seeding list" to break ties that are not resolvable by pairwise comparisons. Such a seeding list is, in effect, an external linear order tiebreaker. For the final round the ranking of the qualification round is used as a seeding list; for the qualification round a seeding list based on the prior qualification system is used [22].

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Suppose that a and c have the same nationality. Athlete a is not predicted to win a medal, while c is on course for the silver. However, if a deliberately performs worse than c in the speed competition and the other ranks are as predicted, c will be the unique gold medal winner with a product score of 12.[2] National loyalty may lead a to *manipulate* in this manner, *spoiling* e's efforts which would otherwise procure a gold medal.

We have seen two potential problems with the proposed competition format for climbing: firstly ties, and secondly a phenomenon that we refer to as manipulation. The rest of the paper further explores these two issues. In Section 2 we define our framework, including formal definitions of manipulation. Section 3 is divided into three subsections in which we present our main results. Our simulation results in the first subsection show that the probability of ties under the proposed method is low, but there is a high potential for manipulation. In the second subsection, we present a hardness result concerning determining one of the types of manipulation, however this only applies asymptotically as the number of athletes and disciplines grows, thus is not applicable to the practical case of sports climbing at the Olympics. We consider the theoretical possibility of methods that do not allow for manipulation in the third subsection. We show that it is impossible to completely rule out manipulation, but we give a possibility for a plausible weakening. Section 4 provides a summary and further discussion; we discuss our framework's position with respect to other literature, possible extensions and other further work, and the relevance of our results to the climbing competition at the 2020 Olympics.

2 DEFINITIONS

Denote by $A = \{a, b, c, \ldots\}$ the set of athletes and by $N = \{1, \ldots, n\}$ the set of disciplines. We suppose that $m \geq 3$ and $n \geq 2$. Denote by $\mathcal{L}$ the set of linear orders and by $\mathcal{W}$ the set of total preorders on A. We use $\succeq$ for a total preorder over the athletes, with $\succ$ the asymmetric part. Each athlete competes in each discipline $i \in N$, resulting in n linear orders $\succ_i$ over A. A *profile* summarising the results for each discipline is some $(\succ_1, \ldots, \succ_n) = \boldsymbol{\succ} \in \mathcal{L}^N$. A *ranking function* f takes a profile and produces a total preorder of athletes: $f\colon \mathcal{L}^N \to \mathcal{W}$. For an ordering $\succeq$ over competitors, the *rank* of $a \in A$ is $r_{\succeq}(a) = |\{x \in A\colon x \succ a\}| + 1$. To simplify notation, for a discipline $i \in N$ we write r_i for $r_{\succeq_i}$.[3] We refer to athletes ranked first in the output as *winners*. Denote the distinct pairs of athletes by $D = \{(x, y) \mid x, y \in A, x \neq y\}$. Given a profile $\boldsymbol{\succ}$, define $\mathrm{ct}_{\boldsymbol{\succ}}\colon D \to \mathbb{Z}$ by $\mathrm{ct}_{\boldsymbol{\succ}}(x, y) = |\{i \in N \mid r_i(x) < r_i(y)\}| - |\{i \in N \mid r_i(y) < r_i(x)\}|$. For an arbitrary profile $\boldsymbol{\succ}$ the *weak majority relation* $T_{\boldsymbol{\succ}} \subseteq A \times A$ is defined by $xT_{\boldsymbol{\succ}}y$ iff $\mathrm{ct}_{\boldsymbol{\succ}}(x, y) \geq 0$. This is complete regardless of the parity of $|A|$, but may not be transitive; for a binary relation R we write R^+ for the transitive closure of R. We call the proposed ranking function *inverse-Borda-Nash*[4], denoted by $\mathsf{bn}\colon \mathcal{L}^N \to \mathcal{W}$. Write $\mathrm{prod}_N(x) = \prod_{i \in N} r_i(x)$, when N is clear from context we will leave out this subscript. Define the binary relation $Q \subseteq A \times A$ by xQy iff $(\mathrm{prod}(x) < \mathrm{prod}(y)$ or $(\mathrm{prod}(x) = \mathrm{prod}(y)$ and $xT_{\boldsymbol{\succ}}y))$. Define $\mathsf{bn}(\boldsymbol{\succ}) = Q^+$; because Q is complete this is a total preorder.

2.1 Basic desiderata

An athlete $x \in A$ clearly beats $y \in A$ in $\boldsymbol{\succ}$ if for all $i \in N$, $x \succ_i y$. We say f satisfies the *clear winner condition* if whenever x clearly beats y in $\boldsymbol{\succ}$, then for $\succeq = f(\boldsymbol{\succ})$ it is the case that $x \succ y$. We say f is *neutral* if for any permutation $\sigma\colon A \to A$, given $\boldsymbol{\succ}$ and $\boldsymbol{\succ}'$ such that for all $a, b \in A$, $i \in N$ $a \succ_i b \Leftrightarrow a \succ'_i b$, then $af(\boldsymbol{\succ})b \Leftrightarrow af(\boldsymbol{\succ}')b$. Our last basic desideratum limits how much a single discipline determines the winner. The gold is determined by $i \in N$ if, for any profile $\boldsymbol{\succ}$, $r_i(x) = 1$ implies $r_{\succeq}(x) = 1$, where $\succeq = f(\boldsymbol{\succ})$. We say f is *non-determined* if the gold is not determined by any $i \in N$.[5]

2.2 Formal definitions of manipulation

Manipulation in social choice theory traditionally involves insincere representation of preference [11, 27], and allows for any possible change to one of the orderings in the profile. Our situation is slightly different: a potential manipulator has a position within each ordering and can manipulate, individually, by changing her position in multiple orderings; and she is restricted in the type of change she can make to each ordering; specifically, she can only make her own ranking worse: an athlete cannot perform better than her best. Ruling out *purely individual* manipulation amounts to a version of monotonicity that is satisfied by Inverse-Borda-Nash. However, an athlete may be able to *altruistically* manipulate for a teammate—in particular, this may be possible without worsening their own output ranking.

Definition 2.1 (Non-sacrificial manipulation). Take a ranking function f. Let $f(\boldsymbol{\succ}) = \succeq$ and $f(\boldsymbol{\succ}') = \succeq'$, and $a, b \in A$. Athlete a can manipulate without sacrifice, for athlete b, from the profile $\boldsymbol{\succ}$ to the profile $\boldsymbol{\succ}'$ if

(1) for all $i \in N$, $x \in A \backslash \{a\}$ and $y \in A$, $x \succ_i y$ implies $x \succ'_i y$
(2) $r_{\succeq'}(b) < r_{\succeq}(b)$
(3) $r_{\succeq'}(a) \leq r_{\succeq}(a)$.

Condition (1) implies that only a changes her ranking in the profile, and that she can only worsen it. Condition (2) implies that b receives a strictly better ranking in the output. Condition (3) implies that a doesn't receive a worse ranking in the output. We say that a manipulation is *strictly* without sacrifice if it also satisfies

(4) $|\{x \in A : x \succeq' a\}| \leq |\{x \in A : x \succeq a\}|$.

The strictness condition makes it harder to manipulate: the idea is that an athlete prefers to be uniquely ranked in a position than to

[2] This would be feasible in practice. The speed-climbing final proceeds as a kind of knockout tournament. Under the assumption that a defeats g and that c defeats f in the quarter finals, a will face c in the semifinals; if a deliberately loses then c will be guaranteed to be ranked at second or better at speed-climbing [15].

[3] Note that $\succ_i$ inverses the natural ordering on ranks: for all $x, y \in A$ and $i \in N$, $x \succ_i y$ iff $r_i(x) < r_i(y)$; and that the output can contain ties, but if two competitors are ranked first, no competitor is ranked second.

[4] Our nomenclature invokes Borda scores and the Nash product. The Nash product is sometimes described as a compromise between the utility maximisation of additive methods and the maximin of egalitarian methods. This is not the case for the proposed method because the Borda scores are *inverted*. According to inverse-Borda-Nash, an athlete with rankings (1,1,4) beats an athlete with (2,2,2); whereas for traditional Borda scores the opposite is true: (19,19,19) would be considered better than (20,20,17). This is considered an advantage because it favours specialists—it is preferred that the winner of the combined format is a potential winner of world-cups in some individual discipline, rather than a generalist [22]. We are not aware of a precedent for this, perhaps because it becomes "anti-fairness" when applied to social welfare.

[5] These three desiderata correspond to axioms from social choice theory. The clear winner condition is called, for e.g., the "Pareto criterion" [6, p. 42]. "Neutral" is standard terminology in social choice theory. A discipline that determines the gold may be thought of as a "weak top-dictator": a voter whose top ranked alternative is among the top ranked alternatives in the output.

share this ranking with multiple athletes. Indeed, this small change makes the difference between an impossibility and possibility result.

The example of manipulation in the introduction is *not* a manipulation without sacrifice—instead, a poorly ranked athlete "spoils" the fair result concerning other, better ranked, athletes.

Definition 2.2 (Spoiler manipulation). Take a ranking function f. Let $f(\succ) = \geq$ and $f(\succ') = \geq'$, and $a, b \in A$. Athlete a can spoil, for athlete b, from the profile $\succ$ to the profile $\succ'$ if

(1) for all $i \in N$, $x \in A \backslash \{a\}$ and $y \in A$, $x \succ_i y$ implies $x \succ'_i y$
(2) $r_{\geq'}(b) < r_{\geq}(b)$
(5) $r_{\geq'}(b) < r_{\geq}(a)$.

Note that conditions (1) and (2) are as before; condition (5) captures the idea that a is a poorly ranked alternative with respect to the possible ranking of b.

3 RESULTS

Inverse-Borda-Nash satisfies all the basic desiderata—the proof is omitted due to space considerations.

PROPOSITION 3.1. *Inverse-Borda-Nash satisfies the clear winner condition, is neutral, and is non-determined.*

However, our example in the introduction shows that inverse-Borda-Nash may produce ties and is potentially manipulable—undesirable possibilities that we consider further in the next two subsections. In the third subsection we apply a broader analysis and give an impossibility that applies to ranking functions more generally.

3.1 Simulations: the frequency of ties and manipulation

Our simulations suggest that for inverse-Borda-Nash, although ties are unlikely to be a problem, potential for manipulation occurs with a high probability.

We generated profiles with six athletes and three disciplines, the same numbers as in the Olympics sports climbing competition. The generated profiles form three groups: in the first group, for each discipline every possible linear order is equally likely—this is the *impartial culture* [29]. For profiles in the second group there is a positive correlation in an athlete's results across the three disciplines. In the final group there is positive correlation between two disciplines and negative correlation with the third. This third culture conforms best to our actual expectations for the competition because the two disciplines of bouldering and lead climbing have an intersection of athletes at the top level, whereas top level speed climbers do not typically compete in the other disciplines.

For a profile based on the impartial culture we independently select three strict linear orders, each uniformly at random from the set of all possible linear orders. Our positively correlated culture uses the Plackett-Luce ([1959, 1975]) model[6] with initial odds

$$2^1 : 2^2 : 2^3 : 2^4 : 2^5 : 2^6.$$

Label the athletes as a_j for $j \in \{1, 2, 3, 4, 5, 6\}$. Writing $t = \sum_{i=1}^{6} i^2$, each athlete a_j has a $p_j = j^2/t$ probability of being ranked first. The idea is that the initial odds also represent the strengths of the athletes; in particular, we suppose that each athlete is two times stronger than her closest competitor. Given that every athlete in set B defeat all the athletes in $A \backslash B$, the probability that an athlete in $A \backslash B$ defeats the other athletes in $A \backslash B$ only depends upon the strengths of the athletes in $A \backslash B$. So if athlete a_k, $k \neq j$ ranks first, then a_j has a

$$\frac{j^2}{t - k^2} = \frac{p_j}{1 - p_k}$$

probability of being ranked second. If a_k ranks first and a_l ranks second, $l \neq j$, $l \neq k$, then a_j has a

$$\frac{j^2}{t - k^2 - l^2} = \frac{p_j}{1 - p_k - p_l}$$

probability of being ranked third. The positive correlation arises because we suppose that the athletes have the same strengths for each discipline; a profile consists of three independently generated strict linear orders using the same initial odds. A negatively correlated profile is created by taking a positively correlated profile and reversing the strict linear order of the last discipline.[7]

Culture	Ties	Spoiler	Without sacrifice	Strict without	Any manip.
Impartial	632	37,730	47,807	47,326	59,660
Positive cor.	779	13,792	43,723	41,597	46,964
Negative cor.	526	44,826	48,741	48,350	63,151

Table 1: The number of randomly generated profiles that involved ties, were subject to spoiler manipulation, were subject to manipulation without sacrifice, were subject to manipulation strictly without sacrifice, and that were subject to any of the manipulations that we define. We generated 100,000 profiles for each culture.

We randomly generated 100,000 profiles of each type. A profile counts as tied if at least one tie occurs at any ranking level—we do not count the number of distinct ties nor how many athletes are involved in each tie. To count manipulations, we first randomly pair the athletes into three disjoint pairs. A profile counts as manipulable if at least one of the pairs can manipulate. We perform the count separately for spoiler manipulation, manipulation without sacrifice, manipulation completely without sacrifice, and for any type of manipulation. The results are presented in Table 1.

According to our models, it is very unlikely that there will be a tie at any level of the output total preorder in the final round of the competition. We also ran simulations for twenty athlete profiles obtaining similar results.[8] This strongly supports the idea that a tie in the actual competition is very unlikely to occur: note that each of our models exhibits a high degree of symmetry; one would expect that such symmetries would be the most likely to cause tied situations. Indeed, it has been shown that the impartial culture maximises the probability for majority cycles [29], one of the necessary conditions for a tie. However, from our simulations we see more ties for the positively correlated culture: this is perhaps because two opposing criteria need to be fulfilled for there to be a

[6]This model's ability to forecast Formula 1 results has been studied by [14].

[7]The associated culture best represents our, somewhat naive, expectations for the competition—we expect lead climbing and bouldering to be positively correlated with each other and negatively correlated with speed climbing.

[8]For profiles with twenty athletes, of the 100,000 profiles we generated for each culture, 208 profiles had ties for the impartial culture, 1108 profiles had ties for the positively correlated culture, and only 92 profiles had ties for the negatively correlated culture that conforms best to our expectations for the actual competition.

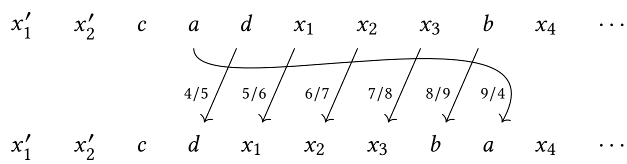

Figure 1: The best ten athletes in the ranking of a single discipline before and after a manipulates for b. The numbers display the ratio between an athlete's product scores before and after this particular manipulation.

tie; there need to be majority cycles, but these must occur among athletes with the same *scores*. Regardless, the incidence of ties is low even for the positive culture. Of the three cultures, we see fewest ties ties in the negatively correlated culture which best represents our expectations for the competition.

In contrast to the low incidence of ties, there does seem to be a high potential for manipulation, of both kinds. For each culture approximately half the profiles are manipulable.[9] We make two further observations: first, the incidence of *non*-strict manipulation without sacrifice is very small—the value obtained when subtracting the value of column five from the value of column four. A loose conclusion is that for inverse-Borda-Nash there is not much difference between the stronger and weaker versions of the without sacrifice axiom. Second, spoiler manipulation seems less likely under the positively correlated culture. An intuitive explanation for the lower incidence of spoiler manipulation for the positively correlated culture is the following: for this culture it is more likely that one athlete in a pair will *always* be ranked above their teammate, in which case the lower ranked athlete cannot spoil. Nevertheless, even for the positive culture there is a non-negligible potential for spoiler manipulation (more than 10% of the generated profiles).

3.2 Complexity results: NP-hardness of manipulation without sacrifice

In this subsection we determine the worst-case complexity of detecting whether or not a manipulation without sacrifice is possible. Given that we have managed to perform simulations, it is obviously easy in practice to determine whether or not a manipulation is possible. Although the hardness result we give is not of direct importance to the case of sports climbing, which involves three disciplines and at most twenty candidates, it may well be important for other applications of our model. The decision problem is:

> Can-manipulate-without-sacrifice (Cmws) :
> **Input**: a profile $\succ \in \mathcal{L}^N$ and two athletes a and b.
> **Question**: Is there a manipulation without sacrifice by a for b from $\succ$ to some other profile?

We show that Cmws is NP-complete, thus under typical assumptions, intractable for large inputs.

Proposition 3.2. *Cmws is NP-complete.*

Proof. For membership, take the manipulated profile as a certificate and verify that this actually is a manipulation, i.e. that b becomes ranked better and that a does not become ranked worse.

For hardness, we provide a polynomial reduction from exact-3-cover, which is the following NP-complete problem:

> **Input**: a set $X = \{x_1, \dots, x_{3t}\}$ and a set $\mathcal{X} = \{X_1, \dots, X_s\}$ of subsets of X such that for each $Y \in \mathcal{X}$, $|Y| = 3$.
> **Question**: is there a subset $\mathcal{X}' \subseteq \mathcal{X}$ such that $\bigcup \mathcal{X}' = X$ and for any distinct pair $X_i, X_j \in \mathcal{X}'$, $X_i \cap X_j = \emptyset$.

Take an instance of exact-3-cover. We use this to create a profile, containing athletes a and b, polynomial in terms of the size of the original instance, such that our manipulation decision problem returns "yes" for this profile if and only if exact-3-cover returns "yes" for the original instance.

The athletes are $\{a,b,c,d\} \cup X \cup X'$. The purpose of these athletes in the reduction is as follows: a is the potential manipulator; b is the athlete she attempts to help; c is an athlete who defeats b according to the starting profile, but who will be tied with b if a manipulates in at least t disciplines; d is an athlete who b defeats according to the starting profile, but who will defeat b if a manipulates in strictly more than t disciplines; each $x_i \in X$ is defeated by a according to the starting profile, but threatens to defeat a in the course of the manipulation—in particular a will only be able to improve the ranking of x_i at most once; and $X' = \{x'_1, \dots, x'_{10}\}$ contains ten dummy athletes who pad rankings and ensure that cycles are of the right length. In the starting profile, each discipline that a potentially changes her ranking in corresponds to some $Y \in \mathcal{X}$. For each of these disciplines, if a changes her ranking then she improves the ranking of each $x_i \in Y$ (though by different amounts: see Figure 1). If we have such a starting profile, then there is a successful manipulation without sacrifice iff exact-3-cover returns "yes" for the original instance.

We construct the starting profile from two parts: a base subprofile and a padding subprofile. In the base subprofile the athletes $\{b,c,d\}$ all receive the same product score, and the athletes $\{a\} \cup X$ all receive the same product score; it also contains all those disciplines for which a might want to worsen her ranking in order to manipulate for b. The padding profile creates the required differences in the product scores of the athletes.

For the base profile, for each triple $\{x_1, x_2, x_3\} \in \mathcal{X}$ we create $t + v + 1$ disciplines and the associated rankings, where v is the smallest natural number such that $t + v + 1$ is divisible by nine. For each triple only one of the created rankings is manipulable by a for b, and we will refer to this ranking as corresponding to the triple, the remaining rankings make athletes within two specific sets have the same product scores. Precisely, two dummy athletes x'_1 and x'_2 are placed at the top of the ranking, then c, a, and d, then (in arbitrary order) the elements in the triple, then b, then (in arbitrary order) the remaining elements of X, and finally the remaining athletes in X'. To ensure that athletes in $\{b,c,d\}$ have the same product scores and that athletes in $\{a\} \cup X$ have the same product scores for this partial profile, we add rankings that cycle through the athletes in these two sets, also requiring that for each of these rankings b performs better than a. Figure 2 demonstrates the cycles, and shows how the dummy athletes in X' allow for b to perform better than a in these cases.

[9] We also tested profiles with twenty athletes for manipulation. Each culture resulted in higher counts of potential manipulation than in the six athlete case. Of course, to fully address the issue of manipulation in the qualification round would require other modifications: here a manipulation is only desirable if it moves the target agent below the sixth place threshold; more fundamentally we require an argument for why the the non-manipulated profile is common knowledge.

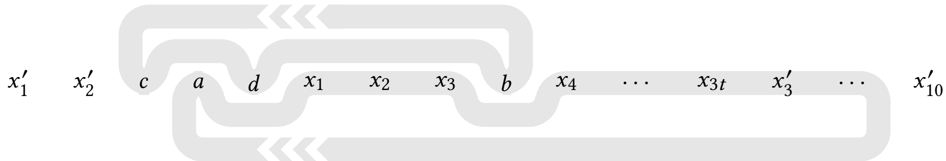

Figure 2: The ranking corresponding to the set $\{x_1, x_2, x_3\} \in \mathcal{X}$. Leftwards athletes perform better than rightwards athletes. The cycling is indicated by the thick arrows. For the rankings other than that displayed, we ensure that a is ranked below b if we cycle by the higher arrow once every three times we cycle by the lower arrow. In the cycle indicated by the lower arrow, of X' we cycle through only some $X'' \subseteq X'$ such that $1 + |X| + |X''|$ is divisible by nine; thus the cycles end at the same point.

For the padding part of the profile, we want to ensure, first, that the following ratios between the product scores of the athletes hold:

(1) $\mathrm{prod}(c) = (8/9)^t \mathrm{prod}(b)$,
(2) $\mathrm{prod}(b) = (9/10)^t \mathrm{prod}(d)$,
(3) $\mathrm{prod}(d) \leq \mathrm{prod}(a)$,
(4) $\mathrm{prod}(a) = (4/9)^t (4/5) \mathrm{prod}(x)$ for all $x \in X$,
(5) $\mathrm{prod}(a) < (4/9)^t \mathrm{prod}(x')$ for all $x' \in X'$;

second, that b is not ranked worse than a in any of the rankings; and third, that the number of disciplines required is bounded by a polynomial in the size of the original input. The construction of the padding profile is facilitated by the following lemma.

Lemma 3.3. *Given disjoint sets $X^{\mathrm{I}}, X^{\mathrm{II}}, X^{\mathrm{III}}$ such that $X^{\mathrm{I}} \cup X^{\mathrm{II}} \cup X^{\mathrm{III}} = X$, athletes $x, y \in X^{\mathrm{II}}$, and integers*

$$p, q \in \left\{ \left|X^{\mathrm{I}}\right| + 1, \left|X^{\mathrm{I}}\right| + 2, \ldots, \left|X^{\mathrm{I}} \cup X^{\mathrm{II}}\right| \right\},$$

there is a profile over an index set I, of with cardinality the lowest common multiple of $\left|X^{\mathrm{I}}\right|$, $\left|X^{\mathrm{II}}\right|$ and $\left|X^{\mathrm{III}}\right|$, such that

$$\begin{aligned} \mathrm{prod}_I(z) &= \mathrm{prod}_I(w), && \text{for } j \in \{\mathrm{I}, \mathrm{III}\},\ z, w \in X^j, \\ \mathrm{prod}_I(z) &< \mathrm{prod}_I(w), && \text{for } j < k,\ z \in X^j,\ w \in X^k, \\ \mathrm{prod}_I(x) &= (p/q)\mathrm{prod}_I(z), && \text{for } z \in X^{\mathrm{II}} \backslash \{x, y\}. \end{aligned}$$

Proof of Lemma 3.3. Create a base ranking starting with elements of X^{I} in arbitrary order, then elements of X^{II} such that x is at position q and y is at position p and with the other elements in arbitrary order, and finally elements of X^{III} in arbitrary order. The profile is built up of cyclings through this ranking, with the base ranking replaced (once) by the ranking where the positions of x and y are swapped. The cycles go individually through X^{I}, X^{II} and X^{III}. ■

In order to make part (1) hold, we apply the lemma t times with the values $X^{\mathrm{I}} = \emptyset$, $X^{\mathrm{II}} = X' \cup \{b, c, d\}$ and $X^3 = X \cup \{a\}$, $x = x_1'$, $y = c$, and $p = 9$ and $q = 8$. Note that b and d have the same product scores in the created profile. This means we can then set the ratio required by (2) between these two athletes, again with repeated applications of the lemma as above except here $x = d$, $y = x_1'$ and $p = 9$ and $q = 10$. For part (4), set $X^{\mathrm{I}} = \{b, c, d\}$, $X^{\mathrm{II}} = X \cup X' \cup \{a\}$ and $X^3 = \emptyset$, and set $x = a$, $y = x_1'$ and $p = 4$, $q = 5$ or $q = 9$. Finally, we can pad with as many instances where $X^{\mathrm{I}} = \{b, c, d\}$, $X^{\mathrm{II}} = X \cup \{a\}$, X^{III} and $p = q$ as required to ensure the desired inequalities of (3) and (5). Note for all of these rankings b is ranked better than a, and that their amount is bounded by some polynomial.

Suppose that we have a "yes" instance to our original problem, so there exists a exact covering $\mathcal{X}'$ of X. Let a worsen her ranking so that she is directly below b in exactly the t disciplines corresponding to $\mathcal{X}'$—an example of how this affects each ranking is displayed in Figure 1. In the resulting profile the product score of c remains the same, the product score of b is multiplied by $(8/9)^t$, and the product score of d is multiplied by $(3/4)^t$: thus the product scores of a, b and c become the same. For each $x \in X$, the product score is decreased by at most $(5/6)$—note this is less of a decrease than $(4/5)$, whereas for a the product score is increased by $(9/4)^t$. For $x' \in X'$ the product score doesn't change. Altogether, this means that c, b and d receive the same, maximal product scores: thus b goes from being ranked second to being ranked first; and a still has a product score strictly less than any $z \in X \cup X'$: thus the manipulation is without sacrifice.

In the other direction, suppose that there is a successful manipulation without sacrifice. As the only athlete that b is ranked worse than in the non-manipulated profile is c, this means that b must be ranked at least as high as c in the manipulated profile, which means that a must change her ranking in at least t disciplines where she is ranked better than b. However, a cannot change her ranking in more of these disciplines, otherwise d would become ranked strictly above b and the manipulation would not be successful. Also, of the disciplines that she changes her ranking in, she can only improve the ranking of any given x_i at most once, otherwise this x_i would multiply her product score by a value less than $(7/8)^2$—itself less than $(4/5)$—so x_i would be ranked better than a and the manipulation would not be without sacrifice. Thus the disciplines in which a does manipulate correspond to an exact covering of the set X. □

3.3 Normative results: an impossibility

We want to define a method that satisfies our desiderata and completely prevents both forms of manipulation. Unfortunately, it is impossible to completely succeed in this task.

Theorem 3.4. *No ranking function prevents spoiler manipulation, prevents manipulation without sacrifice, satisfies the clear winner condition, and is non-determined.*

Proof. We prove the stronger: if both types of manipulation are prevented and the clear winner condition is satisfied then there is a *top-dictator*, a discipline $i \in N$ such that, for $a \in A$ such that $r_i(a) = 1$, $a > x$ for all $x \neq a$. This implies that the function is determined.

Take an arbitrary ranking function f that prevents spoiler manipulation and manipulation without sacrifice and that satisfies the clear winner condition. Consider any profile where a comes first in all disciplines and b comes last. By the clear winner condition a must be ranked first and b last. Now consider moving b up in the first discipline. So long as b does not cross above a, a must still be uniquely ranked first, as otherwise the agent c that b becomes ranked above can spoil for the new winner from P_1 to P_2.

P_1

1	> 1		
a	a	$\mapsto$	a
|	|		|
c	|		
b	|		
|	|		
|	b		

P_2

1	> 1		
a	a	$\mapsto$	a
|	|		|
b	|		
c	|		
|	|		
|	b		

If we rank b above a, either a remains the unique winner or there is some other set of winners. If we continue to rank b first successively for the remaining disciplines, eventually b becomes the unique winner by the clear winner condition—in particular when b is ranked first in all disciplines: thus the second disjunct of the previous sentence must be fulfilled at some point; there is a profile that outputs a top, while if b is moved above a in one discipline the set of winners is X with some $x \in X$ such that $x \neq a$. Label the discipline for which this happens i^* and label the respective profiles as P_3 and P_4. Note that in P_3 athlete b has been moved up to be directly below a, the same argument as above implies that a is still the winner in the output.

P_3

$< i^*$	i^*	$> i^*$		
b	a	a	$\mapsto$	a
a	b	|		|
|	|	|		
|	|	b		

P_4

$< i^*$	i^*	$> i^*$		
b	b	a	$\mapsto$	b
a	a	|		|
|	|	|		
|	|	b		

We know that $a \notin X$, otherwise a could spoil for x from P_3 to P_4. This implies that $b \in X$, as otherwise b could spoil for a from P_4 to P_3; thus $x \notin X$ for $x \neq a, b$, as otherwise b could manipulate without sacrifice for x from the profile where b is the unique winner. In P_4 we can move a down in the profile without changing the output winner b (otherwise a could spoil), call this P_5. Create P_6 from P_5 by moving a up to win in discipline i^*.

P_5

$< i^*$	i^*	$> i^*$		
b	b	|	$\mapsto$	b
|	a	|		|
|	|	|		
|	|	a		
a	|	b		

P_6

$< i^*$	i^*	$> i^*$		
b	a	|	$\mapsto$	a
|	b	|		|
|	|	|		
|	|	a		
a	|	b		

We claim athlete a must the unique winner in P_6. First, note that if neither a nor b were ranked first for P_6, then a can spoil for b from P_6 to P_5. If b is ranked first but not uniquely ranked first, then b can spoil from P_5 to P_6. If b is uniquely ranked first, then at some point in stepwise changes from P_6 to P_3 some other athlete must perform a spoiler manipulation. Thus as b is not ranked first a is amongst the winners. If a were not unique, a could spoil from P_3 to P_6. Take some third alternative $c \neq a, b$. The profile P_7 is obtained from P_6 by moving b and c down in the profile. Here the unique winner is still a, as otherwise b or c could spoil. Create P_8 by moving a to be ranked last in all disciplines except i^*.

P_7

$< i^*$	i^*	$> i^*$		
|	a	|	$\mapsto$	a
|	|	|		|
c	|	c		
b	c	a		
a	b	b		

P_8

$< i^*$	i^*	$> i^*$		
|	a	|	$\mapsto$	a
|	|	|		|
c	|	c		
b	c	b		
a	b	a		

In the profile P_8, alternative c is a clear winner over b, so b cannot be ranked first. If a were not ranked first then b could spoil for a from P_8 to P_7. If any other athlete is ranked first, then a can manipulate without sacrifice from P_7 to P_8. Thus a is the unique winner in P_8.

In general, for any profile where a wins in discipline i^*, a must be uniquely ranked first in the output, as otherwise there would be some chain of changes from P_8 to the profile in question, one of which would be a spoiler manipulation for the new winning athlete. As a is arbitrary, for each alternative x there is a discipline i_x such that whenever x wins in i_x, x is uniquely ranked first. As two alternatives x and y cannot both be ranked first, $i_x = i_y$ for all $x, y \in A$, thus i^* is a top dictator. □

The proof closely follows Reny [24], who presents Arrow's impossibility and the Gibbard-Satterthwaite theorem side-by-side. Although we consider manipulation, the shape of our result is closer to Arrow's result than to the Gibbard-Satterthwaite result. Requiring the impossibility of both forms of manipulation replaces the axiom of independence of irrelevant alternatives (IIA), though this requirement does not *imply* IIA[10], thus our impossibility is not simply a corollary of Arrow's theorem.

We cannot satisfy all our desiderata simultaneously. However, if we weaken manipulation without sacrifice to manipulation strictly without sacrifice there are methods that work. The method we define proceeds in stages, determining the top ranked candidates then removing them from the profile. It may be thought of as a back-to-front version of *instant runoff voting* [33, p. 37] applied using a *majority quota rule*. Also cf. the *Coombs rule* [13]. If an athlete is ranked first in strictly more than half the disciplines, then she is the unique winner with respect to the athletes in the profile. Otherwise, any athlete that has at least one first place ranking in the profile is a joint winner. The winners are removed from the profile, and the procedure repeats. We name this *iterative first place elimination*, ifpe : $\mathcal{L}^N \to \mathcal{W}$. For an arbitrary profile $\succ$, let

$$\mathsf{win}(\succ) = \begin{cases} \{a\} & \text{if } \exists a \in A, |\{i \in N : r_i(a) = 1\}| > n/2, \\ \{x \in A : \exists i \in N, r_i(x) = 1\} & \\ \quad \text{otherwise.} \end{cases}$$

Let $\succ^1 = \succ$. For $t \geq 1$, recursively define $\succ^{t+1}$ as the restriction of $\succ^t$ to $A \backslash \mathsf{win}(\succ^t)$. Writing $\succeq = \text{ifpe}(\succ)$, for $x, y \in A$, define $x \succeq y$ iff there are integers s, t such that $s \leq t$ and $x \in \mathsf{win}(\succ^s)$ and $y \in \mathsf{win}(\succ^t)$.

[10] Consider the discipline aggregator that returns the total preorder where a is ranked uniquely first and all other alternatives jointly second if a is first in all disciplines, and a is ranked second and all other alternatives jointly first otherwise; this violates IIA but prevents both kinds of manipulation.

Proposition 3.5. *Iterative first place elimination prevents spoiler manipulation, prevents manipulation strictly without sacrifice, satisfies the clear winner condition, is neutral, and (for $n \geq 3$) is non-determined.*

Proof. Prevents spoiler manipulation: an athlete cannot affect any of the partial profiles $\succ^t$ starting from $t = 1$ until the profile where she is ranked first in one of the disciplines. Consider those partial profiles in which she is ranked first. There are two possibilities. (1) The athlete is a winner for the partial profile: she cannot spoil because she does as well as the remaining athletes would. (2) The athlete is not a winner for the partial profile, so a different athlete is ranked first in more than half the disciplines: this other athlete is the winner no matter how the putative manipulator changes her ranking.

Prevents manipulation strictly without sacrifice: by the above, an athlete a cannot affect athletes that get better output ranks. Let $\succ$ be the partial profile for which $a \in \mathsf{win}(\succ)$. First suppose a is ranked first in more than half the disciplines: if she performs worse in enough of these she will no longer be the unique winner but such a manipulation is not strict; otherwise she will be removed from the profile in the next step thus any changes to her ranking will not affect the output. Second suppose a is ranked first in less than half the disciplines. If a performs worse in a discipline i for which $r_i(a) > 1$, this will not affect the output ranking as a is removed from the profile in the next round. If a performs worse in a discipline i for which $r_i(a) = 1$, there are three possibilities. (1) A different athlete becomes the unique winner, thus a is ranked lower in the output. (2) A new athlete becomes a winner, in which case the manipulation is not strict. (3) The winners remain the same, thus the same athletes will be removed from this profile and the output will not change.

Clear winner: if a is ranked better than b in all disciplines, then b cannot ranked first in a partial profile if a is still in the partial profile.

Neutrality: permuting the athletes in the profile will result in permuted sets $\mathsf{win}(\succ)$.

Athlete	Ranking in discipline $\{i\}$	$N \setminus \{i\}$
a	2	1
b	1	2
others	≥ 3	≥ 3

Non-determined: here we require the condition that $n \geq 3$; for arbitrary $i \in N$ consider the profile to the left. □

Iterative first place elimination is unsatisfactory because it is *indecisive*, where we use "decisiveness" to refer to a measure of how often ties are produced in the output. A maximally decisive method would always produce a linear order—this recreates the impossibility because it makes manipulation without sacrifice equivalent to the strict version. However, maximal decisiveness is arguably too strong a condition: for a completely symmetric profile, it seems reasonable that conditions external to the profile are used to break the ties.

4 FINAL REMARKS

We propose a novel interpretation of Arrow's traditional social choice framework involving the aggregation of linear orders, under which what are traditionally thought of as candidates are the agents of the model. These agents can strategize by worsening their own position within one or more of the input linear orders. This interpretation captures the problem of aggregating multiple ranked competitions; in particular we consider the method proposed for determining the medal winners for sport climbing at the 2020 Olympics, a method that we call inverse-Borda-Nash. Simulations suggest that although ties are unlikely to occur, inverse-Borda-Nash is potentially manipulable. Although no method can rule out two basic types of manipulation, a small assumption about how athletes are willing to manipulate means that non-manipulable methods are possible. The method demonstrating this possibility is, however, too indecisive to be practical.

Our interpretation is novel to the best of our knowledge. Work concerning manipulation in sports competitions tends to be considered in operations-research (see [31] for a survey) rather than multi-agent settings.[11] We are not aware of any work that explicitly considers candidates as agents in the way that we do—our work is distinct from the traditional presentation of manipulation by strategic candidacy [5, 8, 18].[12] Of course, there are similarities between what we do and other concepts in the literature, e.g. between our definitions of manipulation and *Condorcet independence of irrelevant alternatives* [32] or *one-way monotonicity* [26]; there may be implicit connections that we have missed.

Our interpretation fits well into Arrow's framework. Arguably, the problem of aggregating multiple disciplines is better served by this framework than typical problems of social choice theory. The linear order profile is the input in practice. There are no questions, as there are for social choice theory, about whether eliciting full linear orders is problematic, let alone whether linear order preferences are suitable or even sensible—cf. competing approaches such as approval voting [19] and majority judgment [1].[13] It is not obvious that manipulation is actually undesirable for social choice theory, especially when one considers iterative manipulation [20]—arguably, a better term would be *strategic behaviour*. In contrast, for sports competitions manipulation is aptly named as it is clearly undesirable in and of itself, because it goes against the spirit of the competition or (for the less principled) because it cheapens the spectacle. Concerning information requirements for manipulation, we have proposed that the qualification round can be used as a proxy for the results in the final round. Although this is unrealistic—the

[11] However, there is considerable literature in computational social choice concerning manipulating seedings in tournaments [25, 30].

[12] This does not mean, however, that our work is completely divorced from strategic candidacy. Sometimes candidates have not completely withdrawn from an election race but have merely stopped actively campaigning, in order not to attract votes from a competitor that they prefer to another competitor. This closely parallels the type of manipulation that we consider here. Our ideas may be applied to social choice theory directly, not just the sports competition interpretation.

[13] The use of linear orders is an express desideratum of the IFSC. Rather than linear orders, points could be assigned based upon individual performances, as in, e.g. the pentathlon. Thus the final score for an athlete would be necessarily independent of the performance of other athletes, which rules out our type of manipulation. This approach was discarded by the IFSC because (1) it is too complex for spectators and (2) it is difficult, perhaps impossible, to assign points in a balanced way across the disciplines [22]. We thus take it is as given that the input is ordinal.

Interestingly, Balinski and Laraki [1] use the example of Olympic figure skating as part of their argument against the traditional ordinal approach of social choice. In the past the ranking of skaters was similarly produced by aggregating multiple *ordinal* rankings given by multiple judges. The particular method has since been replaced, and it is argued that this is its ordinal nature was recognised to be unsatisfactory. Balinski and Laraki thus go in the opposite direction to us: they use experience from the Olympics and apply it to social choice theory.

athletes are unlikely to perform exactly the same—it is not less realistic than the traditional Gibbard-Satterthwaite assumption of common knowledge of all preferences of all agents. It should also be noted that in practice there is a small domain of three disciplines and up to twenty athletes, for such small domains assumptions of common knowledge are intuitively more reasonable.

Our interpretation does have a slightly different focus from that of traditional social choice: it stresses the importance of having a minimal *rank*; an athlete is only concerned with the number of athletes ranked *strictly higher* than her in the output total preorder. Another difference is the importance of the decisiveness of the ranking method; how often ties are output at any ranking level. Authors often sidestep the issue of ties in order to obtain their main results, by supposing that there is an external linear order tiebreaker or by restricting the output to linear orders [33, p. 33], but this is obviously unsatisfactory for our purposes because it is the issue of ties itself that we are interested in. Alternative approaches such as using a randomised mechanism to break ties or dealing directly with set-valued outcomes [2] are similarly unsatisfactory. Of course, decisiveness is also required for elections, however once again the size of the domain plays a role: even for a small election involving a hundred voters most rules will distinguish the candidates, but this may no longer be the case for a "population" of three disciplines.

Because preventing manipulation is not equivalent to independence of irrelevant alternatives, Theorem 3.4 is a non-trivial adaptation of Arrow's famous impossibility. For completeness, we note that the impossibility is tight for the four conditions. *Dictatorships*, where the ranking of a single discipline are copied, violate only non-determination. *Constant functions* violate only the clear winner condition, except the function that always ranks every athlete first. (Constant functions also violate neutrality, but this is not included in the impossibility.) Our method of *iterative-first-place-elimination* only allows manipulation without sacrifice. Finally, we sketch an upside-down variant of instant runoff voting that only allows spoiler manipulation: at stage t, remove the athlete who is ranked last in discipline t modulo n, and rank this athlete below the other athletes remaining in the profile.

Our positive result is interesting in part because of its unsuitability: it would certainly produce too many ties to be useful in practice. Completely prohibiting ties recreates the impossibility. It would be interesting to determine if there is a satisfactory middle ground, but there is an intriguing gap in (at least our knowledge of) the literature—we do not know of any suitable definitions of decisiveness that could be brought to task here.

An originating ideas of *computational* social choice is that, even if manipulation is possible in theory, it might be computationally hard to determine a strategy for manipulation, thus manipulation is unfeasible in practice [3]. Because calculating the outcome of inverse-Borda-Nash is polynomial, if we fix the number of disciplines, the complexity of finding a manipulation is polynomial in the number of athletes: one need only check the result for the (less than) 2^n profiles where the manipulator does just worse than her teammate for each possible subset of the disciplines. In particular it is easy to determine whether manipulation in possible for three disciplines. However, note that it is not clear that determining our types of manipulation is easy for a fixed number of athletes, as is known to be the case for traditional definitions of manipulation [7]. Allowing both the number of athletes and disciplines to vary, we have a hardness result for manipulation without sacrifice, but we do not know if the same applies to spoiler manipulation.

An interesting extension of our model would be to apply a *sequential protocol approach*, where one considers partial revelation of the profile in a sequential manner. This is precisely how the Olympic sports climbing event will unfold, though in practice there will be measures put in place to isolate the athletes from the partial results during the progress of competition for a single discipline. Of course, a sequential extension can also be applied to other competition formats. There is already a literature of related results concerning necessary and possible winners (stemming from [17]) to base such study upon. Another avenue for further study would be to introduce the issue of bribery [10]; in particular the notion of swap bribery [9] seems applicable. This would extend the relevance of any results beyond the case of altruistic teammates.

One might postulate a variant of Murphy's law for sport competitions: "if it *can* be manipulated, then at some point it *will* be manipulated". Although this law is almost certainly too strong to be true, we can observe that cheating *does* occurs in the Olympics, including in the form of deliberate bad performances [16, 28]. In another arena, Formula 1, altruistic manipulation (more precisely, spoiler manipulation) has be observed to occur [12].[14] So will there be a wave of manipulation at Tokyo 2020? Probably not. Athletes will be held in isolation during the competition of a single discipline, which in theory limits manipulation according to a sequential protocol approach. What about using qualification results as predictions? We have argued that this is no less realistic than the assumptions made for the Gibbard-Satterthwaite result, but this still does not mean that the prediction will be reliable enough in practice. Lead-climbing routes and boulder problems are not standardised: some problems are easier for tall athletes, while others are easier for short athletes; the qualifying round may favour one type of athlete and the final round another. Also, an athlete may rise to the challenge and perform better in the final round than in the qualifying round, or vice-versa crack under the pressure. The effects of most manipulations will probably be too uncertain to make them worthwhile.

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[14] This paper has been considerably improved thanks to comments and discussions that arose during its revision and presentation at COMSOC 2018. We in particular want to address a remark that was made concerning Formula 1: that the type of manipulation that we consider is less "important" than the phenomenon where a racer blocks the passage of cars of rival teams in order to preserve an advantage for a teammate. In our opinion this exemplifies the distinction between *manipulation* and *strategic behaviour*. In Formula 1 a single "blocking move" is sanctioned [4]. Allowing blocking may be considered desirable because it adds a strategic level for the competitors and improves the spectacle for viewers; one might consider blocking to be strategic behaviour. On the other hand, the type of behaviour we consider—which although rare, also seems to occur in Formula 1 [12]—is *clearly undesirable*—the reviewer used the term "scandalous"—and merits its designation as manipulation. Now, strategic behaviour between teammates plays an important role in many seemingly individualistic competitions, e.g. in various cycling events teammates draft behind each other. However, such strategic behaviours are typically domain specific and thus difficult to consider in the general manner that we treat manipulation; they would arguably be better investigated under the heading of operations research.

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