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Agenda Separability in Judgment Aggregation

Jérôme Lang and **Marija Slavkovik** and **Srdjan Vesic**
LAMSADE, CNRS University of Paris-Dauphine, France, lang@lamsade.dauphine.fr
University of Bergen, Norway, marija.slavkovik@uib.no
CRIL, CNRS – Univ. Artois, France, vesic@cril.fr

Abstract

One of the better studied properties for operators in judgment aggregation is *independence*, which essentially dictates that the collective judgment on one issue should not depend on the individual judgments given on some other issue(s) in the same agenda. Independence, although considered a desirable property, is too strong, because together with mild additional conditions it implies dictatorship. We propose here a weakening of independence, named *agenda separability*: a judgment aggregation rule satisfies it if, whenever the agenda is composed of several independent sub-agendas, the resulting collective judgment sets can be computed separately for each sub-agenda and then put together. We show that this property is discriminant, in the sense that among judgment aggregation rules so far studied in the literature, some satisfy it and some do not. We briefly discuss the implications of agenda separability on the computation of judgment aggregation rules.

1 Introduction

Judgment aggregation consists in finding collective judgments that are representative of a collection of individual judgments on some logically interrelated issues. Judgment aggregation problems originate in political theory and public choice, however they also occur in various areas of artificial intelligence, as a consequence of the increased distributivity of computing systems and social networks, together with the rise of artificial agency. Judgment aggregation generalises voting and preference aggregation (Dietrich and List 2007a; Lang and Slavkovik 2013), and has links with belief revision (Everaere, Konieczny, and Marquis 2015; Pigozzi 2006) as well as abstract argumentation (Caminada and Pigozzi 2011; Awas et al. 2015; Booth 2015; Booth, Awad, and Rahwan 2014). For an overview of applications of judgment aggregation in artificial intelligence see for instance the work by Grossi and Pigozzi (2014) or Endriss (2015).

The main focus of research in judgment aggregation is the development and analysis of judgement aggregation operators. Numerous impossibility results – see the survey by List and Puppe (2009) for an overview – have dashed the hope of finding a universally applicable operator. Consequently, the suitability of an operator for a given judgment aggregation problem has to be identified with respect to the desirable properties that the aggregation process should satisfy.

One of the better studied properties for operators in judgment aggregation is the *independence* property, which essentially dictates that the collective judgment on any one issue in the agenda should not depend on the individual judgments given on any of the other issues in the same agenda. Independence is a desirable property because, among other reasons, it is a necessary condition for strategyproofness (Dietrich and List 2007c), and it leads to rules that are both conceptually simple and easy to compute. However, independence is too strong; in particular, together with mild additional conditions, it implies dictatorship (Dietrich and List 2007a).

We propose a natural weakening of independence, named *agenda separability*. A judgment aggregation rule satisfies it if, whenever the agenda is composed of several independent sub-agendas (with an extreme form of independence being when the sub-agendas are syntactically unrelated to each other), the resulting collective judgment sets can be computed separately for each sub-agenda and then put together. Resorting to syntactically independent sub-languages is reminiscent of Parikh's language splitting (Parikh 1999), where decomposing a logical theory into several subtheories over disjoint sub-languages simplifies many tasks in knowledge representation, such as belief change (Peppas, Chopra, and Foo 2004) or inconsistency handling (Chopra and Parikh 1999).

The agenda separability property is very intuitive and motivations for it can be easily found. For instance, in computational linguistics, we may want to aggregate annotations from several agents about parts of texts (Kruger et al. 2014); then, finding collective annotations about parts of two unrelated texts can (and should) be performed independently. When a rule satisfies agenda separability, it also becomes computationally simpler when applied to decomposable agendas, because the rule can be applied independently to every subagenda of the decomposition. Agenda separability also offers a weak form of strategyproofness: no agent is able to influence the outcome on some issue from one subagenda of the partition by strategically reporting judgments about another subagenda.

Of course, a weakening of independence is meaningful only if there are rules that satisfy it. Not only we show that this is the case, but we also show that agenda separability

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is discriminant, in the sense that among the known judgment aggregation rules, some satisfy it and some do not. This leads us to see agenda independence as a possible means of choosing a judgment aggregation rule against another.

The paper is structured as follows. Section 2 introduces the background. Section 3 discusses the independence property. In Sections 4 and 5 we define two notions of agenda separability, and we identify some rules that satisfy them and some that do not. Section 6 contains a summary and discussion.

2 Preliminaries

Let $\mathcal{L}$ be a set of well-formed propositional logical formulas, including $\top$ (tautology) and $\bot$ (contradiction). An *issue* is a pair of formulas $\varphi, \neg\varphi$ where $\varphi \in \mathcal{L}$ and φ is neither a tautology nor a contradiction. An *agenda* $\mathcal{A}$ is a finite set of issues and has the form $\mathcal{A} = \{\varphi_1, \neg\varphi_1, \ldots, \varphi_m, \neg\varphi_m\}$. The *preagenda* $[\mathcal{A}]$ associated with $\mathcal{A}$ is $[\mathcal{A}] = \{\varphi_1, \ldots, \varphi_m\}$. A *sub-agenda* is a subset of issues from $\mathcal{A}$. A *sub-preagenda* is a subset of $[\mathcal{A}]$. An agenda usually comes with an *integrity constraint* Γ, which is a consistent formula whose role is to filter out inadmissible judgment sets. $(\mathcal{A}, \Gamma)$ is called a *constrained agenda*. As a classical example, given a set of candidates $C = \{x_1, \ldots, x_m\}$, the *preference agenda* over C (Dietrich and List 2007a) is $\mathcal{A}_C = \{x_i P x_j | 1 \leq i < j \leq m\}$, and the associated integrity constraint is $\Gamma_C = \bigwedge_{i,j,k} (x_i P x_j \wedge x_j P x_k \rightarrow x_i P x_k)$. When Γ is not specified, by default it is equal to $\top$.

A *judgment* on $\varphi \in [\mathcal{A}]$ is one of φ or $\neg\varphi$. A *judgment set* J is a subset of $\mathcal{A}$. J is *complete* iff for each $\varphi \in [\mathcal{A}]$, either $\varphi \in J$ or $\neg\varphi \in J$. A judgment set J (and in general, a set of propositional formulas) is Γ-*consistent* if and only if $J \cup \{\Gamma\} \not\models \bot$. Let $\mathcal{J}_{\mathcal{A},\Gamma}$ be the set of all *complete and consistent* judgment sets. To lighten the notations, we will generally say that a judgment set is *consistent* instead of Γ-consistent, and note $\mathcal{J}_{\mathcal{A}}$ instead of $\mathcal{J}_{\mathcal{A},\Gamma}$.

A *profile* $P = \langle J_1, \ldots, J_n \rangle \in \mathcal{J}_{\mathcal{A}}^n$ is a collection of complete and consistent individual judgment sets. We further define $N(P, \varphi) = |\{i \mid \varphi \in J_i\}|$ to be the number of all agents in P whose judgment set includes φ. The order $\succsim_P$ is the weak order over $\mathcal{A}$ defined by $\varphi \succsim_P \phi$ if and only if $N(P, \varphi) \geq N(P, \phi)$.

The *restriction* of $P = \langle J_1, \ldots, J_n \rangle$ over a sub-agenda $\mathcal{A}_1$ of $\mathcal{A}$ is defined as $P_{\downarrow \mathcal{A}_1} = \langle J_1 \cap \mathcal{A}_1, \ldots, J_n \cap \mathcal{A}_1 \rangle$.

Every consistent subset of the agenda $S \subset \mathcal{A}$ can be extended in order to obtain a complete judgment set (there might be several such extensions). For a set S of subsets of agenda , we define $ext(S) = \{J \in \mathcal{J}_{\mathcal{A}} \mid$ there exists $J' \in S$ such that $J' \subseteq J\}$.

A *judgment aggregation rule*, for n agents, is a function R that maps any constrained agenda $(\mathcal{A}, \Gamma)$ and any profile $P \in \mathcal{J}_{\mathcal{A},\Gamma}^n$ to a non-empty set of complete consistent judgment sets over $\mathcal{A}$.[1] If R always outputs a singleton then it is called a *resolute* rule. The majoritarian judgment set associated with profile P contains all elements of the agenda that are supported by a majority of judgment sets in P: $m(P) = \{\varphi \in \mathcal{A} \mid N(P, \varphi) > \frac{n}{2}\}$. A profile P is *majority-consistent* iff $m(P)$ is consistent.

Let $S \subseteq \mathcal{L}$. We define $\mathtt{Atoms}(S)$ as the set of all propositional variables appearing in S. For example, $\mathtt{Atoms}(\{p, q \wedge r, \neg s \rightarrow \neg\neg p\}) = \{p, q, r, s\}$.

Given a set of formulas S and a formula Γ, $S' \subseteq S$ is Γ-consistent if $S' \cup \{\Gamma\}$ is consistent, S' is a maximal Γ-consistent subset of S, if S' is Γ-consistent and there is no $S'' \supset S'$, $S'' \subseteq S$ that is Γ-consistent. . We use $max(S, \subseteq)$ to denote the maximal consistent subsets of S. The set $S' \subseteq S$ is a maxcard Γ-consistent subset of S if S' is Γ-consistent and there exists no Γ-consistent set $S'' \subseteq S$ such that $|S'| < |S''|$. We use $max(S, |.|)$ to denote the maxcard consistent subsets of S.

We now give the definitions of seven judgment aggregation rules. They come from various places in the literature, where they sometimes appear with different names (Lang et al. 2011; Lang and Slavkovik 2013; Nehring and Pivato 2013; Nehring, Pivato, and Puppe 2014; Miller and Osherson 2009; Everaere, Konieczny, and Marquis 2014).

Throughout the subsection, $P = \langle J_1, \ldots, J_n \rangle$ is a profile. For two consistent and complete judgment sets J, J' we denote their Hamming distance as $d_H(J, J') = |J \setminus J'|$.

MC, MCC. The maximum Condorcet rule (MC) and the maxcard Condorcet rule (MCC) rules are defined as follows. For every agenda $\mathcal{A}$, for every profile $P \in \mathcal{J}_{\mathcal{A}}^n$, $\text{MC}(P) = \{ext(S) \mid S \in max(m(P), \subseteq)\}$ and $\text{MCC}(P) = \{ext(S) \mid S \in max(m(P), |.|)\}$.

RA. For $\mathcal{A} = \{\psi_1, \ldots, \psi_{2m}\}$ and a permutation σ of $\{1, \ldots, 2m\}$, let $>_\sigma$ be the linear order on $\mathcal{A}$ defined by $\psi_{\sigma(1)} >_\sigma \ldots >_\sigma \psi_{\sigma(2m)}$. We say that $>_\sigma$ is compatible with $\succsim_P$ if $\psi_{\sigma(1)} \succsim_P \ldots \succsim_P \psi_{\sigma(2m)}$. The ranked agenda rule RA is defined as $J \in \text{RA}(P)$ if and only if there exists a permutation σ such that $>_\sigma$ is compatible with $\succsim_P$ and such that $J = J_\sigma$ is obtained by the following procedure:

- $S := \emptyset$;
- for $j = 1, \ldots, 2m$ do
-     if $S \cup \{\psi_{\sigma(j)}\}$ is consistent, let $S := S \cup \{\psi_{\sigma(j)}\}$;
- $J_\sigma := S$.

$$\mathbf{R}^{d_H,\mathbf{MAX}}(P) = \underset{J \in \mathcal{J}_{\mathcal{A}}}{\text{argmin}} \max_{i=1}^{n} d_H(J_i, J).$$

$\mathbf{R}_S$. A *scoring function* (Dietrich 2014) is defined as $s : \mathcal{J}_{\mathcal{A}} \times \mathcal{A} \rightarrow \mathbb{R}^+$. Given a scoring function s, the judgment aggregation rule R_s is defined as $\text{R}_S(P) = \underset{J \in \mathcal{J}_{\mathcal{A}}}{\text{argmax}} \sum_{J_i \in P} \sum_{\varphi \in J} s(J_i, \varphi)$. If we choose the *reversal* scoring function $s_{rev}(J_i, \varphi)$ as the minimal number of judgment reversals needed in J_i in order to reject φ then we get the *reversal scoring rule* R_{rev} (Dietrich 2014). If we choose the scoring function s defined by $s_{med}(J_i, \varphi) = 1$ if $\varphi \in J_i$ and 0 if $\varphi \notin J_i$ then R_s is exactly the *median* rule, i.e. $R_s \equiv \text{MED}$.

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[1] The reason why the (constrained) agenda is an argument of rules is that the notions we study need a rule to be applied to a variable agenda. We omit writing $\mathcal{A}, \Gamma$ as an argument of R when defining R to improve the readability of the text.

MED(P) =

$$\operatorname*{argmax}_{J\in\mathcal{J}_{\mathcal{A}}}\sum_{\varphi\in J}N(P,\varphi) = \operatorname*{argmin}_{J\in\mathcal{J}_{\mathcal{A}}}\sum_{J_i\in P}d_H(J_i,J).$$

FULL$_H$**.** Given profiles $P = \langle J_1,\ldots,J_n\rangle$ and $Q = \langle J'_1,\ldots,J'_n\rangle$ in $\mathcal{J}^n_{\mathcal{A}}$, let $D_H(P,Q) = \sum_{i=1}^{n} d_H(J_i,J'_i)$.
$\text{FULL}_H(P) = \{ext(m(Q)) \mid Q \in \operatorname*{argmin}_{Q'\in\mathcal{J}^n_{\mathcal{A}}} D_H(P,Q')\}$.

The rules defined here are irresolute, but similarly as in voting theory, can be made resolute by composing them with a tie-breaking mechanism. A simple way of defining a tie-breaking mechanism θ is via a priority relation $>_\theta$ over consistent and complete judgment sets. Given an irresolute rule R and a tie-breaking mechanism θ, the resolute rule R_θ is the rule that, given P, returns the maximal (with respect to $>_\theta$) element of $R(P)$.

3 Relaxing Independence

A judgment aggregation rule F satisfies *independence of irrelevant alternatives* (IIA) if for every two profiles $P, P' \in \mathcal{J}^n_{\mathcal{A}}$, and every $\varphi \in \mathcal{A}$, if $P_{\downarrow\{\varphi,\neg\varphi\}} = P'_{\downarrow\{\varphi,\neg\varphi\}}$, then $\varphi \in F(P)$ iff $\varphi \in F(P')$. Independence is a very strong property: together with three seemingly innocuous properties, namely universal domain (F is defined for every profile), unanimity principle, and collective rationality (F outputs complete and consistent judgment sets), it implies dictatorship (Dietrich and List 2007a).

Now, while it is natural to expect that the individual judgments on logically related issues will influence the choice of collective judgments for those issues, it is also natural to expect that individual judgments over logically unrelated issues will have no impact on them. To illustrate this point, we give an example from a collective decision making problem that occurs in crowdcomputing.

There are a lot of tasks that are rather simple for a human to do, but fairly complicated for a computer, such as labelling images, choosing the best out of several images, identifying music segments etc. These types of tasks are called human intelligence tasks (HITs). Considering the task of cataloguing pictures by location, that is outsourced as HITs to an unspecified, but finite, group of people. The people undertaking these tasks should label each photo in a series and also indicate reasons for their labelling. For example: the photo is of Paris (p) if the Eiffel tower can be seen on it (e) or the Triumphal arc can be seen on it (t); the photo is of Rome (r) if the Colosseum can be seen on it (c) or the Spanish Steps can be seen on it (s). The commissioner of the HITs will aggregate the individual labelings and assign the labels that are collectively supported. The problem of finding which labelings are collectively supported can be solved as a judgment aggregation problem; see the work by Endriss and Fernández (2013) for a similar view of crowdsourcing as a judgment aggregation problem. Assume, for simplicity, that we have three labellers (or agents) and two pictures. Furthermore, the commissioner is only interested in whether the first photo is of Paris and whether the second one is of Rome. The problem for the first photo is represented with the agenda $[\mathcal{A}_1] = \{p, e, t, e \vee t \to p\}$, while the problem for the second photo is represented with the agenda $[\mathcal{A}_2] = \{r, c, s, c \vee s \to r\}$. Observe that $\texttt{Atoms}(\mathcal{A}_1) \cap \texttt{Atoms}(\mathcal{A}_2) = \emptyset$. The agents get the pictures at the same time. Clearly, whether the first picture is of Paris or not has nothing to do with whether the second picture is of Rome or not, consequently we would expect that the collective judgments regarding issues in $\mathcal{A}_1$ depend only on the judgments given for these issues, but not on the individual judgments given for issues in $\mathcal{A}_2$.

In the next section we relax independence along this principle, defining a new property called *agenda separability*.

4 Agenda Separability

Following the idea that only judgments on logically related issues should influence the collective judgment on each issue, we define agenda separability as the property requiring that when two agendas can be split into sub-agendas that are independent from each other, the output judgment sets can be obtained by first applying the rule on each sub-agenda separately and then taking the pairwise unions of judgment sets from the two resulting sets.

A partition $\{\mathcal{A}_1, \mathcal{A}_2\}$ of $\mathcal{A}$ is an *independent partition of* $\mathcal{A}$ if for every $J^1 \in \mathcal{J}_{\mathcal{A}_1}$ and $J^2 \in \mathcal{J}_{\mathcal{A}_2}$, $J^1 \cup J^2$ is Γ-consistent.[2]

Definition 1 (Agenda separability) *We say that rule R satisfies* agenda separability (AS) *if for every agenda $\mathcal{A}$, every independent partition $\{\mathcal{A}_1, \mathcal{A}_2\}$ of $\mathcal{A}$, and all profiles $P \in \mathcal{J}^n_{\mathcal{A}}$, we have*

$$R(P) = \{J^1 \cup J^2 \mid J^1 \in R(P_{\downarrow\mathcal{A}_1}) \text{ and } J^2 \in R(P_{\downarrow\mathcal{A}_2})\}.$$

If R is a resolute rule, then the last line of the definition simplifies into $R(P) = R(P_{\downarrow\mathcal{A}_1}) \cup R(P_{\downarrow\mathcal{A}_2})$.

Also, by associativity of $\cup$, this notion generalises to agendas that can be partitioned into a collection $\{\mathcal{A}_1, \ldots, \mathcal{A}_k\}$ such that for every $J_1 \in \mathcal{J}_{\mathcal{A}_1}, \ldots, J_k \in \mathcal{J}_{\mathcal{A}_k}$, $J_1 \cup \ldots \cup J_k$ is consistent. In that case,

$$R(P) = \left\{ \bigcup_{i=1}^{k} J^i \;\middle|\; J^1 \in R(P_{\downarrow\mathcal{A}_1}), \ldots, J^k \in R(P_{\downarrow\mathcal{A}_k}) \right\}.$$

IIA is defined for resolute rules only. We show that agenda separability restricted to resolute rules is a weakening of IIA.

Proposition 1 *Any resolute judgment aggregation rule that satisfies IIA is agenda separable.*

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[2] A stronger notion of independence, which makes sense only when $\Gamma = \top$, is *syntactical agenda independence*: a partition $\{\mathcal{A}_1, \mathcal{A}_2\}$ of $\mathcal{A}$ is *syntactically independent* if $\texttt{Atoms}(\mathcal{A}_1) \cap \texttt{Atoms}(\mathcal{A}_2) = \emptyset$. Clearly, syntactical agenda independence implies agenda independence, because $\texttt{Atoms}(\mathcal{A}_1) \cap \texttt{Atoms}(\mathcal{A}_2) = \emptyset$ implies that $\mathcal{A}_1$ and $\mathcal{A}_2$ are independent. Note that the implication is strict: for example, let $\mathcal{A} = \{x, \neg x, x \leftrightarrow y, \neg(x \leftrightarrow y)\} = \mathcal{A}_1 \cup \mathcal{A}_2$, $\Gamma = \top$, $\mathcal{A}_1 = \{x, \neg x\}$ and $\mathcal{A}_2 = \{x \leftrightarrow y, \neg(x \leftrightarrow y)\}$. $\{\mathcal{A}_1, \mathcal{A}_2\}$ is an independent partition of $\mathcal{A}$ although $\texttt{Atoms}(\mathcal{A}_1) \cap \texttt{Atoms}(\mathcal{A}_2) \neq \emptyset$.

Proof. If a resolute rule R satisfies IIA, we can write $R(P) = \bigcup_{i=1}^{m} F_i(P_{\downarrow\{\varphi_i,\neg\varphi_i\}})$ where $[\mathcal{A}] = \{\varphi_1,\ldots\varphi_m\}$ and $F_1,\ldots,F_m$ are resolute rules. Let $\{\mathcal{A}_1,\mathcal{A}_2\}$ be an independent partition of $\mathcal{A}$. Without loss of generality, assume $[\mathcal{A}_1] = \{\varphi_1,\ldots,\varphi_k\}$ and $[\mathcal{A}_2] = \{\varphi_{k+1},\ldots,\varphi_m\}$. We have $R(P) = \bigcup_{i=1}^{k} F_i(P_{\downarrow\{\varphi_i,\neg\varphi_i\}}) \cup \bigcup_{i=k+1}^{m} F_i(P_{\downarrow\{\varphi_i,\neg\varphi_i\}}) = R(P_{\downarrow\mathcal{A}_1}) \cup R(P_{\downarrow\mathcal{A}_2})$. □

We shall see that the reverse implication does not hold.

Definition 2 *The scoring function s is* separable *if for every $\mathcal{A}$ and every independent partition $\{\mathcal{A}_1,\mathcal{A}_2\}$ of $\mathcal{A}$, for $i \in \{1,2\}$, and every $J \in \mathcal{J}_{\mathcal{A}}$ and $\varphi \in \mathcal{A}_i$, we have $s(J,\varphi) = s(J \cap \mathcal{A}_i,\varphi)$.*

We omit the easy proofs of the next two results.

Proposition 2 *If s is a separable scoring function, then* R_S *is agenda separable.*

Corollary 1 MED *and* R_{rev} *are agenda separable.*

Proposition 3 MC, MCC, RA, *and* FULL$_H$ *are agenda separable.* $\mathrm{R}^{d_H,\mathrm{MAX}}$ *is not agenda separable.*

Proof. For MC and RA, this will be a consequence of a stronger result proven in Section 5, therefore we give a proof only for MCC and FULL$_H$. Let $\{\mathcal{A}_1,\mathcal{A}_2\}$ be an independent partition of $\mathcal{A}$.

MCC. Denote $B_1 = m(P_1)$, $B_2 = m(P_2)$ and $B = m(P)$. Let $\Pi_{P_1,P_2} = \{J^1 \cup J^2 \mid J^1 \in \mathrm{MCC}(P_1) \text{ and } J^2 \in \mathrm{MCC}(P_2)\}$. We first show that $\mathrm{MCC}(P) \subseteq \Pi_{P_1,P_2}$. Let $J_* \in \mathrm{MCC}(P)$; thus $J_* \in ext(max(B,|.|))$. Let $J_*^1 = J_* \cap \mathcal{A}_1$ and $J_*^2 = J_* \cap \mathcal{A}_2$. J_*^1 and J_*^2 are consistent, because J_* is consistent. Assume $J_*^1 \notin ext(max(B_1,|.|))$. Since J_*^1 is consistent, there exists $J_{**}^1 \in ext(max(B_1,|.|))$ such that $|J_{**}^1| > |J_*^1|$. Let $J_{**} = J_{**}^1 \cup J_*^2$. Because $\{\mathcal{A}_1,\mathcal{A}_2\}$ is an independent partition of $\mathcal{A}$, the consistency of J_{**}^1 and of J_*^2 implies the consistency of J_{**}. But then $|J_{**}| > |J_*|$, which contradicts $J_* \in ext(max(B,|.|))$. Therefore, $J_*^1 \in ext(max(B_1,|.|))$. Similarly, $J_*^2 \in ext(max(B_2,|.|))$. Thus, $J_* \in \Pi_{P_1,P_2}$.

Now we show that $\Pi_{P_1,P_2} \subseteq \mathrm{MCC}(P)$. Let $J^1 \in \mathrm{MCC}(P_1)$ and $J^2 \in \mathrm{MCC}(P_2)$, that is, $J^1 \in ext(max(B_1,|.|))$ and $J^2 \in ext(max(B_2,|.|))$. Let us show that $J = J^1 \cup J^2 \in ext(max(B,|.|))$. Because $\{\mathcal{A}_1,\mathcal{A}_2\}$ is an independent partition of $\mathcal{A}$, J is consistent. Suppose that there exists $J_* \in ext(max(B,|.|))$ such that $|J_*| > |J|$. Let $J_*^1 = J_* \cap B_1$ and $J_*^2 = J_* \cap B_2$. Both J_*^1 and J_*^2 are consistent, and $|J_*| > |J|$ implies that $|J_{**}^1| > |J_*^1|$ or $|J_{**}^2| > |J_*^2|$, which contradicts $J_*^1 \notin ext(max(B_1,|.|))$ and $J_*^2 \notin ext(max(B_2,|.|))$. Thus, it must be that $J \in ext(max(B,|.|))$ and, consequently, $J \in \mathrm{MCC}(P)$.

FULL$_H$. Let $X \subseteq \mathcal{J}_{\mathcal{A}}^n$ be the set of all profiles Q such that $ext(m(Q)) \subseteq \mathcal{J}_{\mathcal{A}}$, $\texttt{CMC}(P) = \mathrm{argmin}_{Q \in X} D_H(P,Q)$, and $\texttt{UA}_{12} = \{J^1 \cup J^2 \mid J^1 \in \mathrm{FULL}_H(P_{\downarrow\mathcal{A}_1}) \text{ and } \mathrm{FULL}_H(P_{\downarrow\mathcal{A}_2})\}$.

We first show that $\mathrm{FULL}_H(P) \subseteq \texttt{UA}_{12}$. Let $J_\circ \in \mathrm{FULL}_H(P)$. Let $J_\circ^1 = J_\circ \cap \mathcal{A}_1$ and $J_\circ^2 = J_\circ \cap \mathcal{A}_2$. Since $J_\circ \in \mathrm{FULL}_H(P)$ then there exists $Q \in \texttt{CMC}(P)$ such that $J_\circ \in ext(m(Q))$. Let us show that $Q_{\downarrow\mathcal{A}_1} \in \texttt{CMC}(P_{\downarrow\mathcal{A}_1})$. Suppose that $Q_{\downarrow\mathcal{A}_1} \notin \texttt{CMC}(P_{\downarrow\mathcal{A}_1})$. Then, there exists a majority-consistent $Q_1^* \in \mathcal{J}_{\mathcal{A}_1}^n$, $Q_1^* = \langle I_1^*,\ldots,I_n^*\rangle$ such that $D_H(Q_1^*,P_{\downarrow\mathcal{A}_1}) < D_H(Q_{\downarrow\mathcal{A}_1},P_{\downarrow\mathcal{A}_1})$. Let $Q = \langle I_1,\ldots,I_n\rangle$. Let $Q_{\downarrow\mathcal{A}_1} = \langle I_1^1,\ldots,I_n^1\rangle$ and $Q_{\downarrow\mathcal{A}_2} = \langle I_1^2,\ldots,I_n^2\rangle$. Define $Q^* = \langle I_1^* \cup I_1^2,\ldots,I_n^* \cup I_n^2\rangle$. Because $\{\mathcal{A}_1,\mathcal{A}_2\}$ is an independent partition of $\mathcal{A}$, Q^* is a majority-consistent profile. Note also that $D_H(Q^*,P) < D_H(Q,P)$. Contradiction. Thus, $Q_{\downarrow\mathcal{A}_1} \in \texttt{CMC}(P_{\downarrow\mathcal{A}_1})$, and for the same reasons, $Q_{\downarrow\mathcal{A}_2} \in \texttt{CMC}(P_{\downarrow\mathcal{A}_2})$. Therefore, $J_\circ^1 \in \mathrm{FULL}_H(P_{\downarrow\mathcal{A}_1})$, $J_\circ^2 \in \mathrm{FULL}_H(P_{\downarrow\mathcal{A}_2})$, and $\mathrm{FULL}_H \subseteq \texttt{UA}_{12}$.

Agents	p	q	$p \wedge q$	t
J_1	+	+	+	+
J_2	+	-	-	+
J_3	-	+	-	-
	P_1			P_2

Figure 1: Counter example to $\mathrm{R}^{d_H,\mathrm{MAX}}$ being agenda separable.

We now show that $\texttt{UA}_{12} \subseteq \mathrm{FULL}_H$. Let $J_\circ^1 \in \mathrm{FULL}_H(P_{\downarrow\mathcal{A}_1})$ and $J_\circ^2 \in \mathrm{FULL}_H(P_{\downarrow\mathcal{A}_2})$. Thus, there exist profiles $Q_1 \in \texttt{CMC}(P_{\downarrow\mathcal{A}_1})$ and $Q_2 \in \texttt{CMC}(P_{\downarrow\mathcal{A}_2})$ such that $J_\circ^1 \in ext(m(Q_1))$ and $J_\circ^2 \in ext(m(Q_2))$. Let $Q_1 = \langle Q_1^1,\ldots,Q_n^1\rangle$, $Q_2 = \langle Q_1^2,\ldots,Q_n^2\rangle$, and $Q = \langle Q_1^1 \cup Q_1^2,\ldots,Q_n^1 \cup Q_n^2\rangle$. Because $\{\mathcal{A}_1,\mathcal{A}_2\}$ is an independent partition of $\mathcal{A}$, Q is majority-consistent.

Let us show that $Q \in \texttt{CMC}(P)$. Assume that $Q \notin \texttt{CMC}(P)$. Then there exists $Q^* \in \texttt{CMC}(P)$ s.t. $D_H(Q^*,P) < D_H(Q,P)$. Observe that $D_H(Q^*{}_{\downarrow\mathcal{A}_1},P) + D_H(Q^*{}_{\downarrow\mathcal{A}_2},P) < D_H(Q_{\downarrow\mathcal{A}_1},P) + D_H(Q_{\downarrow\mathcal{A}_2},P)$. This means that $D_H(Q^*{}_{\downarrow\mathcal{A}_1},P) < D_H(Q_{\downarrow\mathcal{A}_1},P)$ or $D_H(Q^*{}_{\downarrow\mathcal{A}_2},P) < D_H(Q_{\downarrow\mathcal{A}_2},P)$. Recall that $Q_{\downarrow\mathcal{A}_1} = Q_1$ and $Q_{\downarrow\mathcal{A}_2} = Q_2$. Thus, $D_H(Q^*{}_{\downarrow\mathcal{A}_1},P) < D_H(Q_1,P)$ or $D_H(Q^*{}_{\downarrow\mathcal{A}_2},P) < D_H(Q_2,P)$, which, together with the fact that $Q^*{}_{\downarrow\mathcal{A}_1}$ and $Q^*{}_{\downarrow\mathcal{A}_2}$ are majority-consistent, contradicts $Q_1 \in \texttt{CMC}(P_{\downarrow\mathcal{A}_1})$ and $Q_2 \in \texttt{CMC}(P_{\downarrow\mathcal{A}_2})$. Thus, $Q \in \texttt{CMC}(P)$. Note that $J_\circ^1 \cup J_\circ^2 \in m(Q)$. This implies that $J_\circ^1 \cup J_\circ^2 \in \mathrm{FULL}_H(P)$.

$\mathbf{R}^{d_H,\mathrm{MAX}}$. We provide a counter example.Let $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$ with $[\mathcal{A}_1] = \{p,q,p \wedge q\}$ and $[\mathcal{A}_2] = \{t\}$. Consider the profile P from Figure 1 with $P_1 = P_{\downarrow\mathcal{A}_1}$ and $P_2 = P_{\downarrow\mathcal{A}_2}$. We obtain $\mathrm{R}^{d_H,\mathrm{MAX}}(P) = \{\{\neg p, q, \neg(p \wedge q), t\}\}$. However $\mathrm{R}^{d_H,\mathrm{MAX}}(P_2) = \{\{t\}, \{\neg t\}\}$ and $\mathrm{R}^{d_H,\mathrm{MAX}}(P_1) = \{\{\neg p, q, \neg(p \wedge q)\}, \{p, q, (p \wedge q)\}, \{p, \neg q, \neg(p \wedge q)\}\}$. □

The fact that a rule satisfies agenda separability does not imply that a resolute rule obtained by composing it with a tie-breaking mechanism satisfies agenda separability as well. For instance, if tie-breaking favours $\{\neg a\}$ over $\{a\}$ when $\mathcal{A} = \{a\}$, $\{\neg b\}$ over $\{b\}$ when $\mathcal{A} = \{b\}$, and $\{a,b\}$ over all other judgment sets when $\mathcal{A} = \{a,b\}$, and if P contains one judgment set $\{a,b\}$ and one judgment set $\{\neg a,\neg b\}$, then for any one of our rules, and with $\mathcal{A}_1 = \{a,\neg a\}$ and $\mathcal{A}_2 = \{b,\neg b\}$, we have $R(P_{\downarrow\mathcal{A}_1}) = \{\neg a\}$, $R(P_{\downarrow\mathcal{A}_2}) = \{\neg b\}$, and $R(P) = \{a,b\}$. However, if the tie-breaking priority relation $>_\theta$ satisfies the following decomposability property, then agenda separability of an irresolute rule implies agenda separability of its composition with θ.

A tie-breaking priority relation $>_\theta$ is *agenda separable* if for every agenda $\mathcal{A}$, for every independent partition $\{\mathcal{A}_1, \mathcal{A}_2\}$ of $\mathcal{A}$, and every $J^1_*, J^1_\circ \in \mathcal{J}_{\mathcal{A}_1}$, and $J^2_*, J^2_\circ \in \mathcal{J}_{\mathcal{A}_2}$, $J^1_* >_\theta J^1_\circ$ and $J^2_* >_\theta J^2_\circ$ imply $J^1_* \cup J^2_* >_\theta J^1_\circ \cup J^2_\circ$.

Observation 1 *If $>_\theta$ is an agenda separable tie-breaking priority relation and R is agenda separable, then R_θ is agenda separable.*

Let $>_\theta$ be an agenda separable tie-breaking priority relation, then RA$_\theta$ is agenda separable. However, since it satisfies universal domain, unanimity principle (Lang et al. 2011), and collective rationality, then it does not satisfy IIA. Hence, the implication stated in Proposition 1 is strict.

Lastly, we would like to state two observations about the properties of rules that are agenda separable.

Observation 2 *Let K be a constant and say that agenda $\mathcal{A}$ is K-decomposable if $\mathcal{A}$ can be partitioned into p syntactically unrelated agendas $\mathcal{A}_1, \ldots, \mathcal{A}_p$ such that for all i $|\texttt{Atoms}(\mathcal{A}_i)| \leq K$. If a rule satisfies agenda separability, then the collective judgment sets can be computed in time $O(2^K n)$ whenever the agenda is K-decomposable.*

In other terms, computing these rules is parameterized tractable when he parameter is the degree K of decomposability, which is a complexity gap, since winner determination for these rules is Θ^2_p-hard or even Π^2_p-hard (Lang and Slavkovik 2014; Endriss and de Haan 2015).

Moreover, agenda separability allows for a weak form of strategyproofness. Indeed, if A can be partitioned into p syntactically unrelated agendas $\mathcal{A}_1, \ldots, \mathcal{A}_p$, then no agent is able to influence the outcome on some issue in $\mathcal{A}_i$ by reporting strategic judgments about issues of $\mathcal{A}_j$ for $j \neq i$.

5 Overlapping Agenda Separability

In this section, we consider a stricter property than agenda separability. We first need the notion of independent overlapping decomposition.

Definition 3 (Independent overlapping decomposition) *Let $\mathcal{A}$ be an agenda and let $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$ (but not necessarily $\mathcal{A}_1 \cap \mathcal{A}_2 = \emptyset$). We say that $\{\mathcal{A}_1, \mathcal{A}_2\}$ is an* independent overlapping decomposition *(IOD) of $\mathcal{A}$ if and only if for every $J^1 \in \mathcal{J}_{\mathcal{A}_1}$, for every $J^2 \in \mathcal{J}_{\mathcal{A}_2}$*

$$\textit{if} \quad J^1 \cap \mathcal{A}_2 = J^2 \cap \mathcal{A}_1 \quad \textit{then} \quad J^1 \cup J^2 \in \mathcal{J}_{\mathcal{A}}.$$

Example 1 *Let $[\mathcal{A}] = \{p, \neg p \vee t, p \leftrightarrow q\}$, $[\mathcal{A}_1] = \{p, \neg p \vee t\}$ and $[\mathcal{A}_2] = \{\neg p \vee t, p \leftrightarrow q\}$. Note that $\{\mathcal{A}_1, \mathcal{A}_2\}$ is an independent overlapping decomposition of $\mathcal{A}$.*

Observation 3 *Every independent partition is an independent overlapping decomposition.*

Example 1 shows that the contrary of the previous observation does not hold. Indeed, as soon as the intersection of the two sub-agendas is non-empty, they do not form an independent partition.

There is a clear connection between independent overlapping decompositions and conditional independence in propositional logic (Darwiche 1997; Lang, Liberatore, and Marquis 2002); we do not give technical details here, but we mention that this connection gives us several characterizations as well as complexity results for finding independent overlapping decompositions.

We can now introduce the definition of overlapping agenda separability.

Definition 4 (Overlapping agenda separability) *We say that rule R satisfies* overlapping agenda separability *(OAS) if for every agenda $\mathcal{A}$ and every independent overlapping decomposition $\{\mathcal{A}_1, \mathcal{A}_2\}$ of $\mathcal{A}$, for every profile P over $\mathcal{A}$ it holds that: if for every $J^1 \in R(P_{\downarrow \mathcal{A}_1})$, for every $J^2 \in R(P_{\downarrow \mathcal{A}_2})$, we have $J^1 \cap \mathcal{A}_2 = J^2 \cap \mathcal{A}_1$ then $R(P) = \{J^1 \cup J^2 \mid J^1 \in R(P_{\downarrow \mathcal{A}_1})$ and $J^2 \in R(P_{\downarrow \mathcal{A}_2})\}$.*

Observation 4 *Overlapping agenda separability implies agenda separability.*

Proof. Let $\{\mathcal{A}_1, \mathcal{A}_2\}$ be an independent partition of $\mathcal{A}$. From Observation 3 $\{\mathcal{A}_1, \mathcal{A}_2\}$ is an IOD. Since $\mathcal{A}_1 \cap \mathcal{A}_2 = \emptyset$, condition $J^1 \cap \mathcal{A}_2 = J^2 \cap \mathcal{A}_1$ is satisfied for every J^1, J^2. Thus, $R(P) = \{J^1 \cup J^2 \mid J^1 \in R(P_{\downarrow \mathcal{A}_1})$ and $J^2 \in R(P_{\downarrow \mathcal{A}_2})\}$. □

Proposition 4 MC *and* RA *satisfy* OAS.

Proof.
MC. Suppose that for every $J^1 \in \mathrm{MC}(P_1)$, for every $J^2 \in \mathrm{MC}(P_2)$, $J^1 \cap \mathcal{A}_2 = J^2 \cap \mathcal{A}_1$. Let $\Pi_{P_1, P_2} = \{J^1 \cup J^2 \mid J^1 \in \mathrm{MC}(P_1)$ and $J^2 \in \mathrm{MC}(P_2)\}$.

We first show that $\mathrm{MC}(P) \subseteq \Pi_{P_1, P_2}$. Let $J \in \mathrm{MC}(P)$. Denote $J^1 = J \cap \mathcal{A}_1$ and $J^2 = J \cap \mathcal{A}_2$. We claim that $J^1 \in \mathrm{MC}(P_1)$ and $J^2 \in \mathrm{MC}(P_2)$. Note that J^1 and J^2 are consistent. By means of contradiction, and without loss of generality, assume $J^1 \notin \mathrm{MC}(P_1)$. Thus, there exists $J^1_\star \in \mathcal{J}_{\mathcal{A}_1}$ such that $J^1 \cap m(P) \subset J^1_\star \cap m(P)$. Denote $J_\star = J^1_\star \cup J^2$. Observe that $J_\star$ is consistent. Furthermore, $J \cap m(P) \subset J_\star \cap m(P)$, thus $J \notin \mathrm{MC}(P)$, contradiction.

We now show that $\Pi_{P_1, P_2} \subseteq \mathrm{MC}(P)$. Let $J^1 \in \mathrm{MC}(P_1)$ and $J^2 \in \mathrm{MC}(P_2)$. Denote $J = J^1 \cup J^2$. Since $\{\mathcal{A}_1, \mathcal{A}_2\}$ is an IOD, J is consistent. Suppose $J \notin \mathrm{MC}(P)$. Thus, there exists $J' \in \mathcal{J}_{\mathcal{A}}$ such that $J \cap m(P) \subset J' \cap m(P)$. Let $\varphi \in (J' \cap m(P)) \setminus (J \cap m(P))$. Without loss of generality, assume $\varphi \in \mathcal{A}_1$. Denote $J^1_\star = J' \cap \mathcal{A}_1$. Note that $J^1_\star$ is consistent and $J^1 \cap m(P) \subset J^1_\star \cap m(P)$, contradiction.

RA. We give only a proof sketch.

Suppose that for every $J^1 \in \mathrm{RA}(P_1)$, for every $J^2 \in \mathrm{RA}(P_2)$, $J^1 \cap \mathcal{A}_2 = J^2 \cap \mathcal{A}_1$. Let $\Pi_{P_1, P_2} = \{J^1 \cup J^2 \mid J^1 \in \mathrm{RA}(P_1)$ and $J^2 \in \mathrm{RA}(P_2)\}$. Let $J^1 \in \mathrm{RA}(P_1)$, $J^2 \in \mathrm{RA}(P_2)$. Denote $J = J^1 \cup J^2$. We claim that $J \in \mathrm{RA}(P)$. Because $J^1 \in \mathrm{RA}(P_1)$, there is an order $>_{\sigma^1}$ on $\mathcal{A}_1$, refining $\succsim_{P_1}$ such that $J^1 = J_{\sigma^1}$. Similarly, there is an order $>_{\sigma^2}$ on $\mathcal{A}_2$, refining $\succsim_{P_2}$, such that $J^2 = J_{\sigma^2}$. We first claim that without loss of generality, we can assume that $>_{\sigma^1}$ and $>_{\sigma^2}$ coincide on $\mathcal{A}_1 \cap \mathcal{A}_2$. For this we construct σ'' on $\mathcal{A}_2$, refining $\succsim_{P_2}$, such that $>_{\sigma''}$ coincides with $>_{\sigma^1}$ on $\mathcal{A}_1 \cap \mathcal{A}_2$ and $J_{\sigma''} = J_{\sigma^2} = J^2$.

Now, let $>_\sigma$ be an order on $\mathcal{A}$ refining $\succsim_P$ and extending both $>_{\sigma^1}$ and $>_{\sigma^2}$. Let $\mathcal{A} = \{\alpha_1, \ldots, \alpha_{2m}\}$. Without loss of generality, suppose $\alpha_1 >_\sigma \ldots >_\sigma \alpha_{2m}$. Let $S_i \subseteq \mathcal{A}$ be

	p	$p\to q$	$p\to r$	q	r	s	$s\to q$	$s\to r$
J_1	+	+	+	+	+	+	+	+
J_2	-	+	+	-	-	-	+	+
J_3	+	-	-	-	-	+	-	-

Figure 2: The counter example used to show that several rules do not satisfy OAS.

the set obtained at the step i of construction of $J = J_\sigma$. We show by induction on i that

$$(H_i) \quad \forall j \in \{1, \dots, i\} \quad \text{we have} \quad \alpha_j \in S_i \text{ iff } \alpha_j \in J^1 \cup J^2.$$

From (H_{2m}), we obtain $J = J^1 \cup J^2$.

We now show that $\mathrm{RA}(P) \subseteq \Pi_{P_1,P_2}$. Suppose $J \in \mathrm{RA}(P)$. Let $>_\sigma$ be an order on $\mathcal{A}$ such that $J = J_\sigma$. Without loss of generality, suppose $\alpha_1 >_\sigma \dots >_\sigma \alpha_{2m}$. Denote by $>_{\sigma^1}$ (resp. $>_{\sigma^2}$) the restriction of $>_\sigma$ on $\mathcal{A}_1$ (resp. $\mathcal{A}_2$). Let $J^1 = J_{\sigma^1}$ and $J^2 = J_{\sigma^2}$. Observe that $J^1 \cap \mathcal{A}_2 = J^2 \cap \mathcal{A}_1$. Since $\{\mathcal{A}_1, \mathcal{A}_2\}$ is an IOD, $J^1 \cup J^2$ is consistent.

Let $S_i \subseteq \mathcal{A}$ be the set obtained at the step i of construction of $J = J_\sigma$. We show by induction on i that

$$(H_i) \quad \forall j \in \{1, \dots, i\} \quad \text{we have} \quad \alpha_j \in S_i \text{ iff } \alpha_j \in J^1 \cup J^2.$$

By putting $i = 2m$, we obtain $J = J^1 \cup J^2$. ☐

Proposition 5 *MCC, MED, FULL$_H$, and R$_{rev}$ do not satisfy OAS.*

Proof.
MCC, **MED** and **FULL$_H$**. We now provide a counter-example to show that MCC and MED do not satisfy OAS. Let $[\mathcal{A}_1] = \{p, p \to q, p \to r, q, r\}$, $[\mathcal{A}_2] = \{q, r, s, s \to q, s \to r\}$, and $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$. Observe that $\{\mathcal{A}_1, \mathcal{A}_2\}$ is an IOD of $\mathcal{A}$. Consider the profile from Figure 2. We obtain $\mathrm{MCC}(P_1) = \mathrm{MED}(P_1) = \mathrm{FULL}_H(P_1) = \{\ \{\ \neg p,\ \ p \to q,\ \ p \to r,\ \ \neg q,\ \ \neg r, \}\ \}$, and $\mathrm{MCC}(P_2) = \mathrm{MED}(P_2) = \mathrm{FULL}_H(P_2) = \{\ \{\ \neg s,\ \ s \to q,\ \ s \to r,\ \ \neg q,\ \ \neg r, \}\ \}$. However, $\mathrm{MCC}(P) = \mathrm{MED}(P) = \mathrm{FULL}_H(P) = \{\{\neg p, p \to q, p \to r, \neg q, \neg r, \neg s, s \to q, s \to r\}, \{p, p \to q, p \to r, q, r, s, s \to q, s \to r\}\}$.

R$_{rev}$. The proof is omitted due to space limitations. ☐

The preference agenda (Dietrich and List 2007b) associated with a set of alternatives $C = \{x_1, \dots, x_q\}$ is $\mathcal{A}_C = \{x_i P x_j \mid 1 \le i < j \le q\}$. When $j > i$, $x_i P x_j$ is not a proposition of $\mathcal{A}_C$, but we write $x_j P x_i$ as a shorthand for $\neg(x_j P x_i)$. Conversely, given a judgment set J on $\mathcal{A}_C$, the binary relation $\succ_J$ over C is defined by: for all $x_i, x_j \in C$, $x_i \succ_J x_j$ if $x_i P x_j \in J$ and $x_j \succ_J x_i$ if $\neg x_i P x_j \in J$.

Observation 5 *For any $m \ge 3$, there exists no (non-trivial) independent overlapping decomposition of the preference agenda over the set of alternatives $C = \{x_1, \dots, x_m\}$.*

Proof. We first establish the following lemma: if $\{\mathcal{A}_1, \mathcal{A}_2\}$ is an independent overlapping decomposition, then for all x_i, x_j, x_k, $x_i P x_j$ and $x_i P x_k$ are both in $\mathcal{A}_1$ or both in $\mathcal{A}_2$. Assume that it is not the case: without loss of generality, $x_i P x_j \in \mathcal{A}_1$ and $x_i P x_k \in \mathcal{A}_2$. Also without loss of generality, assume $x_j P x_k \in \mathcal{A}_1$. Let J_1 and J_2 be two consistent judgment sets over $\mathcal{A}_1$ and $\mathcal{A}_2$ such that J_1 contains $\{x_i P x_j, x_j P x_k\}$, J_2 contains $x_k P x_i$, and J_1 and J_2 are completed in an arbitrary way such that $J_1 \cap \mathcal{A}_2 = J_2 \cap \mathcal{A}_1$; $J_1 \cup J_2$ is an inconsistent judgment set over $\mathcal{A}_1 \cup \mathcal{A}_2$, which contradicts the assumption that $\{\mathcal{A}_1, \mathcal{A}_2\}$ is an independent overlapping decomposition.

Assume without loss of generality that $x_1 P x_2 \in \mathcal{A}_1$. Let $x', x'' \in \{x_1, \dots, x_k\}$. If $x' = x_1$ or $x'' = x_1$ then the above lemma implies that $x' P x'' \in \mathcal{A}_1$. If neither $x' = x_1$ nor $x'' = x_1$, then the above lemma implies that $x_1 P x' \in \mathcal{A}_1$, and applying the lemma again leads to $x' P x'' \in \mathcal{A}_1$. This being true for all x', x'', we have $\mathcal{A}_1 = \mathcal{A}$, and $\{\mathcal{A}_1, \mathcal{A}_2\}$ is a trivial decomposition. ☐

6 Discussion

We proposed a new property for judgment aggregation, namely agenda separability. It is a relaxation of the classical independence property, and unlike it, it is satisfied by several non-degenerate judgment aggregation rules. We have defined a stronger version of agenda separability, namely overlapping agenda separability, which is even more discriminant, since we have identified only two of the previously studied judgment aggregation rules that satisfy it, namely MC and RA. Note that RA satisfied furthermore unanimity principle (Lang et al. 2011). Also, two rules were left out of this paper due to space limitations: the judgment aggregation version of the Young rule, which does not satisfy agenda separability, and the 'geodesic' distance-base rule of Duddy and Piggins (2012), which satisfies agenda separability but not overlapping agenda separability.

A possible reason why agenda separability has not been studied sooner is that it is not applicable to common agendas such as the preference agenda, simply because they are not decomposable (cf. Observation 5). A similar observation would hold for other agendas of interest, such as those used for the aggregation of equivalence relations or for committee elections. However, agenda separability does apply to variants of these problems. Suppose for instance that we have to elect a committee made of K men and K women; then agenda separability applies and says that the election of the K men and the K women do not interfere.

This notion of agenda separability should not be confused with a notion of separability, also known as consistency or reinforcement, considered in voting theory (Young 1975) and generalized to judgment aggregation (Lang et al. 2011): these notions say that if a *profile* P can be decomposed into two subprofiles P_1 and P_2 for which the output is the same, then this should also be the output for P.

An ambitious issue for further work would be characterizing the set of rules that satisfy agenda separability, or one of its variants.

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