In Praise of Nonanonymity: Nonsubstitutable Voting

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Abstract In voting theory, it is generally assumed that voters are substitutable with each other, a property referred to as *anonymity* (which I will also call *nonsubstitutability* because of the possible ambiguity of the former): the outcome should remain unchanged after a permutation of agents' identities. Nonsubstitutability is generally taken for granted. While it is certainly highly desirable in political elections, I try to argue here that there are a whole lot of contexts where substitutability leads to questionable outcomes, and I suggest a simple way of generalizing almost all common voting rules to nonsubstitutable settings.

In the rest of this position paper, $N = \{1, \ldots, n\}$ is a set of agents and A is a finite set of alternatives. An ordinal profile is a collection $(V_1, \ldots, V_n)$ of linear orders on A, and in this case $rank(x, V_i)$ is the rank of x in V_i and $N(a \succ b)$ is the set of agents who prefer a to b. An approval profile is a collection $(V_1, \ldots, V_n)$ of subsets of A, and in this case $App(a)$ is the set of agents who approve a.

1 Three Examples

1.1 Elections in Shawington

Until now, Shawington, the federal capital of the United States of Planet Mars, did not have a mayor. Now it has to elect one. As we know, Shawington is split in three ethnic communities: the Greens, the Purples, and the Blues. Each community represents one third of the population, with 100 voters each. The candidates are a, b, c and d. The preferences of the population are as follows:

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60 Greens	$a \succ b \succ c \succ d$
10 Greens	$a \succ b \succ d \succ c$
30 Greens	$a \succ c \succ d \succ b$
40 Purples	$a \succ b \succ c \succ d$
30 Purples	$a \succ b \succ d \succ c$
30 Purples	$a \succ d \succ c \succ b$
70 Blues	$d \succ b \succ c \succ a$
30 Blues	$d \succ a \succ c \succ b$

The Shawingtonians decide to use a positional scoring rule associated with a vector $(s_1, s_2, s_3, 0)$, with $s_1 \geq s_2 \geq s_3$ and $s_1 > 0$. The scores of the 4 candidates are as follows:

a	$200s_1 + 30s_2$
b	$210s_2$
c	$30s_2 + 230s_3$
d	$100s_1 + 30s_2 + 70s_3$

Since a Lorenz-dominates (or stochastically dominates) all other candidates, it is always a winner (and a single winner except if $s_1 = s_2 = s_3$, in which case it is tied with c). Is it the best outcome? Not sure, as the cumulated scores by population are as follows (we show in boldface the minimum value in each row):

	Greens	Purples	Blues
a	$100s_1$	$100s_1$	$\mathbf{30s_2}$
b	$\mathbf{70s_2}$	$\mathbf{70s_2}$	$\mathbf{70s_2}$
c	$30s_2 + 60s_3$	$\mathbf{70s_3}$	$100s_3$
d	$\mathbf{40s_3}$	$30s_2 + 30s_3$	$100s_1$

It seems that b is a good trade-off: all populations are equally satisfied. It does not mean that *voters* are equally satisfied, though: b is the worst alternative for 30 Greens, 30 Purples and 30 Blues. But each of the three communities is solidary enough so that the dissatisfaction of 30% of the community be compensated by other means; on the other hand, choosing a is really unfair to the Blues, and no compensation from the other communities is possible: in other terms, on planet Mars, utility may be transferrable within a single community but not between communities.[1]

[1] Readers who feel uncomfortable with grouping voters by race, religion, nationality, ethnicity, or any such criterion can consider instead a similar example where professors are grouped by department when a university makes a policy decision. It may seem acceptable to be fair across departments, rather than fair across individual faculty members. Thanks to Nisarg Shah for this suggestion.

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1.2 Family Smith Watching Television

Family Smith lives in Shawington. Every day, they watch a movie on TV. There are three channels $c1$, $c2$, $c3$, and the preferences of the family members for every day of the week are as follows (we recall that Martian weeks contain three days):

	Earthday	Arrowday	Senday
Ann	$c1 \succ c2 \succ c3$	$c3 \succ c2 \succ c1$	$c1 \succ c2 \succ c3$
Betty	$c1 \succ c3 \succ c2$	$c3 \succ c2 \succ c1$	$c1 \succ c2 \succ c3$
Charles	$c2 \succ c3 \succ c1$	$c1 \succ c2 \succ c3$	$c3 \succ c1 \succ c2$

Let us first assume that the TV programs for the whole week are known at the beginning of the week. A fair decision with a reasonable trade-off between efficiency and fairness seems to be watching $c2$ on Earthday, $c3$ on Arrowday, and $c1$ on Senday.

Now, if the programs for each day are known only on the same day, planning ahead is not possible. On Earthday, $c1$ seems a reasonable decision. Next, when choosing the program for on Arrowday, it makes sense to introduce a bias towards Charles (who had to watch a movie he did not like on Earthday) and choose $c2$, and then $c1$ on Senday (while choosing $c3$ on Arrowday would likely lead to choosing $c3$ on Senday).

These two scenarios are respectively *off-line* and *on-line* temporal voting scenarios.

Off-line temporal voting can be seen as a standard multiple election (Brams et al. 1998) and can be formulated more generally as a fair public decision making problem (Conitzer et al. 2017), with the issues being the different days of the week. On the other hand, its structure is very close to the structure of our previous example (voting by community): what plays the role of the Greens, the Purples and the Blues are here the ‘the Anns’ (Ann on Earthday, Ann on Arrowday and Ann on Senday), ‘the Bettys’ and ‘the Charles’. Utility is transferrable between the three Anns (respectively, the three Bettys, the three Charles).[2]

On-line temporal voting is a very different problem that has received only little attention, one exception being (Freeman et al. 2017), who deal with it by giving at each step more weight to the voters who were less satisfied in the previous steps.

[2]That utility is transferrable between the three Anns, etc. may be debatable under certain circumstances: if Ann may prefer (2, 2, 2) to (3, 3, 0), i.e., she may be adverse to utility dispersion (a timewise version of risk aversion). One may also argue that if the time scale is long, agents may discount future utilities; this fits the setting, however: after scores have been transformed into utlities, we simply apply a discounting factor to them.

1.3 Fair Public Decision Making in Shawington

Shawington has a budget to be spent on public projects, and it wants to decide which project to build according to a *participatory budgeting* process, following other cities such as Paris.[3] Paris' participatory budgeting works as follows: each voter is allowed to support 6 projects for the whole city and 6 for her district ("arrondissement"). Shawington takes a slightly different view: it is not true that only the inhabitants of a district are concerned by the projects to be realized in the district. For instance, when choosing which facility to build in a district d, it is not unreasonable to give more weight to the inhabitants of district d, some lower weight to those who live in a district close to d, and yet a smaller weight to all other inhabitants. What's more, it is a good idea to be fair to the inhabitants of the various districts—again, to communities. The recent approach to fair public decision making (Freeman et al. 2017) does take fairness issues into account but does not differentiate between voters for a given project, nor considers communities of voters.

Yet other examples have to do with *epistemic social choice*: first, some agents may have more expertise on some issues than in some others; second, their opinions may often be positively correlated, for instance because they have similar sources, so that the weight of two correlated voters should count less than the sum of their weights. This idea is developed further in Ani and Nehring (2014).

2 Choquet Voting

The common point of Examples 1, 2 and 3 is that voters' utilities should not be simply added, but aggregated in a more subtle way. We outline here a general model for voting, which handles most of the variants of Examples 1, 2 and 3. It is based on *Choquet integrals* (see Grabisch and Labreuche 2010 for a good survey), widely known in decision under uncertainty and in multicriteria decision making, but much less in voting theory, probably because of this obsession for anonymity.

A *(normalized) capacity* on N is a function $\mu : 2^N \to [0, 1]$ satisfying $\mu(\emptyset) = 0$, $\mu(N) = 1$, and monotonicity ($S \subseteq T$ implies $\mu(S) \leq \mu(T)$). Let $\vec{u} = (u_1, \ldots, u_n) \in (I\!R^+)^n$. The *Choquet integral* of $\vec{u}$ w.r.t. μ is defined by

$$C_\mu(\vec{u}) = \sum_{i=1}^{n} u_{\sigma(i)} \mu(\sigma(i)|A(i-1))$$

where

- σ is a permutation of N such that $u_{\sigma(1)} \leq \ldots \leq u_{\sigma(n)}$;
- $A(0) = \emptyset$ and for all $i \in N$, $A(i) = \{\sigma(1), \ldots, \sigma(i)\}$ and $\mu(\sigma(i)|A(i-1)) = \mu(A(i)) - \mu(A(i-1)) = \mu(A(i-1) \cup \{\sigma(i)\}) - \mu(A(i-1))$.

[3] https://www.paris.fr/actualites/the-participatory-budget-of-the-city-of-paris-4151.

In words, $\mu(\sigma(i)|A(i-1))$ is the marginal importance contribution of agent $\sigma(i)$ to the set of agents $\{\sigma(1), \ldots, \sigma(i-1)\}$.

If μ is additive, then C_μ is simply a weighted average. If μ is anonymous, that is, if $\mu(S)$ depends only on $|S|$, C_μ is an *ordered weighted average*. If μ is dichotomous and $\vec{u} \in \{0, 1\}^n$, C_μ is a *simple game*.

Surprisingly (or perhaps not), all commonly used voting rules can be generalized to nonanonymous settings in a natural way via Choquet integrals. For approval voting, the μ-approval voting rule AV_μ is simply defined as

$$AV_\mu(V) = \text{argmax}_{x \in A} \mu(App(x))$$

If F_s is a positional scoring rule induced by scoring vector s, then for each ordinal profile V,

$$F_{s,\mu}(V) = \text{argmax}_{x \in A} C_\mu(s_{rank(x,V_1)}, \ldots, s_{rank(x,V_n)})$$

Rules based on iterated elimination of alternatives such as STV can also be easily generalized. For instance, for STV, the definition is as usual, replacing plurality scores by μ-plurality scores.

The μ-majority graph $M_{V,\mu}$ associated with V contains, for each pair (a, b) of alternatives, an edge (a, b) if and only if $\mu(N(a \succ b)) > \mu(N(b \succ a))$. The pairwise comparison matrix associated with V is defined by $W_V(a, b) = \mu(N(a \succ b)) - \mu(N(b \succ a))$. The μ-majority graph and μ-pairwise majority matrix being defined, rules based on the majority graph or (more generally) on the pairwise comparison matrix can be defined as usual.

If μ is anonymous and additive, that is, $\mu(S) = |S|$, then we find the classical versions of the voting rules. If μ is additive (but not necessarily anonymous), we find the versions of the rules with weighted voters. If μ is anonymous (but not necessarily additive), we find ranked-dependent versions of the rules; see Goldsmith et al. (2014), García-Lapresta and Martínez-Panero (2017) for two independent investigations of this family of rules.

Let us come back to our first introductory example (the election of the mayor of Shawington). We define the following capacity: for each $S \subseteq N$, let $S = S_G \cup S_P \cup S_B$, where S_G is the set of Green voters in S, etc. Finally,

$$\mu'(S) = \max(|S_G|, |S_P|, |S_B|)$$

Under a scoring rule, after some simple calculations, we find that the score of an alternative is the minimum, over all three communities, of the score they obtain for this community. Take Borda as an example. As a gets 300 among the Greens, 300 among the Purples and 60 among the Blues, its Borda$_\mu$ score is 60. b gets 140 among the Greens, 140 among the Purples and 140 among the Blues: its Borda$_\mu$ score is 140. We let reader check that b is the winner. As a slightly more sophisticated capacity, take

$$\mu'(S) = 7\max(|S_G|, |S_P|, |S_B|) + 2\,\text{med}(|S_G|, |S_P|, |S_B|) + \min(|S_G|, |S_P|, |S_B|)$$

The intuition is as follows: the least satisfied community counts 7 times more than the most satisfied community, and the one in the middle twice as much. We let the reader check that the winner is still b, but this time by a small margin.

We let the reader check that the other two examples can also be expressed as Choquet voting.

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