# Dealing with multi-source information in possibilistic logic

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**Abstract.** In this paper, both the uncertainty and the origin of pieces of information is handled in an extended possibilistic logic framework. Each formula is associated with a set (a fuzzy set more generally) which gathers labels of sources according to which the formula is (more or less) certainly true. In case of a single source of information, possibilistic logic is recovered. Soundness and completeness results of possibilistic logic are extended. Besides, the combination of the information provided by different sources (fusion), taking possibly into account the relative reliability of the sources, is discussed.

## 1 Motivations

Often, the available information comes from several sources rather than from a unique source. The information provided by each source may be incomplete and pervaded with uncertainty. The global information given by the different sources may be partially inconsistent due to the presence of conflicting pieces of information coming from various sources, even if each source provides a consistent information. In a given framework for modelling uncertainty, there exist rules for combining conflicting pieces of information, but it is not kept track of the origin of the information. No difference is made between a piece provided by one source only and a piece of information asserted by all the sources (if we except a possible reinforcement effect of the certainty). Thus it may be useful to keep the sources of information distinct in the reasoning process. While the ideas of attaching to a logical formula, a set of justifications, an hypothetical context, or a time interval have been investigated at length in truth maintenance systems (TMS), in assumption-based TMS, and in reified temporal logic respectively, it seems that the explicit handling of the origin of the information in reasoning has retained little attention.

## 2 Possibilistic Logic: background

Let $\Omega$ be the set of interpretations of a propositional or first-order logical language $\mathcal{L}$. A *possibility distribution* on $\Omega$ is a function $\pi$ from $\Omega$ to [0,1] which reflects the available knowledge; $\pi(\omega)$ estimates to what extent it is possible that the interpretation $\omega$ corresponds to the one underlying the real world; $\pi$ encodes a *preferential ordering* among interpretations [5]. $\pi$ is *normalized* iff $\exists\omega\in\Omega$ such that $\pi(\omega) = 1$. It expresses that at least one interpretation is fully possible when $\Omega$ is exhaustive.

From $\pi$, a *possibility* $\Pi$ and a dual *necessity measure* N, from $\mathcal{L}$ to [0,1], are defined [8,3] :

$$(\forall\ \varphi \in \mathcal{L})\quad \Pi(\varphi) = \sup\{\pi(\omega) \mid \omega \models \varphi\}$$
$$N(\varphi) = 1 - \Pi(\neg\varphi) = \inf\{1 - \pi(\omega) \mid \omega \models \neg\varphi\}.$$

$\Pi(\varphi)$ estimates the compatibility of $\varphi$ with the available knowledge, $N(\varphi)$ the extent to which $\varphi$ is entailed by this knowledge. The characteristic axioms of a possibility measure $\Pi$ are, in the finite case :

(i) $\Pi(\bot) = 0$ ($\bot$: contradiction)
(ii) $\forall\varphi, \forall\psi, \Pi(\varphi \vee \psi) = \max(\Pi(\varphi),\Pi(\psi))$.

Note that we *only* have $\Pi(\varphi\wedge\psi) \le \min(\Pi(\varphi),\Pi(\psi))$ in general; (ii) is equivalent to

$$\forall\varphi, \forall\psi, N(\varphi \wedge \psi) = \min(N(\varphi), N(\psi))$$

When $\pi$ is normalized, $N(\varphi) > 0 \Rightarrow \Pi(\varphi) = 1$. Besides, $N(\varphi \vee \psi) \ge \max(N(\varphi), N(\psi))$ only. Practically, given the available knowledge, $N(\varphi) = 1$ ($\Leftrightarrow \Pi(\neg\varphi) = 0$) means that $\varphi$ is certainly true; $1 > N(\varphi) > 0$ ($\Leftrightarrow 0 < \Pi(\neg p) < 1$) that $\varphi$ is somewhat certain and $\neg\varphi$ not certain at all; $N(\varphi) = N(\neg\varphi) = 0$ ($\Leftrightarrow \Pi(\varphi) = \Pi(\neg\varphi) = 1$) means total ignorance.

A *necessity-valued formula* (nvf) is a pair $(\varphi\ \alpha)$, where $\varphi$ is a formula of $\mathcal{L}$, and $\alpha \in [0,1]$ is a lower bound of a necessity measure. Such formulae are interpreted by means of possibility distributions. The satisfaction of a nvf by a possibility distribution is defined by :

$$\pi \models (\varphi\ \alpha) \quad \text{iff} \quad N(\varphi) \ge \alpha$$

where N is induced by $\pi$. Let $\mathcal{F} = \{(\varphi_1\ \alpha_1), \ldots, (\varphi_n\ \alpha_n)\}$ a set of nvf's, consituting a knowledge base, then the notion of logical consequence is defined by

$$\mathcal{F} \models (\varphi\ \alpha) \quad \text{iff} \quad \forall\pi, \pi \models \mathcal{F} \text{ implies } \pi \models (\varphi\ \alpha)$$

where $\quad \pi \models \mathcal{F} \quad \text{iff} \quad \forall\ i \in \{1, \ldots, n\}, \pi \models (\varphi_i\ \alpha_i)$

i.e. the set of possibility distributions satisfying $\mathcal{F}$ is included in the set of those satisfying $(\varphi\ \alpha)$.

The consistency of $\mathcal{F}$ is estimated by the extent to which there is at least an interpretation completely possible for $\mathcal{F}$, i.e. there exists a normalized possibility distribution satisfying $\mathcal{F}$; the quantity

$$\mathrm{Inc}(\mathcal{F}) = 1 - \sup_{\pi \models \mathcal{F}} \sup_{\omega\in\Omega} \pi(\omega)$$

is called the *inconsistency degree* of $\mathcal{F}$. If $\mathrm{Inc}(\mathcal{F}) = 0$, $\mathcal{F}$ is *completely consistent*; indeed $\mathrm{Inc}(\mathcal{F}) = 0$ iff the knowledge base obtained from $\mathcal{F}$ by ignoring the valuations is consistent. If $\mathrm{Inc}(\mathcal{F}) = 1$ $\mathcal{F}$ is *completely inconsistent*, and if $0 < \mathrm{Inc}(\mathcal{F}) < 1$ then $\mathcal{F}$ is *partially inconsistent*. Then we have [7,2]

- $\mathrm{Inc}(\mathcal{F}) = \inf\{N(\bot) \mid \pi \models \mathcal{F}\}$ where N is induced by $\pi$.
- Let $\mathcal{F} = \{(\varphi_1\ \alpha_1), ..., (\varphi_n\ \alpha_n)\}$, and

$$\pi^*_{\mathcal{F}}(\omega) = \min\{1 - \alpha_i \mid \omega \models \neg\varphi_i,\ i = 1,\dots, n\}$$
$$= 1 \text{ if } \forall i,\ \omega \models \varphi_i\ ;$$

then $\pi \models \mathcal{F}$ iff $\pi \leq \pi^*_{\mathcal{F}}$. $\pi^*_{\mathcal{F}}$ is the least specific (i.e. the largest) possibility distribution satisfying $\mathcal{F}$. Moreover $\mathcal{F} \models (\varphi\ \alpha)$ iff $\pi^*_{\mathcal{F}} \models (\varphi\ \alpha)$. Besides $\mathrm{Inc}(\mathcal{F}) = 1 - \sup_{\omega\in\Omega} \pi^*_{\mathcal{F}}(\omega)$.

$\mathrm{Inc}(\mathcal{F})$ can be seen as a threshold below which any deduction from $\mathcal{F}$ is trivial. Indeed, if $\mathrm{Inc}(\mathcal{F}) = \alpha$, then $\forall\pi \models \mathcal{F}$, $N(\bot) \geq \alpha$ and a fortiori $\forall\varphi$, $N(\varphi) \geq N(\bot) \geq \alpha$ where N is induced by $\pi$; thus, any deduction $\mathcal{F} \models (\varphi\ \beta)$ with $\beta \leq \alpha$ is trivial. Allowing non-trivial deductions only makes the consequence operator nonmonotonic [5]. Deduction and refutation are extended to possibilistic logic [7][2] by

- $\mathcal{F} \cup \{(\varphi\ 1)\} \models (\psi\ \alpha)$ iff $\mathcal{F} \models (\varphi \to \psi\ \ \alpha)$.
- $\mathcal{F} \models (\varphi\ \alpha)$ iff $\mathcal{F} \cup \{(\neg\varphi\ 1)\} \models (\bot\ \alpha)$.

Thus, if we want to know whether $(\varphi\ \alpha)$ is a logical consequence of $\mathcal{F}$ or not, it is sufficient to compute the inconsistency degree of $\mathcal{F} \cup \{(\neg\varphi\ 1)\}$, which is equal to the largest $\alpha$ such that $\mathcal{F} \models (\varphi\ \alpha)$. A *necessity-valued* clause (nvc) is a nvf $(c\ \alpha)$ where c is a clause. If $(\varphi\ \alpha)$ is a nvf and if $\{c_1, \dots, c_n\}$ is a clausal form of $\varphi$ then a clausal form of $(\varphi\ \alpha)$ is $\{(c_1\ \alpha), \dots, (c_n\ \alpha)\}$. The resolution rule is then :

$$(c_1\ \alpha_1)\ ,\ (c_2\ \alpha_2) \vdash (c'\ \min(\alpha_1, \alpha_2))$$

where c' is a resolvent of clauses $c_1$ and $c_2$. Possibilistic resolution for nvc's is proved to be sound and complete for refutation [7]. Let us consider an example.

$\mathcal{F}^1 = \{(\neg p \vee r\ 0.7), (\neg p \vee q\ 1), (\neg q \vee r\ 0.4), (p\ 0.5), (q\ 0.8)\}$

It induces the constraints $\forall\ \pi \models \mathcal{F}^1$,

$\pi \models (\neg p \vee r\ 0.7) \Leftrightarrow N(\neg p \vee r) \geq 0.7 \Leftrightarrow \Pi(p \wedge \neg r) \leq 0.3 \Leftrightarrow \forall\ \omega \models p \wedge \neg r,\ \pi(\omega) \leq 0.3$; and similarly $\forall\ \omega \models p \wedge \neg q,\ \pi(\omega) = 0$ ; $\forall\ \omega \models q \wedge \neg r,\ \pi(\omega) \leq 0.6$ ; $\forall\ \omega \models \neg p,\ \pi(\omega) \leq 0.5$ ; $\forall\ \omega \models \neg q,\ \pi(\omega) \leq 0.2$. $\pi^*_{\mathcal{F}1}$ is thus given by

$\pi^*_{\mathcal{F}1}(p \wedge q \wedge r) = 1$ ;

$\pi^*_{\mathcal{F}1}(\neg p \wedge q \wedge r) = \pi^*_{\mathcal{F}1}(\neg p \wedge q \wedge \neg r) = 0.5$ ;

$\pi^*_{\mathcal{F}1}(p \wedge q \wedge \neg r) = 0.3$ ;

$\pi^*_{\mathcal{F}1}(\neg p \wedge \neg q \wedge r) = \pi^*_{\mathcal{F}1}(\neg p \wedge \neg q \wedge \neg r) = 0.2$ ;

$\pi^*_{\mathcal{F}1}(p \wedge \neg q \wedge r) = \pi^*_{\mathcal{F}1}(p \wedge \neg q \wedge \neg r) = 0$.

We have the following derivation, when looking for the certainty of r, and thus adding $(\neg r\ 1)$ to $\mathcal{F}^1$

$$(\neg p \vee r\ \ 0.7), (\neg r\ 1) \vdash (\neg p\ 0.7)$$
$$(\neg p\ 0.7), (p\ 0.5) \vdash (\bot\ 0.5)$$

i.e. $N(r) \geq 0.5$ and indeed it can be checked that using $\pi^*_{\mathcal{F}1}$, we have for the associated possibility measure $\Pi^*(\neg r)=0.5$. Then $N^*(r)=0.5$, and $\forall\pi \leq \pi^*_{\mathcal{F}1}$, $N(r) \geq 0.5$.

## 3 Multi-Source Possibilistic Logic

The semantics of possibilistic logic only requires the definition of necessity measures on a logical language $\mathcal{L}$, and in order to define these necessity measures from $\mathcal{L}$ to [0,1], we only needed three operations on [0,1] : the minimum and maximum operators (which underlie the *ordering structure*) and the *order reversing operation* $(1 - (\cdot))$. A straightforward generalization is to map possibility distributions, as well as possibility and necessity measures, no longer into [0,1] but into any *complete distributive lattice* L. In the following, we take $L = [0,1]^{\mathcal{S}}$ where $\mathcal{S}$ is a given set, interpreted as the set of sources of information. L is equipped with the fuzzy set intersection $(\cap)$, union $(\cup)$ and complementation $(\overline{\phantom{x}})$, pointwisely defined by means of the operations min, max and $1 - (.)$ respectively (the ordering being the fuzzy set inclusion defined by the inequality between membership functions) ; formally, $\forall x = (x_1,\dots,x_m),\ y = (y_1,\dots,y_m) \in L$,

$x \cup y = (\max(x_1,y_1),\dots,\max(x_m,y_m))$

$x \cap y = (\min(x_1,y_1),\dots,\min(x_m,y_m))$

$\bar{x} = (1 - x_1,\dots,1 - x_m)$.

where $\mathcal{S} = \{s_1, \dots, s_m\}$ and $x_i \in [0,1]$, $y_i \in [0,1]$. Now we actualize the definitions of Section 2, in the framework of fuzzy set-valued possibilistic logic, keeping in mind the multi-source interpretation. A fuzzy set A of $\mathcal{S}$ will be denoted by $\{\mu_A(s_j) / s_j\ ,\ j = 1,m\}$ where $\mu_A(s_j)$ is the membership degree of $s_j$. Then

- $\pi$ will denote a multi-source possibility distribution defined from the set of interpretations $\Omega$ to L =

$[0,1]^{\mathcal{S}}$, i.e. $\pi(\omega)$ is the fuzzy set $\{\pi^j(\omega) / s_j,\ j = 1,m\}$ of $\mathcal{S}$, where the degree attached to $s_j$ is interpreted as the degree of possibility $\pi^j(\omega)$ of the interpretation $\omega$ according to the source $s_j$. $\pi$ is said to be normalized iff $\bigcup_{\omega\in\Omega} \pi(\omega) = \mathcal{S}$, i.e. $\forall j=1,m$, $\max_{\omega\in\Omega} \pi^j(\omega) = 1$ (since $\mathcal{S} = \{1/s_1,\ldots,1/s_m\}$). This is equivalent, if $\Omega$ is finite, to $\exists\omega$, $\pi^j(\omega) = 1$, i.e. each $\pi^j$ is normalized; it means that each source is fully consistent;

- the possibility measure $\Pi$ associated with $\pi$ is defined by the fuzzy set $\Pi(\varphi) = \bigcup\{\pi(\omega), \omega \models \varphi\}$
 $= \{(\max_{\omega\models\varphi} \pi^j(\omega))/s_j,\ j=1,m\} = \{\Pi^j(\varphi)/s_j,\ j=1,m\}$
 ($\Pi^j$ is the scalar possibility measure induced by $\pi^j$)
- by duality $N(\varphi)$ is defined by the fuzzy set
 $N(\varphi) = \overline{\Pi(\neg\varphi)} = \bigcap\{\overline{\pi(\omega)}, \omega \models \neg\varphi\}$
 $= \{(\min_{\omega\models\neg\varphi} 1-\pi^j(\omega))/s_j,j=1,m\} = \{N^j(\varphi)/s_j,j=1,m\}$
 Clearly, we can write
 $\Pi(\varphi \vee \psi) = \Pi(\varphi) \cup \Pi(\psi)$ ; $N(\varphi \wedge \psi) = N(\varphi) \cap N(\psi)$.

The semantics of multi-source possibilistic logic is then easily defined, as a very natural generalization of possibilistic logic's. Let us consider the knowledge base $\mathcal{F} = \{(\varphi_i\ A_i), i = 1,n\}$, where $A_i$ denotes a fuzzy set of $\mathcal{S}$ interpreted as $A_i = \{\mu_{A_i}(s_j)/s_j,\ j=1,m\}$ with $\mu_{A_i}(s_j) \leq N^j(\varphi_i)$, i.e. $A_i$ provides lower bounds on the extent to which $\varphi_i$ is necessarily true according to each source $s_j$. Then the least specific multi-source possibility distribution is given by

$$\begin{aligned}\pi^*_{\mathcal{F}}(\omega) &= \bigcap\{\overline{A_i}, \omega \models \neg\varphi_i,\ i=1,n\}\\ &= \{(\min_{i,\omega\models\neg\varphi_i} 1 - \mu_{A_i}(s_j)) / s_j,\ j=1,m\}\\ &= \{\pi^{j*}_{\mathcal{F}}(\omega) / s_j,\ j=1,m\}\end{aligned}$$

where $\pi^{j*}_{\mathcal{F}}$ is the least specific possibility distribution representing the semantics of the information provided by source $s_j$. The fuzzy set of inconsistent sources with respect to $\mathcal{F}$ is given by $Inc(\mathcal{F}) = \overline{\bigcup\{\pi^*_{\mathcal{F}}(\omega), \omega \in \Omega\}} = \bigcap\{\overline{\pi^*_{\mathcal{F}}(\omega)}, \omega\in\Omega\}=\{(1 - \sup_{\omega\in\Omega} \pi^{j*}_{\mathcal{F}}(\omega)) / s_j,j=1,m\}$.

Clearly, instead of focusing on the set of sources for which a given formula $\varphi$ is somewhat certain, we may dually consider the set of formulas which are somewhat certainly true for a given source $s_j$. As made clear by the above expressions, the two points of view are perfectly equivalent. Indeed the fuzzy sets $\pi$, $\Pi$, $N$, $\pi^*_{\mathcal{F}}$, $Inc(\mathcal{F})$ introduced above, can be viewed as vectors of the corresponding scalar values for j = 1,m.

Formally, the multi-source extensions of deduction and refutation theorems hold, i.e. we respectively have

- $\mathcal{F} \cup \{(\varphi\ \mathcal{S})\} \models (\psi\ A)$ iff $\mathcal{F} \models (\varphi \to \psi\ A)$
- $\mathcal{F} \models (\varphi\ A)$ iff $\mathcal{F} \cup \{(\neg\varphi\ \mathcal{S})\} \models (\bot\ A)$.

as well as counterparts of the other results of Sec. 2. Clausal forms also extend to the multi-source case. The *resolution* rule (c A), (c' A') $\vdash$ (c'' $A \cap A'$) obtains, where c'' is a resolvent of clauses c and c'. It is obvious when A and A' are non-fuzzy; in terms of fuzzy sets it reads

$$(c\ \{\mu_A(s_j) / s_j, j=1,m\}), (c'\ \{\mu_{A'}(s_j) / s_j, j=1,m\})$$
$$\vdash\ (c''\ \{\min(\mu_A(s_j), \mu_{A'}(s_j)) / s_j, j=1,m\}).$$

In order to get a sound and complete procedure we must add the *combination* rule (c A), (c A') $\vdash$ (c $A \cup A'$), which states that if the clause c is considered as somewhat certainly true by the two fuzzy sets of sources A and A', c is still similarly considered by their union. More precisely, the greatest lower bound according to each source is retained as expected, i.e. we obtain

$$(c\ \{\max(\mu_A(s_j), \mu_{A'}(s_j)) / s_j,\ j = 1,m\}).$$

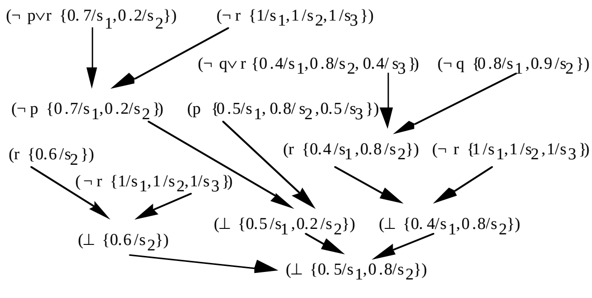
**Example**. We consider the knowledge base $\mathcal{F}^1$ of Section 2, provided by source $s_1$ together with the two knowledge bases $\mathcal{F}^2$ and $\mathcal{F}^3$ given by $s_2$ and $s_3$,

$\mathcal{F}^2 = \{(\neg p \vee q\ 1), (\neg q \vee r\ 0.8), (\neg p \vee r\ 0.2), (p\ 0.8), (q\ 0.9), (r\ 0.6)\}$ ;
$\mathcal{F}^3 = \{(\neg q \vee r\ 0.4), (\neg p \vee \neg r\ 0.3), (p\ 0.5)\}$.

Altogether it makes the following multi-source knowledge base (zero membership degrees are omitted)

$\mathcal{F} = \{(\neg p\vee q\ \{1/s_1,1/s_2\}), (\neg p\vee r\ \{0.7/s_1,0.2/s_2\}), (\neg q\vee r\ \{0.4/s_1,0.8/s_2,0.4/s_3\}),(\neg p\vee\neg r\ \{0.3/s_3\}), (p\ \{0.5/s_1,0.8/s_2,0.5/s_3\}),(q\ \{0.8/s_1,0.9/s_2\}), (r\ \{0.6/s_2\})\}$.

Then by resolution and combination we can compute the multi-source certainty attached to r. So we proceed by refutation by adding $(\neg r\ \mathcal{S}) = (\neg r\ \{s_1,s_2,s_3\}) = (\neg r\ \{1/s_1, 1/s_2, 1/s_3\})$ to $\mathcal{F}$. We get




Thus $N(r) \supseteq \{0.5/s_1, 0.8/s_2\}$ (where $\supseteq$ denote fuzzy set inclusion, i.e. it means $N^1(r) \geq 0.5$, $N^2(r) \geq 0.8$). It can be checked that $\pi^*_{\mathcal{F}}$ is normalized since $\pi^{j*}_{\mathcal{F}}$ is so, for j=1,3 ; it means that each source gives consistent information. However it does not mean that the sources are consistent altogether. Indeed from $\mathcal{F}$ we can also prove $N(\neg r) \supseteq \{0.3/s_3\}$, i.e. $s_3$ is in conflict with $\{s_1,s_2\}$ with respect to r. Thus by distinguishing between the sources, we avoid inconsistency problems (while dealing with $\mathcal{F}^1 \cup \mathcal{F}^2 \cup \mathcal{F}^3$ in our example would create an inconsistent possibilistic knowledge base).

We do not insist here on the case where a source provides inconsistent information by itself. The treatment of this situation is an immediate by-product of the capability of possibilistic logic to handle inconsistency [7][5][2] as briefly recalled in Section 2.

## 4 Fusion of Sources and Information Combination

The necessity measure defined from a multi-source possibility distribution is equivalent to a vector of scalar-valued necessity measures, each of them representing a source. Given m sources of information, a natural question is then to know if it is possible to replace these m sources by an equivalent fictitious source. More generally, the pieces of information provided by the different sources have to be combined, taking into account the relative reliability of the sources, in order to provide the user with synthetic conclusions. The problem of the fusion of m sources, is partially answered, in the possibilistic framework, by the following result (see [4] for the proof)

• The *only* functions f from $[0,1]^m$ to [0,1], satisfying the idempotency constraint $f(a, \ldots, a) = a$, such that the function defined by

$$\forall \varphi,\ N(\varphi) = f(N^1(\varphi), \ldots, N^m(\varphi))$$

is still a *scalar* necessity function, are of the form

$$N(\varphi) = \min(g_1(N^1(\varphi)), \ldots, g_m(N^m(\varphi)))$$

where the $g_j$'s are functions from [0,1] to [0,1] such that $\forall j,\ g_j(1) = 1$ and $\exists k,\ g_k(0) = 0$. The $g_i$'s can be chosen as non-decreasing.

Clearly, the idempotency constraint ensures that if all the sources agree on the level of certainty of a proposition, the result of the combination is what each source tells, i.e. $\forall\ a \in [0,1],\ N^1(\varphi) = \ldots = N^m(\varphi) = a \Rightarrow N(\varphi) = a$. This is equivalent to the following combination in terms of possibility distributions

$$\pi(\omega) = \max(h_1(\pi^1(\omega)), \ldots, h_m(\pi^m(\omega)))$$

with $\forall j,\ h_j(0) = 0$ and $\exists k,\ h_k(1) = 1$. More precisely we should have $h_j(x) = 1 - g_j(1 - x)$.

This result is in agreement with the fact that in classical logic the intersection of deductively closed knowledge bases is itself deductively closed. Indeed the intersection of the closed knowledge bases corresponds to the union of their set of models (assuming $N^j(\varphi) \in \{0,1\}$, and choosing $\forall j,\ h^j(1) = 1$, since $N^j(\varphi) = 1 \Leftrightarrow \varphi$ belongs to the deductive closure of $\mathcal{F}^j$).

An example of function $g_j$ is $g_j(x) = \max(x, 1 - \lambda_j)$ with the normalization condition $\max_{j=1,m} \lambda_j = 1$. It leads to a weighted minimum combination for N, i.e.

$$N(\varphi) = \min_{j=1,m} \max(N^j(\varphi), 1 - \lambda_j).$$

In terms of possibility distributions, it is equivalent to $\pi = \max_{j=1,m} \min(\pi^j,\lambda_j)$, i.e. $\pi$ is obtained as a weighted union of the possibility distributions associated with each source. It is a weighted version of the fuzzy set union. Note that $\pi$ remains normalized as soon as all the $\pi^j$ are. The weight $\lambda_j$ can be interpreted as the relative level of reliability of source $s_j$. Indeed, if all the $\lambda_j$ are equal, we have $\forall j,\ \lambda_j = 1$ due to normalization and the fuzzy set union on the $\pi^j$ is recovered (and the min operation for the $N^j$) ; if $\lambda_j = 0$ the information provided by the source $s_j$ is not taken into account. For intermediary $\lambda_j$, only sufficiently certain information is taken into account. Clearly, this consensus based on a (weighted) union of the possibility distributions, or equivalently on the min-combination of the certainty degrees, is a very cautious combination since, when all the $\lambda_j$ are equal to 1, only the least informative lower bound, is retained as an estimate of the certainty of a formula, among the lower bounds provided by the sources. In other words, the opinion of the source which is the least certain prevails. In case of conflict between self-consistent sources concerning a formula $\varphi$, i.e. $\exists k, \ell,\ N^k(\varphi) > 0$ (then $N^k(\neg\varphi) = 0$) and $N^\ell(\neg\varphi) > 0$ (then $N^\ell(\varphi) = 0$), this leads to $N(\varphi) = 0 = N(\neg\varphi)$ when $\forall j,\ \lambda_j = 1$ ; when the sources have unequal reliability, in case of conflict, the certainty degrees of the most reliable sources are decreased, but not necessarily down to 0, as it can be checked.

Many other combinations may be performed, at the semantic level, on the possibility distributions representing the information provided by each source. Let $*$ be the such a combination different from the weighted union, i.e. $\pi = \pi_1 * \ldots * \pi_m$. Then as seen above, there does not exist a function f such that the necessity $N(\varphi)$ induced by $\pi$ can be expressed in terms of the $N^i(\varphi)$ in a compositional way for any $\varphi$. This situation is to be related to the fact that in classical logic, the union of deductively closed knowledge bases is generally not deductively closed.

The union of knowledge bases corresponds to the intersection of the corresponding sets of models. Thus a worth-considering combination operation on the $\pi^j$ is the fuzzy set intersection, and more generally the weighted fuzzy intersection [4] (taking into account reliability levels $\lambda_j$) defined by

$$\pi = \min_{j=1,m} \max(\pi^j, 1 - \lambda_j) \text{ with } \max_{j=1,m} \lambda_j = 1.$$

For $\lambda_j = 1$, $\forall j$, the fuzzy set intersection is recovered. Clearly $\pi$ may then be subnormalized even if all the $\pi^j$ are normalized. If $\pi$ is not normalized, it expresses a conflict between the sources about some formula, as it is the case between $\{s_1,s_2\}$ and $s_3$ about r in the example of Section 3. In the particular case of a source k which is in complete agreement with the others, but better informed, i.e. $\exists k$, such that $\pi^k \leq \min_{j\neq k} \pi^j$, we have $\pi = \pi^k$ and $N = N^k$. More generally, it can be shown that the weighted intersection of the possibility distributions representing the semantics of the $\mathcal{F}^j$ is nothing but the possibility distribution expressing the semantics of the knowledge base obtained from $\mathcal{F}^1,\ldots,\mathcal{F}^m$ by computing the weighted max of the certainty degrees attached to the formulas, i.e. the knowledge base made of the pairs $(\varphi \ \max_{j=1,m} \min(\alpha^j,\lambda_j))$ where $(\varphi \ \alpha^j)$ belongs to $\mathcal{F}^j$ (with possibly $\alpha_j = 0$). In this combination process the largest lower bound, among the lower bounds provided by the sources, is retained as the estimate of the certainty of a formula, provided that this lower bound is not greater than the level of reliability of the source which provides it. The fact that the weighted union of the possibilistic deductive closures of the $\mathcal{F}^i$'s is not closed just points out that the lower bounds computed on the formulas will not be optimal, but are likely to be improved through possibilistic deduction. Obviously when $\pi$ is not normalized, the knowledge base built as said above will be (partially) inconsistent [7] [2], i.e. will contain $(\varphi \ \alpha)$ and $(\neg\varphi \ \alpha')$ with $\alpha > 0$, $\alpha' > 0$ for some formula $\varphi$.

## 5 - Concluding remarks

The logic presented here is similar to a so-called *timed possibilistic logic* [1] where each formula is associated with a (possibly fuzzy) set of time instants at which this formula is more or less certainly true. But the problem of combining the pieces of information coming from different sources more or less reliable has no counterpart in the temporal interpretation.

The proposed framework suggests a methodology for merging several knowledge bases into a single one while retaining the origin of each piece of information. This is done by attaching to any formula a tag where the name of the sources that supplied this information appears ; both the level of support of the formula by a source, and the reliability of the source itself can be handled. Our approach can be cast in the setting of labelled deduction systems [6], as possibilistic logic itself.

Our framework has some potential to deal with inconsistency due to the presence of conflicting sources : Firstly by structuring a knowledge base into separate consistent parts ; in that case, the deduction methods deal with these parts in parallel rather than making separate inferences from each sub-base. Secondly the inconsistency between sources can be resolved for a given query using one of the combination modes presented in the previous section. The choice of a mode depends on whether one must be cautious, or can be adventurous in the given situation. More generally revision procedures for inconsistent knowledge bases should explicitly involve the origin of the pieces of information and the reliability of the sources.

## References


[1] D. Dubois, J. Lang, and H. Prade. Timed possibilistic logic. *Fundamenta Informaticae*, XV:211-234, 1991.

[2] D. Dubois, J. Lang, and H. Prade. Possibilistic logic. In D.M. Gabbay, editor, *Handbook of Logic for Artificial Intelligence*, Oxford University Press, 1992, to appear.

[3] D. Dubois, and H. Prade. Necessity measures and the resolution principle. *IEEE Trans. on Systems, Man and Cybernetics*, 17:474-478, 1987.

[4] D. Dubois, and H. Prade. Aggregation of possibility measures. In J. Kacprzyk and M. Fedrizzi, editors, *Multiperson Decision Making Using Fuzzy Sets and Possibility Theory*, Kluwer Academic Publ., pp. 55-63, 1990.

[5] D. Dubois, and H. Prade. Possibilistic logic, preference models, non-monotonicity and related issues. Proc. 12th Inter. Joint Conf. on A.I., Sydney, pp. 419-424, 1991.

[6] D.M. Gabbay. *Labelled Deduction Systems*. Oxford University Press, to appear.

[7] J. Lang, D. Dubois, and H. Prade. A logic of graded possibility and certainty coping with partial inconsistency. *Proc. 7th Conf. on Uncertainty in A.I.,* Los Angeles, pp. 188-196. Morgan & Kaufmann, 1991.

[8] L.A. Zadeh. Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems*, 1(1):3-28, 1978.