A modal logic for epistemic tests

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Abstract. We study a modal logic of knowledge and action, focussing on tests actions. Such knowledge-gathering actions increase the agents' knowledge. We propose a semantics, and associate an axiomatics and a rewriting-based proof procedure.

1 Introduction

In order to be able to evolve in her/his environment, an intelligent agent must interact with the latter. For instance, imagine a robot that wants to open a door that might be locked. If the robot is shrewd enough, it starts by checking whether the door is effectively locked or not. This action establishes truth or falsehood of the formula "the door is locked". We call *test* such an action.

Consider a logical circuit with various components. If the circuit is broken down, several configurations of faulty components are possible. Consider three possible configurations: φ_1: components c_1 and c_2 are broken down; φ_2: only the component c_2 and φ_3: components c_1 and c_3.

Suppose there is only one possible configuration of breakdown. Which test sequence enables us to know in which configuration we are? In this example, if we test component c_1 and component c_2, we know in which configuration we are. Hence we should be able to prove that these tests are sufficient.

A test is a particular interaction between the agent and the world. It is one-sided form of communication: the agent acquires knowledge about the environment, but does not change the environment. We make the following simplifying assumptions

- the world does not evolve while the test is carried out;
- tests do not modify the environment;
- tests are reliable;
- every proposition can be tested successfully.

Database querying is a basic example of such tests.

What do we test? We can test objects of the world, for example a battery or a computer program. Here, we will be interested rather in testing facts: *test if* the battery is defective.

A test plan is a sequence of test. In our logic validating a test plan comes down to studying the validity of a logical formula.

Tests are actions. We shall integrate tests consequently into an action logic. In order to be able to reason both about knowledge and about actions, our logic comprises an epistemic component and a dynamic component. In section 2 we present a basic dynamic epistemic logic EDL. In section 3 we integrate actions of the type "test if a proposition is true". We give an axiomatisation and a proof procedure. In section 4 we investigate the opposite concept of uninformative actions. Finally, in section 5, we show a separation theorem. In section 6 we integrate a solution to the Frame Problem into EDL. Section 7 discurses related work.

2 The basic logic

Our basic logic EDL combines dynamic logic and epistemic logic. The language is built from a set of atomic formulas FML_0, a set of atomic actions ACT_0, the classical logic operators $\rightarrow, \wedge, \vee, \neg$, the action operators λ, $\cup$, $;$, the modal operator $[.]$ and the epistemic operator $\mathbf{K}$. Actions and complex formulas are defined by recursion. $\alpha;\beta$ means "execute α and then β", $\alpha \cup \beta$ means "choose nondeterministically between α and β, and then execute the chosen action", and λ is the action to "do nothing".

We read the formula $\mathbf{K}p$ "the agent knows p" and $[\alpha]A$ as "A holds after the execution of action α". $\neg\mathbf{K}p \wedge \neg\mathbf{K}\neg p$ means that the agent ignores whether A. $\langle\alpha\rangle A$ is an abbreviation of $\neg[\alpha]\neg A$. In particular, $[\alpha]\bot$ means that α is inexecutable and $\langle\alpha\rangle\top$ means that α is executable. We say that a formula from *FML* is *objective* if it contains no occurrence of $\mathbf{K}$.

Our axiomatisation contains a modal logic for knowledge and a *K* modal logic for actions. We require the logic of knowledge to be at least as strong as *KT* (i.e. to validate the principle $\mathbf{K}A \rightarrow A$).

N($\mathbf{K}$)	$\dfrac{A}{\mathbf{K}A}$	N($[\alpha]$)	$\dfrac{A}{[\alpha]A}$
K($\mathbf{K}$)	$(\mathbf{K}A \wedge \mathbf{K}(A \rightarrow C)) \rightarrow \mathbf{K}C$		
T($\mathbf{K}$)	$\mathbf{K}A \rightarrow A$		
K($[\alpha]$)	$([\alpha]A \wedge [\alpha](A \rightarrow C)) \rightarrow [\alpha]C$		
Def(λ)	$[\lambda]A \equiv A$		
Def($;$)	$[\alpha;\beta]A \equiv [\alpha][\beta]A$		
Def($\cup$)	$[\alpha \cup \beta]A \equiv ([\alpha]A \wedge [\beta]A)$		
Acq($[\alpha], \mathbf{K}$)	$\mathbf{K}[\alpha]C \rightarrow [\alpha]\mathbf{K}C$		

All the axioms are standard, except Acq($[\alpha], \mathbf{K}$). It means if the agent knows what will be true after a test then he knows he does not loose this knowledge after the test.

Remark The only case where Acq($[\alpha], \mathbf{K}$) cannot be accepted is when the action modifies the agent's integrity. For instance, if the agent is a robot and it knows that erasing its memorry, it will forget about all facts. In this paper we do not consider such actions.

The semantics is in terms of possible states. We interpret the knowledge of the agent at a possible world w by a set of worlds associated to w; called his belief state. Actions are interpreted as transition relations on worlds.

We define a model as a quadruple $M = \langle W, R_{\mathbf{K}}, \{R_\alpha : \alpha \in ACT_0\}, V\rangle$ where W is a set of possible worlds, $R_{\mathbf{K}} \subseteq W \times W$ and every $R_\alpha \subseteq W \times W$ are accessibility relation., and V associates to each world an interpretation. We shall sometimes identify $R_{\mathbf{K}}$ and R_α with mappings $R_{\mathbf{K}} : W \longrightarrow 2^W$ and $R_\alpha : W \longrightarrow 2^W$. We require

- $R_\lambda\{w\} = \{w\}$,

- $R_{\alpha;\beta} = R_\alpha \circ R_\beta$,
- $R_{\alpha\cup\beta} = R_\alpha \cup R_\beta$,
- $R_\alpha \circ R_{\mathbf{K}} \subseteq R_{\mathbf{K}} \circ R_\alpha$.

The last condition corresponds to Acq($[\alpha], \mathbf{K}$).

The truth conditions are defined as usual, in particular:

- $\models_{M,w} \mathbf{K}A$ if $\models_{M,v} A$ for every $v \in R_{\mathbf{K}}(w)$
- $\models_{M,w} [\alpha]A$ if $\models_{M,w'} A$ for every $w' \in R_\alpha(w)$

We call EDL our basic epistemic dynamic logic. EDL is sound, complete and decidable.

3 Adding tests to EDL

Now we extend EDL with the action "test if a formula is true", expressed by a new action operator: $??$. We read the formula $[A??]\mathbf{K}A$ "after testing if A, the agent knows A".

In our introductory example, imagine we cannot test c_2 but only $c_1 \vee c_2$ or $c_1 \to c_2$. Then we can write $[c_1 \vee c_2??; c_1 \to c_2??](\mathbf{K}c_2 \vee \mathbf{K}\neg c_2)$. In this paper we suppose that every formula can be tested, even those speaking about hypothetical execution of actions.

3.1 Tests are ...

3.1.1 ... deterministic

Every possible execution of a test will always lead to the same result. Determinism implies that once a test has been executed, it will be worthless executing it again. Van Linder et al. [9] speak about *idempotent* tests.

Det($??$) $\quad \langle A??\rangle C \to [A??]C$

Semantically we have

- $R_{A??}(w)$ is a singleton or the empty set.

3.1.2 ... purely informative

Purely informative actions never change the state of the world, they just change the knowledge. McIlraith [8] speak about *non-intrusive* tests and van Linder et al. [9] about *passive* tests.

Pres($??$) $\quad C \to [A??]C$ if C is an objective formula.

Semantically, this corresponds to the condition

- if $w_1 R_{A??} w_2$ then $V_{w_1} = V_{w_2}$

It follows by standard modal principles that purely informative actions do not reduce the knowledge of an agent.

Proposition 1 *Let C be an objective formula. Then $\mathbf{K}C \to [A??]\mathbf{K}C$ is provable from EDL together with Pres($??$).*

Proof 1 *Suppose C is objective and α is purely informative action. From the axiom $C \to [A??]C$ we get $\mathbf{K}C \to [A??]\mathbf{K}C$ by the standard modal principles N($\mathbf{K}$) and K($\mathbf{K}$). By our basic axiom Acq($[\alpha], \mathbf{K}$): $\mathbf{K}[A??]C \to [A??]\mathbf{K}C$ we obtain the desired $\mathbf{K}C \to [A??]\mathbf{K}C$.* $\diamond$

3.1.3 ... reliable

We suppose that the test of A always leads to the knowledge of A.

Reliab($??$) $\quad A \to [A??]\mathbf{K}A$

Semantically, this axiom corresponds to the condition

- if $\models_{M,w} A$ and $w(R_{A??} \circ R_{\mathbf{K}})w'$ then $\models_{M,w'} A$

Proposition 2 *The formula $[A??](\mathbf{K}A \vee \mathbf{K}\neg A)$ is provable from Reliab($??$).*

3.1.4 ... executable

We suppose that testing is always executable: there are no *applicability conditions*.

Exec($??$) $\quad [A??]C \to \langle A??\rangle C$

Semantically, this corresponds to the condition

- for every A, $R_{A??}$ is serial.

Remark We do not to accept applicability conditions, because we want a logic as simple as possible. Note that in most cases an applicability condition can be rewritten. If the test of A has applicability condition $Cond_A$, then we can write : $Cond_A \leftrightarrow \langle A??\rangle\top$.
In this case a satisfactory test plan must be executable: if π is a plan, we must prove the executability of π i.e. that $\langle\pi\rangle\top$.

3.1.5 ... predictable

If A is true then testing if A eliminates exactly those accessible worlds where A is false. For Scherl and Levesque [10] this is the basic axiom for knowledge-gathering axioms.

Perm($??, \mathbf{K}$) $\quad A \to ([A??]\mathbf{K}C \equiv \mathbf{K}(A \to [A??]C)$

Semantically, this corresponds to the condition

$$\text{If } wR_{A??}w' \text{ then } R_{\mathbf{K}}(w') = \bigcup_{v \in R_{\mathbf{K}}(w), \models_{M,v} A \text{ iff } \models_{M,w} A} R_{A??}(v)$$

3.1.6 Insensitive to negation

Neg($??$) $\quad [A??]C \equiv [\neg A??]C$

Semantically, this corresponds to the condition

- $R_{A??} = R_{\neg A??}$.

3.2 Automated theorem proving

In the rest of the section we suppose $ACT_0 = \emptyset$, i.e. we only consider test actions.

Proposition 3 *The following equivalences are theorems.*

1. $[A??](C_1 \wedge C_2) \equiv ([A??]C_1 \wedge [A??]C_2)$
2. $[A??](C_1 \vee C_2) \equiv ([A??]C_1 \vee [A??]C_2)$
3. $[A??]\neg C \equiv \neg[A??]C$
4. $[A??]\mathbf{K}C \equiv ((A \to \mathbf{K}(A \to [A??]C)) \wedge (\neg A \to \mathbf{K}(\neg A \to [A??]C)$
5. $[A??]C \equiv C$ *if C is objective.*

Proof 2 *1 is provable with axiom K($[\alpha]$), 2 with* determinism *Det($??$), 3 with* determinism *and* executability *Exec($??$). For 4 we need* predictability *Perm($??, \mathbf{K}$) and* insensivity to negation *Neg($??$). Finally for 5 we use Pres($??$) and Det($??$).* $\diamond$

Hence we can 'push down' the modal operator of test from the left to the right through all the other connectives $\mathbf{K}, \neg, \wedge, \vee$. When $[A??]$ reaches an objective formula then we can apply $[A??]C \equiv C$ and thus eliminate one modal operator of test from the formula. (We suppose here that the other action construction operators have been eliminated.)

Iterating these rewrite steps we can obtain formulas without occurrences of test operators. It is easy to prove that this rewriting system is confluent and terminating.

Now, we can reduce the problem of proving theorems in our logic to that of proving theorems in the standard modal logic for knowledge. The reduction is done by rewrite rules.

Theorem 1 *Suppose* $\mathrm{ACT}_0 = \emptyset$. *Then for every formula A there exists a formula A' without action operators such that $A \equiv A'$ is a theorem of EDL.*

3.3 Soundness and completeness

Theorem 2 *EDL plus the axioms for tests of section 3.1 is sound.*

Proof 3 *Each of the axioms is valid, and the inference rules preserve validity.* $\diamond$

The above theorem gives us completeness for the logic where we have only tests.

Theorem 3 *Suppose* $\mathrm{ACT}_0 = \emptyset$. *Then EDL plus the axioms for tests of section 3.1 is complete.*

Proof 4 *Let A be consistent. According to theorem 1 of section 3.2 there exists a formula A' without action operators such that $A \equiv A'$ is a theorem. Hence A' is consistent. Now A' is in the language of KT, and given that the axiomatics of our logic contains that of the epistemic logic KT, A' is as well consistent in KT. Via the completeness of KT there must therefore exist a KT-model containing a state w where A' is true. Then it is straightforward to extend that model to an EDL-model where A' is true in w. Finally, given that (due to soundness) the equivalences that we have used to rewrite formulas are valid, that model must also satisfy A in w.* $\diamond$

3.4 Example

Now we can handle our running example in our logic. A possible plan π is to first test component c_1 and then component c_2: $\pi = c_1??; c_2??$

We must prove that for every state of affairs, if the agent executes π then she/he knows the configuration of breakdown. In other words, we must prove that

$$T \vdash \Sigma_{Init} \to [\pi](\mathbf{K}\varphi_1 \vee \mathbf{K}\varphi_2 \vee \mathbf{K}\varphi_3)$$

where Σ_{Init} is the initial situation and T is the nonlogical theory of the domain, consisting of the following set T of domains axioms:

$$T = \{ ((\neg c_1 \wedge \neg c_2) \to \varphi_1), \\ (\neg c_2 \to \varphi_2), \\ ((\neg c_1 \wedge \neg c_3) \to \varphi_3)\}.$$

(Note that $T \vdash \mathbf{K}(\neg c_2 \to \varphi_2)$ by N($\mathbf{K}$).)

We can rewrite $[\pi]\mathbf{K}\varphi_1$ as

1. $[c_1??; c_2??]\mathbf{K}\varphi_1$
2. $[c_1??]((\neg c_2 \vee \mathbf{K}(c_2 \to [c_2??]\varphi_1)) \wedge (c_2 \vee \mathbf{K}(\neg c_2 \to [c_2??]\varphi_1)))$
3. $[c_1??]((\neg c_2 \vee \mathbf{K}(c_2 \to \varphi_1)) \wedge (c_2 \vee \mathbf{K}(\neg c_2 \to \varphi_1)))$
4. $(\neg c_2 \vee [c_1??]\mathbf{K}(c_2 \to \varphi_1)) \wedge (c_2 \vee [c_1??]\mathbf{K}(\neg c_2 \to \varphi_1))$
5. $(\neg c_1 \vee \neg c_2 \vee \mathbf{K}((c_1 \wedge c_2) \to \varphi_1) \wedge (c_1 \vee \neg c_2 \vee \mathbf{K}((\neg c_1 \wedge c_2) \to \varphi_1) \wedge (\neg c_1 \vee c_2 \vee \mathbf{K}((c_1 \wedge \neg c_2) \to \varphi_1) \wedge (c_1 \vee c_2 \vee \mathbf{K}((\neg c_1 \wedge \neg c_2) \to \varphi_1)$

Suppose e.g. $\Sigma_{Init} = \neg c_1 \wedge \neg c_2$. Then now, it is easy to deduce $T \vdash \Sigma_{Init} \to [\pi]\mathbf{K}\varphi_1$.

We can prove in the same way $T \vdash [c_1 \vee c_2??; c_1 \to c_2??]\mathbf{K}\varphi_2$

4 Uninformative actions

Our logic allows to explicitly distinguish between the ontic and the epistemic effects of actions, where ontic (respectively epistemic) effects are meant to be effects on the physical world only (resp. on the epistemic state of the agent only). As a consequence, our logic allows to speak about observability, since observability is primarily a matter of interactions between ontic and epistemic effects. Two particular classes of actions are *uninformative actions*, whose effects are purely ontic, and *purely informative actions*, or pure knowledge gatering actions, whose effects are purely epistemic. Our test actions $A??$ are examples of purely informative actions.

Formally an action α is uninformative if

Perm($[\alpha], \mathbf{K}$) $\langle\alpha\rangle\mathbf{K}A \to \mathbf{K}[\alpha]A$

holds for every formula A. This expresses that the agent cannot observe anything after α is performed: indeed if he cannot predict before α is performed that A will hold after α is performed, then he will not know A after α is performed.

Intuitively any action α can be decomposed into two actions, namely $\alpha = \alpha^o \circ \alpha^e$, where α^e is purely informative (epistemic) and α^o uninformative (ontic).

Given that $\alpha = \alpha^o \circ \alpha^e$, what is the relation between α^o and α^e? Ideally, all changes brought about by α^o are perceived through α^e. We call actions of this kind (i.e., actions informing the agent about all changes they cause) *fully informant*. Formally, α is fully informant iff for any objective formula A, we have that $A \to [\alpha](\neg A \to \mathbf{K}\neg A)$ holds. Purely informative actions α are fully informant because the ontic part is empty, i.e. $\alpha^o = \lambda$.

5 An alternative test action

In this section we split the test action in a composition of uninformative actions. To that end we introduce into our langage a "testing that" operator '?' mapping a formula to an action. We read $A?$ as "establishing A" or "testing that A is true".

It is then possible to define the action $A??$ of checking testing if A as an abbreviation of the complex action $A? \cup (\neg A)?$. This can be corresponds to the axiom

Def($??$) $\qquad [A??]C \equiv ([A?]C \wedge [\neg A?]C)$

This formally expresses that to check whether A is true amounts to nondeterministically choose between trying to establish that A and trying to establish that $\neg A$.

The other way round we can choose '$??$' to be primitive. In this case we can define '?' by

Def(?) $[A?]C \equiv [A??](A \to C)$.

We give the following axiomatics for '?'.

Det(?) $\langle A?\rangle C \to [A?]C$
Pres(?) $C \to [A?]C$ if C is an objective formula
Exec(?) $A \to \langle A?\rangle \top$
Reliab(?) $[A?]\mathbf{K}A$
Perm(?, **K**) $\langle A?\rangle \mathbf{K}C \to \mathbf{K}[A?]C$

Det(?), Pres(?), Exec(?), Reliab(?) respectively stand for *determinism*, *purely informative*, *executability*, *reliablility*. The first three have the same semantical constaint than for '??'. Reliab(?) corresponds to

- if $w(R_{A?} \circ R_{\mathbf{K}})w'$ then $\models_{M,w'} A$

The last axiom says that test '?' are uninformative. It is simpler than Perm(??, **K**). Semantically, the axiom corresponds to the condition

- if $R_\alpha(w) \neq \emptyset$ and $w_1 R_{\mathbf{K}} \circ R_\alpha w_2$ then $w_1 R_\alpha \circ R_{\mathbf{K}} w_2$

Theorem 4 *EDL plus the axioms for '??' plus Def(?) is equivalent to EDL plus the axioms for '?' plus Def(??).*

5.1 Automated theorem proving

Proposition 4 *The following equivalences are theorems of EDL .*

1. $[A?]\neg C \equiv (A \to \neg[A?]C)$
2. $[A?](C_1 \wedge C_2) \equiv ([A?]C_1 \wedge [A?]C_2)$
3. $[A?](C_1 \vee C_2) \equiv ([A?]C_1 \vee [A?]C_2)$
4. $[A?]\mathbf{K}C \equiv ([A?]\bot \vee \mathbf{K}[A?]C)$
5. $[A?]C \equiv (A \to C)$ *if* C *is an objective formula*

The proof of these equivalences is similar to that of proposition 3.

5.2 Soundness and completeness

Theorem 5 *EDL plus the axioms for '?' is sound.*

The above theorem gives us completeness for the logic where we have only tests.

Theorem 6 *Suppose* $\mathrm{ACT}_0 = \emptyset$. *Then EDL plus the axioms for tests '?' is complete.*

5.3 Complexity

The fragment of EDL plus the test operator '?' where tests are non-nested has an interesting complexity. In this case our rewriting procedure is a polynomial transformation into the epistemic logic. Hence the problem of deciding theoremhood in that fragment has the same complexity as in epistemic logic.

Consider the case where *S5* our logic of knowledge. Given that the problem of deciding whether a given S5-formula is a theorem is coNP-complete, it follows that the decision problem for the fragment of our logic is also coNP-complete.

5.4 The separation theorem

As said above, we consider every action α to be decomposable into an uninformative α^o and a purely informative α^e such that $\alpha = \alpha^o \circ \alpha^e$. Typically, the α^e are 'testing if' actions. Now we can rewrite these $A??$ into $A?\cup?\neg A$ with Def(??). Given that the latter 'testing that' actions also obey axiom Perm($[\alpha], \mathbf{K}$), we end up with formulas where all the actions are uninformative. For the corresponding logic EDL plus Perm($[\alpha], \mathbf{K}$) we can show a quite strong separation theorem:

Theorem 7 *Suppose all actions in* ACT_0 *are uninformative. Then for every formula* A *there is a formula* A' *such that* $A \equiv A'$ *in EDL plus Perm(*$[\alpha], \mathbf{K}$*) and* A' *contains no epistemic operator in the scope of an action operator.*

6 A solution to the Frame Problem

We must solve the Frame Problem in order to put to work our logic. Basically, we could integrate any solution into our framework, given that we have analysed the epistemic effects of an action in terms of its ontic effects on possible worlds. Scherl and Levesque e.g. used Reiter's solution to the Frame Problem, and applied regression as a reasoning method [10]. We adopt the solution in [1] based on dependence relations, which can be taken over without modifications and which we briefly recall here.

We associate to every atomic action α the set of atomic formulas it influences. Formally, we suppose given a dependency function $DEP : ACT_0 \longrightarrow VAR$. $p \in DEP(\alpha)$ means that α may change the truth value of the atom p. The other way round, if $p \notin DEP(\alpha)$ then α does not change the truth value of p. In other words, DEP represents frame axioms in an economic way. This is expressed by

$Pres_{DEP}([\alpha])$ if $p \notin DEP(\alpha)$ then $p \to [\alpha]p$ and $\neg p \to [\alpha]\neg p$

Semantically, the axiom corresponds to the condition

- For all $w, w' \in W$ and $p \in VAR$, if $w' \in R_\alpha(w)$ and $p \notin DEP(\alpha)$ then $\models_{M,w} p$ iff $\models_{M,w'} p$.

Given a dependence relation DEP, $\models_{DEP}$ is the corresponding extension of the EDL consequence relation.

As a particular case, purely informative actions verify $DEP(\alpha) = \emptyset$.

Combining all these atomic frame axioms by principles of classical logic, we obtain the following.

Proposition 5 *Let* A *be objective.*
If $DEP(\alpha) = \emptyset$ *then* $\models_{DEP} A \to [\alpha]A$.
If $\mathrm{atm}(A)$ *is the set of atoms of* A *and* $\mathrm{atm}(A) \cap DEP(\alpha) = \emptyset$ *then* $\models_{DEP} \mathbf{K}A \to [\alpha]\mathbf{K}A$.

7 Related work

A lot of logics of knowledge and action exist. Closest to ours is the work of Gerbrandy and Groeneveld [2, 4, 5]. Their Dynamic Epistemic Logic has two sorts of test, the first of which is noted $?A$[1] and is the standard dynamic logic test: it "succeeds [...] when A is true, and fails otherwise". Consequently $[?A]C$ is an abbreviation of $A \to C$. The second one is noted $U\alpha$ and "corresponds to [the] agent [...] learning that program α has been executed". (We have slightly

[1] The authors consider several agents and groups of agents. We abstract from that here.

adapted notation.) This means that agents act *a priori* unconsciously and must explicitly learn about the executions of their actions. [2]

$U?A$ is similar to our $A?$. More precisely, our logic can be mapped into Gerbrandy's logic of [2]: our action $A?$ can be translated into their $?A; U?A$.

In [2] there is given an axiomatics, which is similar to ours. Nevertheless there are subtle differences. We have already mentioned the first one: there is a non-epistemic test $?A$ supplementing the epistemic test $U?A$.

The second main difference is that there, the logic of knowledge is K, while ours is at least *KT* . Hence there is no axiom T($\mathbf{K}$). It seems to be problematic to add these axioms to the logic. This will be detailed after our next point.

The third main difference is that there, instead of axiom Exec(?) $A \to \neg[A?]\neg A$ there is an equivalence $A \equiv \neg[A?]\neg A$ (axiom 5 in [2]). This means that an agent can always successfully learn about the execution of test.[3] This leads to difficulties at least if we suppose that the epistemic notion under concern satisfies a consistency requirement as expressed by the modal axiom D($\mathbf{K}$) $\mathbf{K}A \to \neg\mathbf{K}\neg A$ (that is a consequence of axiom T($\mathbf{K}$)). Indeed, suppose p is an atom. Then $[U?p]\mathbf{K}p$ is derivable in their logic, as well as $\mathbf{K}\neg p \to [U?p]\mathbf{K}\neg p$. But from these two we can derive $\mathbf{K}\neg p \to [U?p]\mathbf{K}(p \wedge \neg p)$. While in our logic this means that the test action fails, in theirs the test $U?p$ always succeeds. Therefore axiom D($\mathbf{K}$) cannot be added to their logic as it stands.

Finally a more technical difference are the respective completeness proofs. While ours basically uses a reduction to a modal logic without tests, theirs is a (much longer) Henkin type proof. Nevertheless, our technique also applies to their logic, and permits thus to obtain a much simpler proof. To witness, the K-axiom for $[U\alpha]$ together with the above equivalence $A \equiv \neg[U\alpha]\neg A$ permit to pass the modal operator $[U\alpha]$ through conjunction, disjunction, and negation, and their axiom 7 $[U\alpha]\mathbf{K}A \equiv \mathbf{K}[\alpha][U\alpha]A$ permits to pass through the epistemic operator $\mathbf{K}$. Finally their axiom 6 permits to eliminate the $[U\alpha]$ operator from formulas. Thus one can follow the same line of reasoning as in our completeness proof. This have been done in [3].

In a series of articles Segerberg has developped a logic of belief and action called Doxastic Dynamic Logic (DDL) [12, 11]. There are three types of modalities $+A$, $-A$, and $*A$ the first of which corresponds to our $A?$. He discusses axioms for $+A$ that are similar to ours, but nevertheless closer to Gerbrandy and Groeneveld's work. To witness, he also considers that tests are always executable and deterministic, i.e. he has the axiom $[+A]\neg C \equiv \neg[+A]C$ (his axiom 13), as well as a preservation axiom in terms of equivalence (his axiom 10). Therefore our above remarks also apply to this approach.

Another line of research has been developped in the AI field of reasoning about actions around the concept of knowledge gathering actions [10, 7, 6].

Scherl and Levesque's approach [10] is dosely related to ours. They solution to he frame problem is based on a sitution caculus formula that corresponds to Perm($??, \mathbf{K}$). Their framework is based on Reiter's solution to the Frame Problem and inherits his limitations. In particular, action are constrained to be witheout indirect and indterminate affects. Our framework is more general in that respect.

[2] This leaves more flexibility than our language e.g. to speak about agent i learning that agent j learned that A has been tested (expressible here as $U_aU_b?A$).

[3] This makes it also possible to write the preservation axiom Pres(?) as an equivalence.

The logic AOL of Lakemeyer and Levesque has similarities to our EDL. The main difference is that our logic does not contain the concept of only knowing. To wittness we consider an example given in their paper. "Suppose we have a robot that knows nothing about the initial state of the environment, but that there is asensing action, reading a sonar, which tells the robot when it is getting close to a wall." Let us read the atomic formulas c and s respectively as 'the robot is close to the wall' and 'the sonar works', and let us interpret mc and ma respectively as the atomic actions of moving closer and moving away from the wall. In our language, what they then want to prove is

1. $[c??](\mathbf{K}c \vee \mathbf{K}\neg c)$
2. $\mathbf{K}w \to \mathbf{K}[c??](\mathbf{K}c \vee \mathbf{K}\neg c)$
3. $\mathbf{K}\neg c \to [ma](\mathbf{K}\neg c)$
4. $\mathbf{K}\neg c \to [mc](\neg\mathbf{K}c \wedge \neg\mathbf{K}\neg c)$

Only the last formula requires the non-monotonic only knowing notion.

Van Linder et al. [9] propose two formalizations of the concept of test, respectively in S5 and *KT* . They show that the tests can be seen on the one hand as epistemic operators of update, and on the other hand operators of expansion of knowledge. In both cases, the knowledge of the agent after the execution of a test is obtained by minimizing the changes between the old and the new state of knowledge.

8 Conclusion

We have a logic of knowledge and action EDL, in order to be able to model the test plans of test. We have given an automated theorem proving procedure for a fragment of our logic, and have proved a separation theorem.

If we are only interested in tests of the type '$??$', formulas written using $??$ will explode exponentially if we expand the abbreviation $A??$ to $A? \cup (\neg A)?$.

We plan to continue that work in two directions. First, our logic allows to reason about the evolution of knowledge by tests, but it does not allow planning of test sequences.This might be achieved in a way similar to the approach in [1]. Second, it would be interesting to bring closer our work to logics of the dubious one. There exists, indeed, of dynamic logics in which an action is weight by a probability. In our case, we could wought the test according to its reliability.

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