Erasure-Resilient Wavelet Based Video Transmission System

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Video Compression and Transmission

Wavelet Based Erasure-Resilient Video Transmission System

Joachim M. Buhmann, Marek Karpinski, Yakov Nekrich
CS Report 85257

`http://theory.cs.uni-bonn.de/` $\longrightarrow$ `Publications`

$\longrightarrow$ `2004`

Video Compression and Transmission

Good video compression (high compression rate) and more:

- Rate Scalable Video
- Erasure-Resilient Transmission
- Unequal Loss Protection

Overview

1. Video Compression (basic notions)
2. Erasure-resilient Transmission
   Cauchy-based Reed-Solomon Codes
3. Unequal Loss Protection
   Priority-encoded Transmission
4. Temporal Rate Scalability

Video Coding

Video - Sequence of Frames
Temporal Redundancy - Similarities between Frames

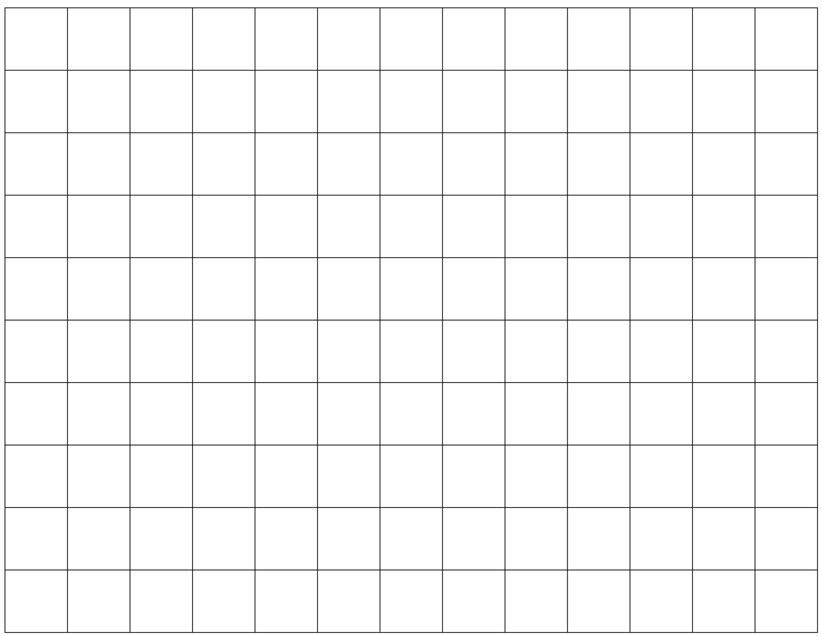
Spatial redundancy - Redundant Information in a Frame

Goals - remove temporal and spatial redundancy

Temporal Redundancy

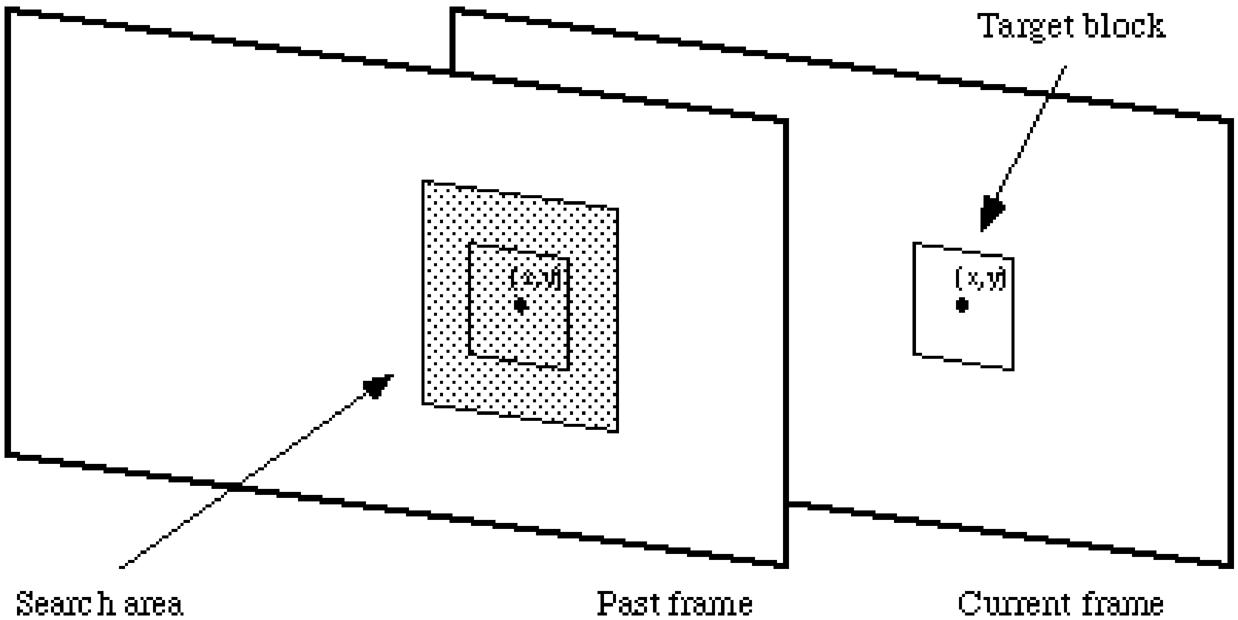
Intra-Frame Coding - Motion Compensation

Block-based Motion Compensation



Typical block size 16×16

Motion Compensation



Compare the target block with all blocks in the search area

Motion Compensation

Usual measure of distance between blocks:
Sum of Absolute Differences
SAD=$\sum_{i=1}^{n} \sum_{j=1}^{n} |R[i,j] - I[i,j]|$
$I[i,j]$ - pixel (i,j) in a macroblock in the target Frame

$R[i,j]$ - pixel (i,j) in a macroblock in the reference Frame

Motion Compensation

Motion Vectors

(X, Y) - upper left corner of B

(X', Y') - upper left corner of B'

(Δ_x, Δ_y) - Motion Vector

$\Delta_x = X - X'$

$\Delta_y = Y - Y'$

Motion Compensation

MC Block:

- with or w/o Motion Compensation
- w/o Motion Compensation:
  compress the block
- with Motion Compensation:
  - compress motion vectors
  - compress the difference between the block and its “best match” in the reference Frame

Analysis

Two Types of Frames

- Intra-Frames (I-Frames)
- Inter-Frames (P-Frames)

Analysis

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- Intra-Frames (I-Frames)
- Inter-Frames (P-Frames)

I P P P P P P P P P P P P . . .

Analysis

Two Types of Frames

- Intra-Frames (I-Frames)
- Inter-Frames (P-Frames)

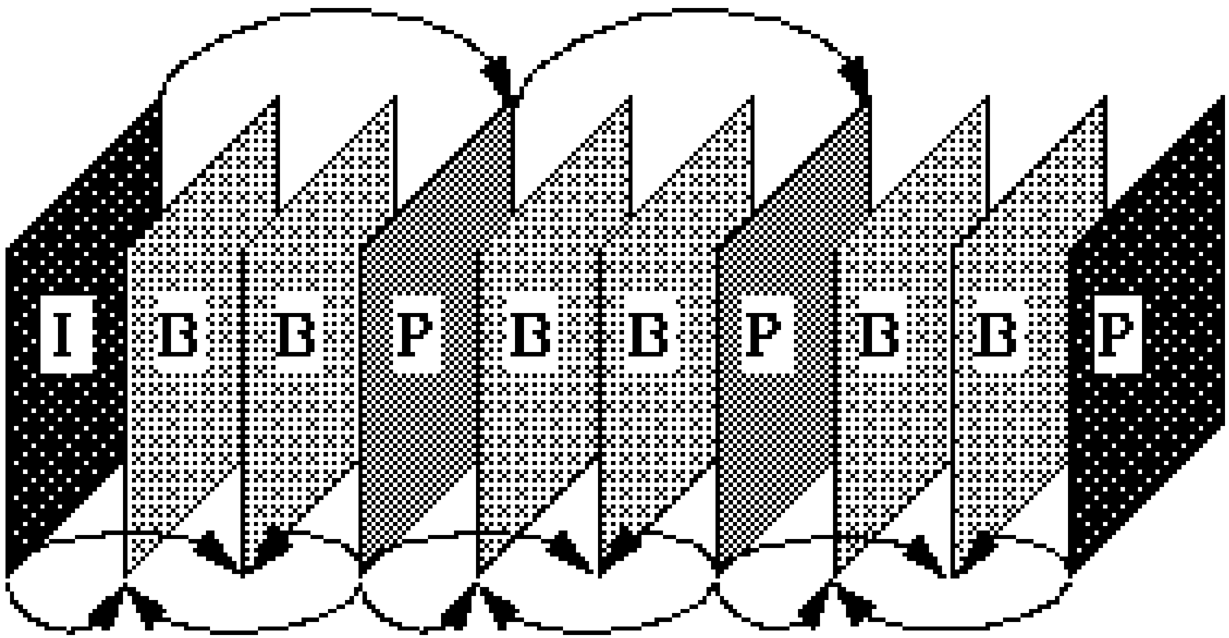
I P P P P P P P P P P P P . . .

I P P P P P P P I P P P P P P P

Motion Compensation

Enhanced Methods:

- bi-directional prediction



Motion Compensation

Enhanced Methods:

- multiple reference Frames
- variable size of the motion block
  (16×16 can be split into four 8×8 blocks etc.)
- half-pel prediction

Motion Compensation

Half-pel Prediction
doubled image

A	B
C	D

A	h_1	B
ν_1	c	ν_2
C	h_2	D

$$h_1 = \lfloor \frac{A+B}{2} + 0.5 \rfloor$$
$$h_2 = \lfloor \frac{C+D}{2} + 0.5 \rfloor$$
$$\nu_1 = \lfloor \frac{A+C}{2} + 0.5 \rfloor$$
$$\nu_2 = \lfloor \frac{B+D}{2} + 0.5 \rfloor$$
$$c == \lfloor \frac{A+B+C+D}{4} + 0.5 \rfloor$$

Image Compression

- Compression of I-Frames
- Compression of the residual signal
  (difference between the P-frame and its prediction)

Wavelet-based image compression

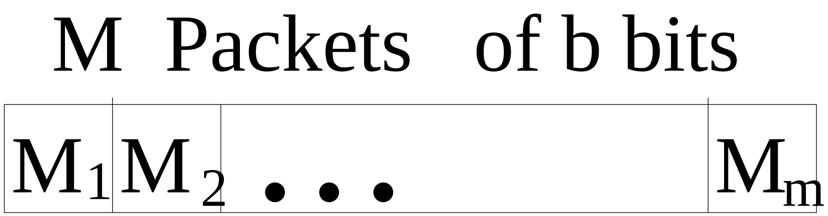
Loss Protection

Forward Error Correction
to protect against *packet losses*

Cauchy-based RS Codes
An XOR-Based Erasure Resilient Coding Scheme
Blömer, Kalfane, Karp, Karpinski, Luby, Zuckerman
Technical Report TR-95-48,
International Computer Science Institute, Berkeley, California, 1995.

`http://citeseer.ist.psu.edu/84162.html`

Loss Protection



Loss Protection

M: m Packets of b bits

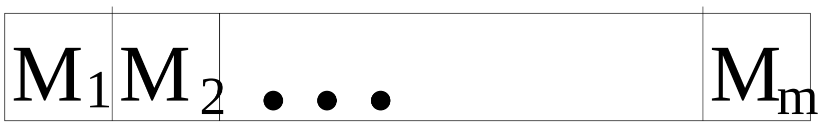
M_1	M_2	. . .	M_m

Encoding: n Packets

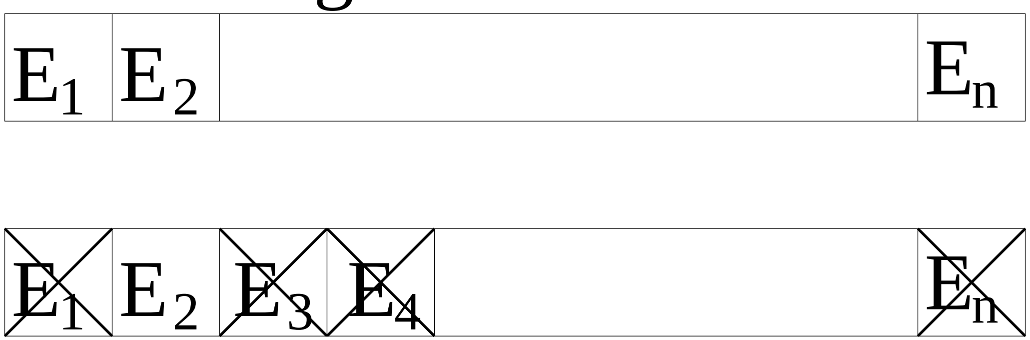
E_1	E_2		E_n

Loss Protection

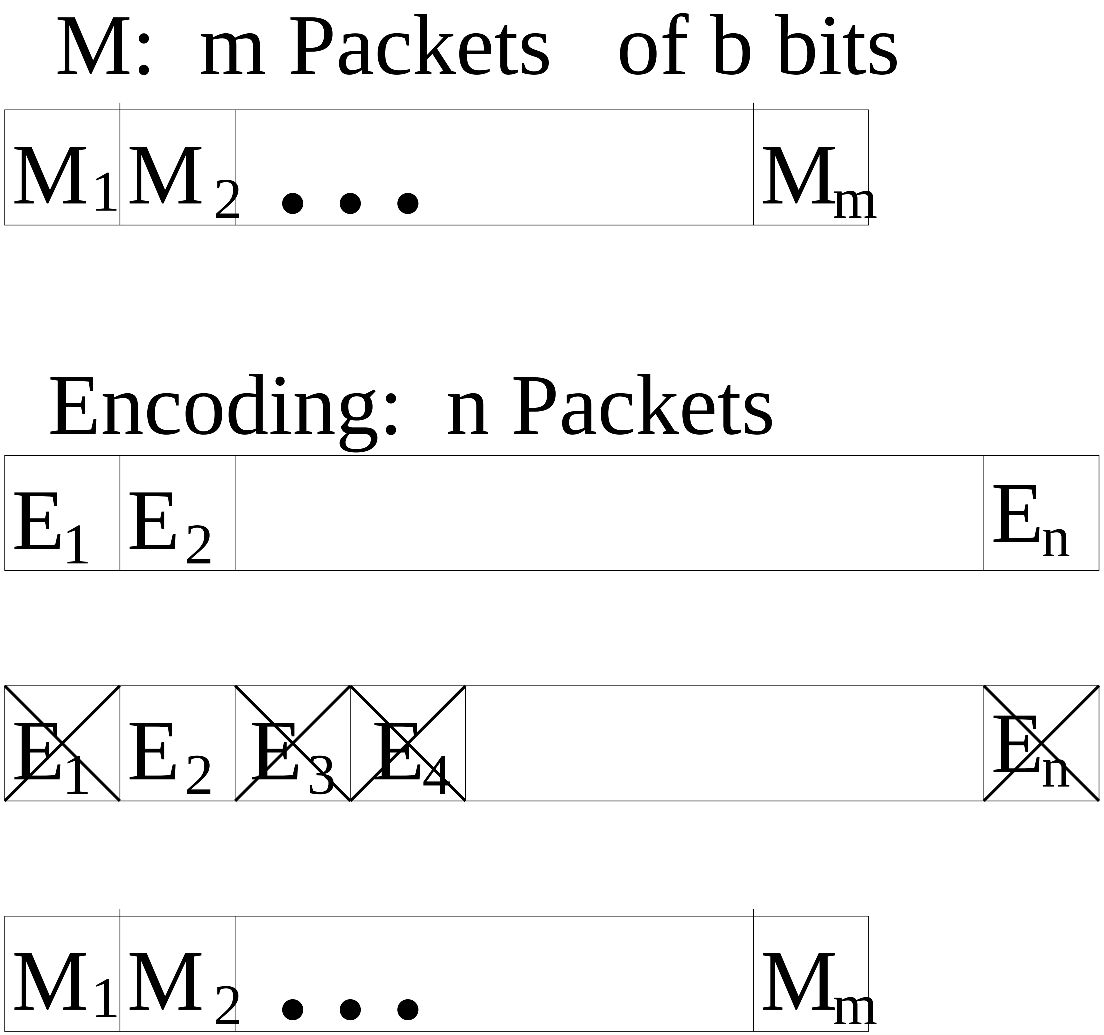
M: m Packets of b bits



Encoding: n Packets



Loss Protection



If $\geq r$ packets are received, M can be reconstructed

Loss Protection

(m, n, b, r) Code
$M_1, M_2, \ldots M_m \Longrightarrow \quad E_1, E_2, \ldots, E_n$
M_i, E_j consist of b bits

arbitrary $E_{i_1}, E_{i_2}, \ldots E_{i_r}$ uniquely determine M

$r = m$ - MDS Code

Loss Protection

Reed-Solomon Codes:
Idea:
M is a vector of el-s from a finite field F
Multiply message M with the *generator matrix*

Vandermonde matrix as the generator matrix

Loss Protection

Main Properties

- Cauchy Matrix instead of Vandermonde matrix

Loss Protection

Main Properties

- Cauchy Matrix instead of Vandermonde matrix
- Matrix Representation of field elements

Loss Protection

Main Properties

- Cauchy Matrix instead of Vandermonde matrix
- Matrix Representation of field elements
- w/o multiplications
  (only XOR and similar operations)

Cauchy Matrix

$X = \{x_1, \ldots, x_m\}$, $Y = \{y_1, \ldots, y_n\}$ - two sets of el-s in a field F

1. $\forall i, 1 \le i \le m$, $\forall j, 1 \le j \le n$, $x_i + y_j \ne 0$
2. $\forall i, 1 \le i \le m$, $\forall j, 1 \le j \le m$, $x_i \ne x_j$; $\forall i, 1 \le i \le n$, $\forall j, 1 \le j \le n$, $y_i \ne y_j$

$$C = \begin{bmatrix} \frac{1}{x_1+y_1} & \frac{1}{x_1+y_2} & \cdots & \frac{1}{x_1+y_n} \\ \frac{1}{x_2+y_1} & \frac{1}{x_2+y_2} & \cdots & \frac{1}{x_2+y_n} \\ & & \ddots & \\ \frac{1}{x_{m-1}+y_1} & \frac{1}{x_{m-1}+y_2} & \cdots & \frac{1}{x_{m-1}+y_n} \\ \frac{1}{x_m+y_1} & \frac{1}{x_m+y_2} & \cdots & \frac{1}{x_m+y_n} \end{bmatrix}$$

Cauchy Matrix

$2^{L-1} \times 2^{L-1}$ Cauchy matrix over GF[2^L]:
$x_i = i - 1$, $i = 1, ..., 2^{L-1}$
$y_i = 2^{L-1} + i - 1$, $i = 1, ..., 2^{L-1}$
In the general case:
X - first m el-s

Y - the following n elements

Cauchy Matrix

Example

$$\begin{pmatrix} 6 & 5 & 2 & 7 & 4 \\ 5 & 6 & 7 & 2 & 3 \end{pmatrix}$$

(2×5) Cauchy Matrix over GF$[2^3]$

$X = \{0, 1\}$ $\qquad$ $Y = \{2, 3, 4, 5, 6\}$

Cauchy Matrix

Properties:

- every square submatrix of C is non-singular
- $det(C) = \frac{\prod_{i<j}(x_i - x_j)\prod_{i<j}(y_i - y_j)}{\prod_{i,j=1}^{n}(x_i + y_j)}$
- inverse matrix of an $n \times n$ matrix can be computed with $O(n^2)$ operations in F

Finite Fields

GF[2^L]

Elements - polynomials

$\sum_{i=0}^{L-1} f_i X^i = f_0 + f_1 X + f_2 X^2 + \ldots + f_{L-1} X^{L-1}$ of degree at most $L-1$

can be specified by a coefficient vector $(f_0, f_1, \ldots, f_{L-1})$

Operations are modulo $p(X)$ where $p(X)$ - irreducible polynomial of degree L

Addition - XOR

Multiplication - complicated

Finite Fields

Matrix Representation:
$\tau(f)$ is an $L \times L$ matrix whose i-th column is the coefficient vector of $X^{i-1}f(X) \mod p(X)$, $i = 1, 2 \ldots, L$

Example

Galois Field $GF[2^3]$
$p(X) = X^3 + X + 1$ is an irreducible polynomial

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Longrightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Finite Fields

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Longrightarrow \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Finite Fields

$\tau(f)$ is a field isomorphism:

- $\tau(0)$ - all-zero matrix
- $\tau(1)$ - identity matrix
- τ is bijective
- $\tau(f+g) = \tau(f) + \tau(g)$
- $\tau(fg) = \tau(f)\tau(g)$

Finite Fields

- polynomial multiplication can be replaced with matrix multiplication
- $M(e_1)$ matrix representation of an element e_1
  $V(e_2)$ - vector representation of an element e_2
  $M(e_1) \times V(e_2) = V(e_1 e_2)$

$$3 * 5 = 4$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} * \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

XOR-based MDS Code

Message $M = (M_1, M_2, \ldots, M_m)$
consists of m packets of L words each
each word consists of w bits
M is an element of $(GF[2])^{mL \times w})$
(M can be viewed as an $mL \times w$ matrix over GF[2])
C - $(n-m) \times m$ Cauchy matrix over $GF[2^L]$
(L must be such that $L \geq \log_2 n$)

Generator matrix - $(I_m|C)$

XOR-based MDS Code

Example

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 6 & 5 & 2 & 7 & 4 \\ 5 & 6 & 7 & 2 & 3 \end{pmatrix}$$

XOR-based MDS Code

Example

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 6 & 5 & 2 & 7 & 4 \\ 5 & 6 & 7 & 2 & 3 \end{pmatrix} * \begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{pmatrix}$$

XOR-based MDS Code

Example

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 6 & 5 & 2 & 7 & 4 \\ 5 & 6 & 7 & 2 & 3 \end{pmatrix} * \begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \\ E_7 \end{pmatrix}$$

$E_i = M_i$, $i = 1, 2, \ldots, 5$

XOR-based MDS Code

Example

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 6 & 5 & 2 & 7 & 4 \\ 5 & 6 & 7 & 2 & 3 \end{pmatrix}^{-1} * \begin{pmatrix} M_1 \\ M_3 \\ M_4 \\ E_6 \\ E_7 \end{pmatrix} = \begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{pmatrix}$$

M_2, M_5 were lost

XOR-based MDS Code

$T = \tau(c_{i,j})$, $i = 1, \ldots n$, $j = 1, \ldots, m$

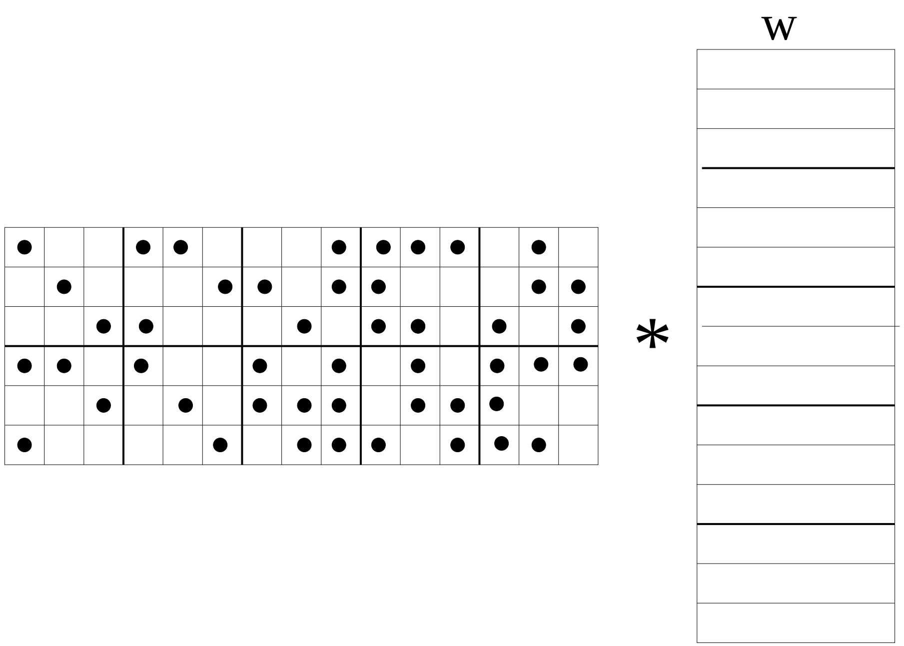
Matrix Representation of each element in the generator matrix

$(n \times m) \rightarrow (nL \times mL)$ matrix

j-th packet E_j consists of rows $r_{jL+1}, r_{jL+2}, \ldots r_{(j+1)L}$ of $T \cdot M$

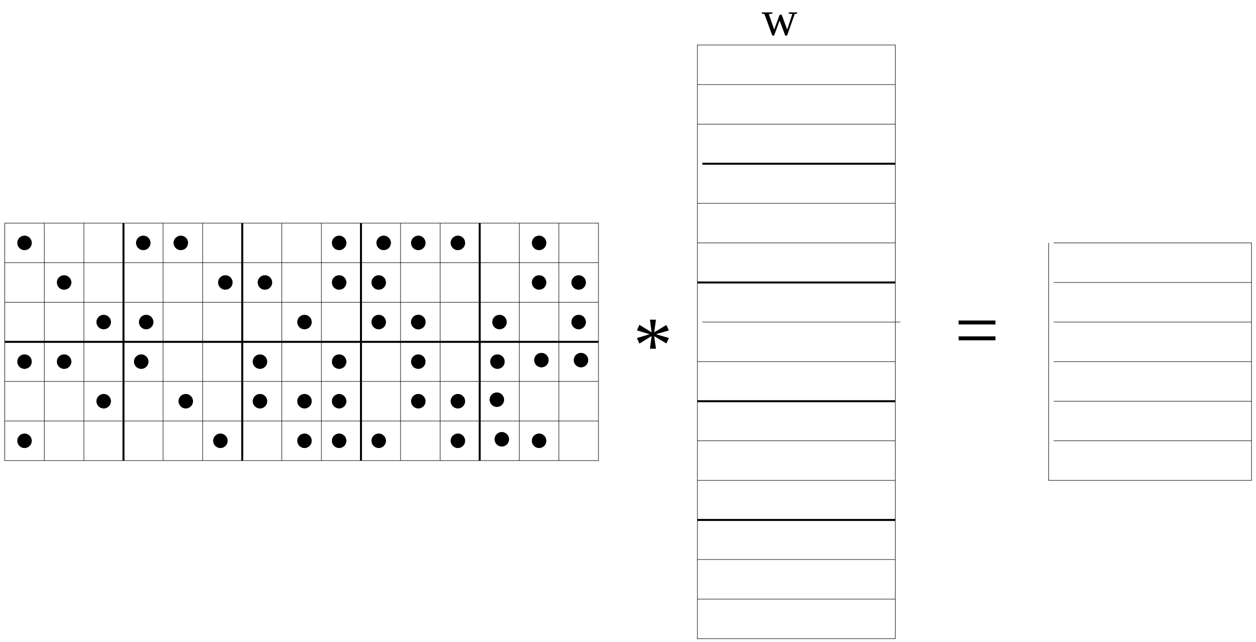
XOR-based MDS Code

$$\begin{pmatrix} 1 & 5 & 2 & 7 & 4 \\ 5 & 1 & 3 & 4 & 7 \end{pmatrix}$$



Each M_i is divided into L slices of w bits.

XOR-based MDS Code



$$E_{6,1} = M_{1,1} + M_{2,1} + M_{2,2} + M_{3,3} + M_{4,1} + M_{4,2} + M_{4,3} + M_{5,2}$$

Decoding

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 6 & 5 & 2 & 7 & 4 \\ 5 & 6 & 7 & 2 & 3 \end{pmatrix} * \begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{pmatrix} = \begin{pmatrix} E_1 \\ \cancel{E}_2 \\ E_3 \\ E_4 \\ \cancel{E}_5 \\ E_6 \\ E_7 \end{pmatrix}$$

$$6M_1 + 5M_2 + 2M_3 + 7M_4 + 4M_5 = E_6$$
$$5M_1 + 6M_2 + 7M_3 + 2M_4 + 3M_5 = E_7$$

Decoding

$$\tilde{E}_6 = E_6 - 7M_4 - 2M_3 - 6M_1$$
$$\tilde{E}_7 = E_7 - 2M_4 - 7M_3 - 5M_1$$
$$5M_2 + 4M_5 = \tilde{E}_6$$

$$6M_2 + 3M_5 = \tilde{E}_7$$

Decoding

$$D = \begin{pmatrix} 5 & 4 \\ 6 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 4 \\ 6 & 3 \end{pmatrix} * \begin{pmatrix} M_2 \\ M_5 \end{pmatrix} = \begin{pmatrix} \tilde{E}_6 \\ \tilde{E}_7 \end{pmatrix}$$

$$\begin{pmatrix} M_2 \\ M_5 \end{pmatrix} = D^{-1} * \begin{pmatrix} \tilde{E}_6 \\ \tilde{E}_7 \end{pmatrix}$$

Decoding

J - set of indices of received redundant packets
I - set of indices of received information packets
$\overline{I}$ - set of indices of lost information packets

1. compute $\tilde{E}_j = E_j - \sum_{i \in I} \tau(c_{ji}) M_i$ for all $j \in J$
2. compute D^{-1} for $D = \tau(c_{ji})$, $j \in J$, $i \in \overline{I}$
3. "combine" $\tilde{E}_j$ into $\tilde{E}$: $jL + i$-th row of $\tilde{E}$ is the i-th row of $\tilde{E}_j$
4. compute $D^{-1}\tilde{E}$

Decoding

Encoding - $O(m(n-m)L^2)$ operations

Decoding - $O(mkL^2)$ operations

Advantages:

- XOR operations only
- Word Parallelism

Implementation

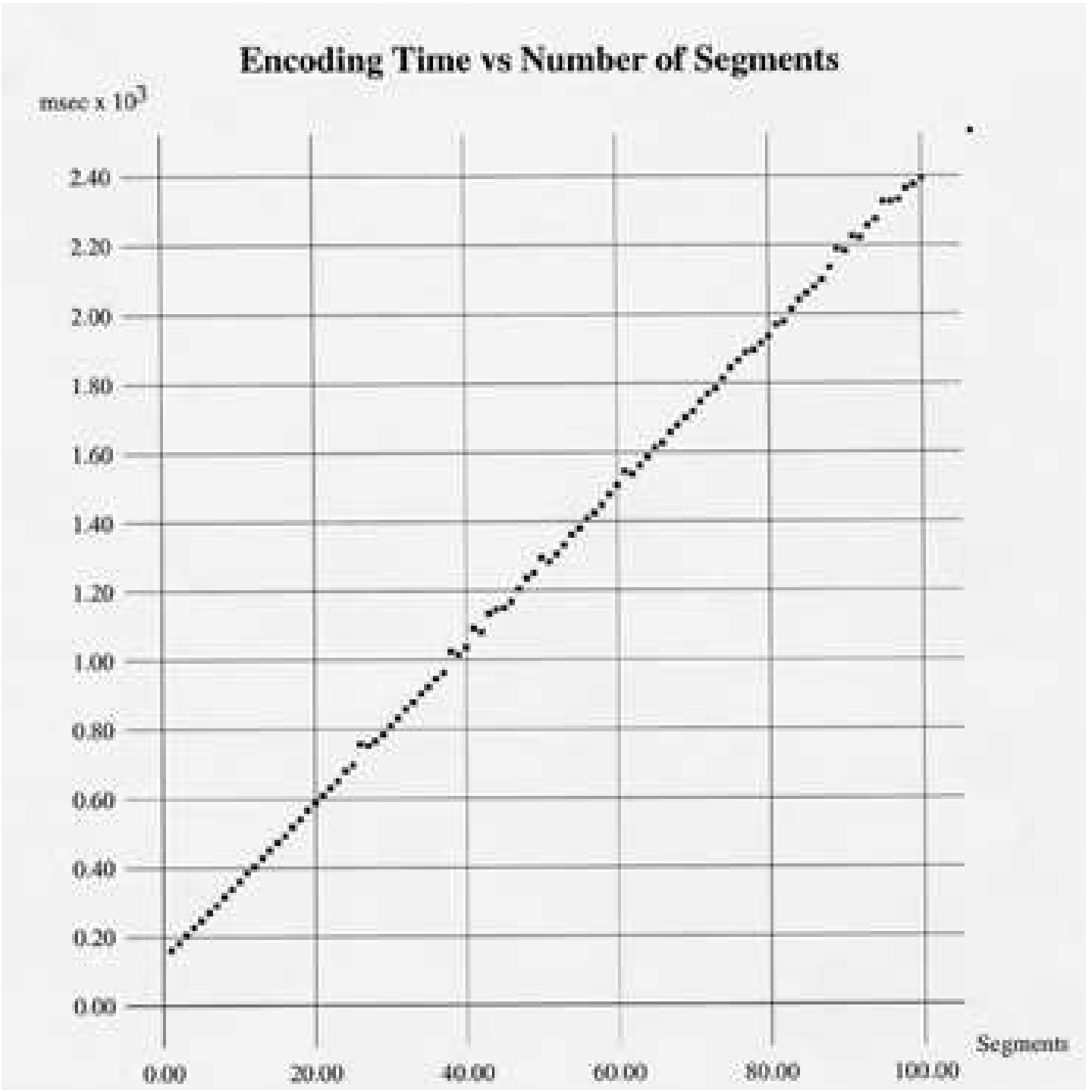
Packet Size b may be $\gg Lw$

Packet is divided into segments of size Lw

Experimental Results

SUN SPARCstation 20
CPU - 61 MHz
64 MB RAM

Loss Protection - Encoding



$L = 10$, $w = 32$

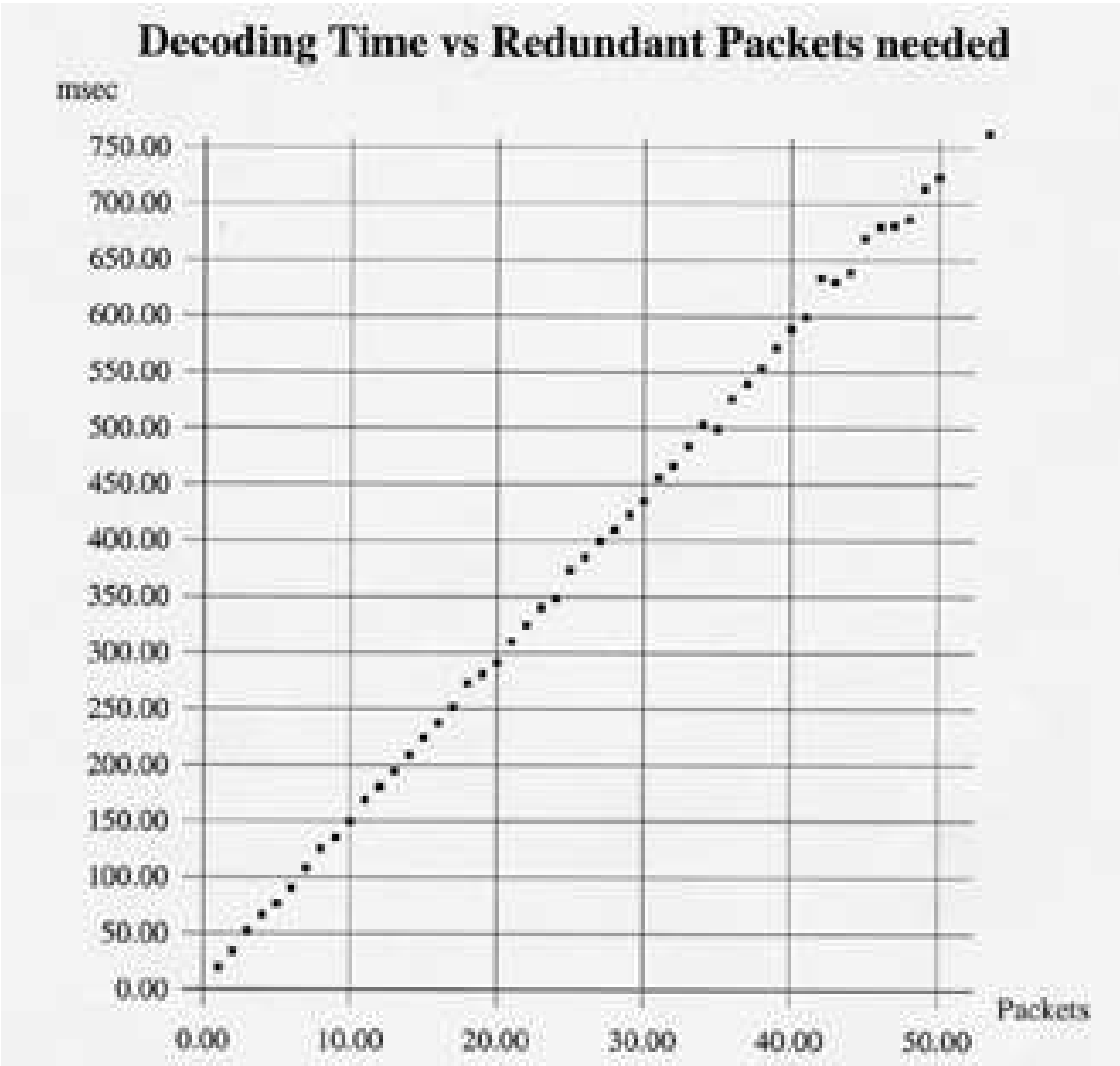
Packets consist of segments of 320 Bits each

$m = 100$

$n = 150$

Number of segments varies between 32000 bits and 3200000 bits

Loss Protection - Decoding



$L = 10$, $w = 32$

Packets consist of segments of 320 Bits each

$m = 100$

Number of redundant packet is between 1 and 50

Unequal Loss Protection

Goal: Graceful Degradation of video quality
Quality of video smoothly decreases with the growing number of packet losses
Priority Encoding Transmission

Different parts of the data stream are assigned different priorities according to their importance

Example: I-Frames are more important than P-Frames

Unequal Loss Protection

Message $w_1, w_2, \ldots w_m$
Priorities $\rho_1, \rho_2, \ldots \rho_m$
Encoding Length nl
(n Packets of length l each)
If $\rho_i \cdot n$ Packets are received, w_i can be decoded

Unequal Loss Protection

$\text{girth}_\rho = \sum \frac{1}{\rho_i}$

girth_ρ is the lower bound of the length of encoding

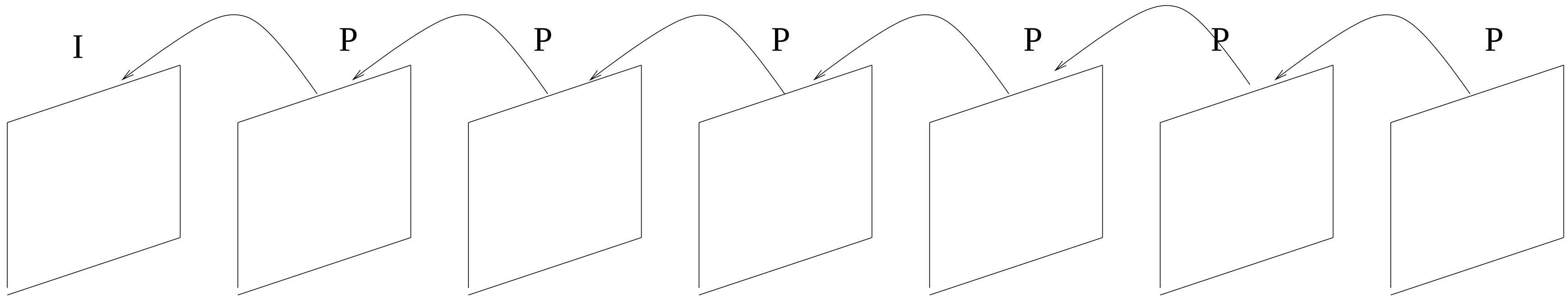
Upper Bound:

$nl \leq \frac{\text{girth}_\rho}{1-d/l} + l$

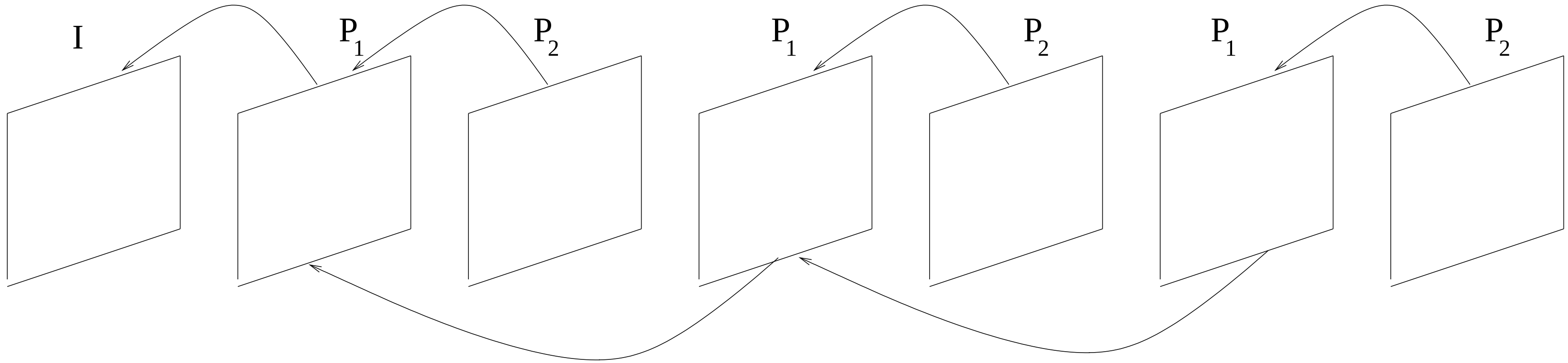
with $\rho'_i \leq \rho_i + l/m$

d - number of *different* values of ρ_i

Temporal Rate Scalability



Temporal Rate Scalability



I-Frames: Redundancy Factor 5
P1-Frames: Redundancy Factor 3
P2-Frames: Redundancy Factor 2

Experimental Results

File	Compression Rate (in kbps)	Loss Rate	average PSNR
coastguard	48	0	27.12
coastguard	48	50%	27.12
coastguard	48	75%	24.55
mother and daughter	24	0	30.56
mother and daughter	24	50%	29.59
mother and daughter	24	75%	21.29
mother and daughter	48	0	33.64
mother and daughter	48	50%	33.60
mother and daughter	48	75%	32.08

Experimental Results