

# Steiner trees and polyhedra

M. Didi Biha<sup>1</sup>, H. Kerivin<sup>2</sup> and A.R. Mahjoub<sup>3</sup>

*1. Laboratoire d'Analyse non linéaire et Géométrie, Université d'Avignon, 339 Chemin des  
Meinajaries, 84911, Avignon, France*

*2. Institute for Mathematics and its Applications, University of Minnesota, 357 Lind Hall, 207  
Church Street S.E., Minneapolis, MN 55455-0436, USA*

*3. LIMOS, CNRS UMR 6158, Université Blaise Pascal Clermont II, Complexe Scientifique des  
Cézeaux, 63177 Aubière Cedex, France*

**Keywords :** Steiner tree, Steiner connected subgraph, Polytope, Facet, Series-parallel Graph.

## Abstract

In this paper we study the dominant of the Steiner tree polytope. We introduce a new class of valid inequalities that generalizes the so-called odd hole, wheel, bipartite, anti-hole and Steiner partition inequalities introduced by Chopra and Rao in [?, ?], and we give sufficient conditions for these inequalities to define facets. We describe some procedures that permit to construct facets from known ones for the dominant of the Steiner tree polytope and the closely related Steiner connected subgraph polytope. Using these methods we give a counterexample to a conjecture of Chopra and Rao [?] on the dominant of the Steiner tree polytope on 2-trees. We also describe the dominant of the Steiner tree polytope and the Steiner connected subgraph polytope on special classes of graphs. In particular we show that if the underlying graph is series-parallel and the terminals satisfy certain conditions, then both polyhedra are given by the trivial inequalities and the Steiner partition inequalities.