Two Edge-Disjoint Hop-Constrained Paths and Polyhedra

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Abstract

Given a graph G with distinguished nodes s and t, a cost on each edge of G, and a fixed integer $L \geq 2$, the Two edge-disjoint Hop-constrained Paths Problem is to find a minimum cost subgraph such that between s and t there exist at least two edge-disjoint paths of length at most L. In this paper, we consider that problem from a polyhedral point of view. We give an integer programming formulation for the problem when $L = 2, 3$. An extension of this result to the more general case where the number of required paths is arbitrary and $L = 2, 3$, is also given. We discuss the associated polytope, $P(G, L)$, for $L = 2, 3$. In particular, we show in this case that the linear relaxation of $P(G, L)$, $Q(G, L)$, given by the trivial, the st-cut, and the so-called L-path-cut inequalities, is integral. As a consequence, we obtain a polynomial time cutting plane algorithm for the problem when $L = 2, 3$. We also give necessary and sufficient conditions for these inequalities to define facets of $P(G, L)$ for $L \geq 2$ when G is complete. We finally investigate the dominant of $P(G, L)$ and give a complete description of this polyhedron for $L \geq 2$, when $P(G, L) = Q(G, L)$.